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HILBERT AND POINCARÉ PROBLEMS FOR SEMI-LINEAR EQUATIONS IN RECTIFIABLE DOMAINS

Vladimir Ryazanov

Abstract. The study of the boundary value problem with arbitrary measurable data originated in the dissertation of Luzin where he investigated the Dirichlet problem for harmonic functions in the unit disk. Recently, in [26], we studied the Hilbert, Poincaré and Neumann boundary value problems with arbitrary measurable data for generalized analytic and generalized harmonic functions and provided applications to relevant problems in mathematical physics. The present paper is devoted to the study of the boundary value problem with arbitrary measurable boundary data in a domain with rectifiable boundary corresponding to semi-linear equation with suitable nonlinear source. We construct a completely continuous operator and generate nonclassical solutions to the Hilbert and Poincaré boundary value problems with arbitrary measurable data for Vekua type and Poisson equations, respectively. Based on that, we prove the existence of solutions of the Hilbert boundary value problem for the nonlinear Vekua type equation with arbitrary measurable data in a domain with rectifiable boundary. It is necessary to point out that our approach differs from the classical variational approach in PDE as it is based on the geometric interpretation of boundary values as angular (along non-tangential paths) limits. The latter makes it possible to also obtain a theorem on the boundary value problem for directional derivatives, and, in particular, of the Neumann problem with arbitrary measurable data for the Poisson equation with nonlinear sources in any Jordan domain with rectifiable boundary. As a result we arrive at applications to some problems of mathematical physics.

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Key words and phrases. Dirichlet, Hilbert, Neumann and Poincaré boundary value problems; generalized analytic and generalized harmonic functions with sources; semi-linear Poisson equations; nonlinear Vekua type equations.

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