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WEIGHTED FOURTH ORDER EQUATION OF KIRCHHOFF TYPE IN DIMENSION 4 WITH NON-LINEAR EXPONENTIAL GROWTH

RACHED JAIDANE

ABSTRACT. In this work, we are concerned with the existence of a ground state solution for a Kirchhoff weighted problem under boundary Dirichlet condition in the unit ball of \mathbb{R}^4 . The nonlinearities have critical growth in view of Adams' inequalities. To prove the existence result, we use Pass Mountain Theorem. The main difficulty is the loss of compactness due to the critical exponential growth of the nonlinear term f. The associated energy function does not satisfy the condition of compactness. We provide a new condition for growth and we stress its importance to check the minmax compactness level.

1. Introduction

In this paper, we consider the non local fourth order elliptic equation:

(1.1)
$$\begin{cases} g\bigg(\int_{B} (w(x)|\Delta u|^{2} + |\nabla u|^{2} + V(x)u^{2}) dx\bigg) \\ \times [\Delta(w(x)\Delta u) - \Delta u + V(x)u)] = f(x,u) & \text{in } B, \\ u = \frac{\partial u}{\partial n} = 0 & \text{on } \partial B, \end{cases}$$

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