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ON A SEMILINEAR FOURTH ORDER ELLIPTIC PROBLEM WITH ASYMMETRIC NONLINEARITY

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ABSTRACT. In this work, we address the existence of solutions for a biharmonic elliptic equation with homogeneous Navier boundary condition. The problem is asymmetric and has linear behavior on $-\infty$ and superlinear on $+\infty$. To obtain the results we apply topological methods.

1. Introduction and main results

In this paper, we investigate the existence of solutions for the following biharmonic elliptic equation with Navier boundary condition:

(1.1)
$$\begin{cases} \Delta^2 u = \lambda_1^2 u + u_+^p + f(x) & \text{in } \Omega, \\ u = \Delta u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ (N > 5) is a smooth bounded domain, λ_1 is the first eigenvalue of the Laplacian operator $-\Delta$ with the Dirichlet boundary condition, $u_+ = \max\{u, 0\}$ and p > 1. We assume that the function f satisfies the hypotheses

(H₁) $f \in L^r(\Omega)$ for some r > N/2;

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