

SOME EXISTENCE RESULTS FOR ELLIPTIC SYSTEMS WITH EXPONENTIAL NONLINEARITIES AND CONVECTION TERMS IN DIMENSION TWO

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ABSTRACT. In this paper, we establish the existence of solutions to a class of elliptic systems. The nonlinearities include exponential growth terms and convection terms. The exponential growth term means it could be critical growth at ∞ . The Trudinger–Moser inequality is used to deal with it. The convection term means it involves the gradient of unknown function. The strong convergence of sequences is employed to overcome the difficulties caused by convection terms. The variational methods are invalid and the Galerkin method and an approximation scheme are applied to obtain four different solutions. Our results supplements those from [11].

1. Introduction

We consider the following elliptic system

$$(1.1) \quad \begin{cases} -\Delta u = \lambda(u^{r_1} + a|\nabla u|^{s_1}) + f(v) & \text{in } \Omega, \\ -\Delta v = \mu(v^{r_2} + b|\nabla v|^{s_2}) + g(u) & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega, \end{cases}$$

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