

## ABSOLUTE NORMALIZED NORMS IN $\mathbb{R}^2$ AND HEINZ MEANS CONSTANT

ZHAN-FEI ZUO — YI-MIN HUANG

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**ABSTRACT.** In this paper, we calculate the precise values of the Heinz means constant under the absolute normalized norms in  $\mathbb{R}^2$ . The conclusions do not only contain some previous results, but also give the exact values of the Heinz means constant for some new concrete Banach spaces.

### 1. Introduction

Let  $X$  be a Banach space with the unit ball  $B_X$  and the unit sphere  $S_X$ . Some geometric constants for the space  $X$  have been investigated, for instance the *Baronti* constant  $A_2(X)$  and the *Alonso* constant  $T(X)$  etc. (see [1], [3]). The relationship between these constants and the important geometric properties of the Banach space, for example, uniformly non-squareness and normal structure have been extensively studied (see [9], [10], [18]–[20]). Therefore, the calculation of the geometric constants for some concrete spaces is very important in studying geometric properties of Banach spaces. Recently, M. Dinarvand [6] introduced the *Heinz means constant*, for  $0 \leq \nu \leq 1$ ,

$$H_\nu(X) = \sup_{x, y \in S_X} M_\nu(\|x + y\|, \|x - y\|),$$

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