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ON DETERMINING THE HOMOLOGICAL CONLEY INDEX OF POINCARÉ MAPS IN AUTONOMOUS SYSTEMS

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Dedicated to the memory of Professor Andrzej Granas

Abstract. A theorem on computation of the homological Conley index of an isolated invariant set of the Poincaré map associated to a section in a rotating local dynamical system ϕ is proved. Let (N, L) be an index pair for a discretization ϕ^h of ϕ, where $h > 0$, and let S denote the invariant part of $N \setminus L$; it follows that the section S_0 of S is an isolated invariant set of the Poincaré map. The theorem asserts that if the sections N_0 of N and L_0 of L are ANRs, the homology classes $[u_j]$ of some cycles u_j form a basis of $H(N_0, L_0)$, and for some scalars a_{ij}, the cycles u_j and $\sum a_{ij}u_i$ are homologous in the covering pair $\big(\widetilde{N}, \widetilde{L}\big)$ of (N, L) and the homology relation is preserved in $\big(\widetilde{N}, \widetilde{L}\big)$ under the transformation induced by ϕ^t for $t \in [0, h]$ then the homological Conley index of S_0 is equal to the Leray reduction of the matrix $[a_{ij}]$. In particular, no information on the values of the Poincaré map or its approximations is required. In a special case of the system generated by a T-periodic non-autonomous ordinary differential equation with rational $T/h > 1$, the theorem was proved in the paper M. Mrozek, R. Srzednicki, and F. Weilandt, SIAM J. Appl. Dyn. Syst. **14** (2015), 1348–1386, and it motivated a construction of an algorithm for determining the index.

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