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WEAKLY ALMOST PERIODIC FUNCTIONS INVARIANT MEANS AND FIXED POINT PROPERTIES IN LOCALLY CONVEX TOPOLOGICAL VECTOR SPACES

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This paper is dedicated to Seydina Al-Hassane Salame with admiration

Abstract. In this paper, we study and establish a positive answer to a long-standing open problem raised by A.T.-M. Lau in 1976. It is about whether the left amenability property of the Banach algebra WAP(S), of all weakly almost periodic functions, on a given semitopological semigroup S is equivalent to the existence of a common fixed point of any separately weakly continuous and weakly quasi-equicontinuous nonexpansive action of S on a nonempty weakly compact convex subset of a separated locally convex space. We establish here an affirmative answer; and among other things, we show that the affine counterpart of this question holds and also the AP(S) formulation of this problem is true.

1. Introduction

It has been a long-time open problem whether the existence of a left invariant mean on the Banach algebra of weakly almost periodic functions of a given semitopological semigroup is equivalent to a certain fixed point property for semigroups of nonexpansive self-mappings of a weakly compact convex set. During a 1976 seminar on fixed point theory and its applications at Dalhousie University in Halifax (Canada), Lau raised the question, see [11, Problem 5], whether every

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