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CONCENTRATING SOLUTIONS FOR AN ANISOTROPIC PLANAR ELLIPTIC NEUMANN PROBLEM WITH HARDY–HÉNON WEIGHT AND LARGE EXPONENT

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Abstract. Let Ω be a bounded domain in $\mathbb{R}^2$ with smooth boundary, we study the following anisotropic elliptic Neumann problem with Hardy–Hénon weight

$$\begin{cases} -\nabla(a(x)\nabla u) + a(x)u = a(x)|x-q|^{2\alpha}u^p, \ u>0 & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \nu} = 0 & \text{on } \partial\Omega, \end{cases}$$

where ν denotes the outer unit normal vector to $\partial\Omega$, $q \in \overline{\Omega}$, $\alpha \in (-1,+\infty) \setminus \mathbb{N}$, $p>1$ is a large exponent and $a(x)$ is a positive smooth function. We investigate the effect of the interaction between anisotropic coefficient $a(x)$ and singular source q on the existence of concentrating solutions. We show that if $q \in \Omega$ is a strict local maximum point of $a(x)$, there exists a family of positive solutions with arbitrarily many interior spikes accumulating to q; while, if $q \in \partial\Omega$ is a strict local maximum point of $a(x)$ and satisfies $\langle \nabla a(q), \nu(q)\rangle = 0$, such a problem has a family of positive solutions with arbitrarily many mixed interior and boundary spikes accumulating to q. In particular, we find that concentration at singular source q is always possible whether $q \in \overline{\Omega}$ is an isolated local maximum point of $a(x)$ or not.

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