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THE CHOQUARD LOGARITHMIC EQUATION INVOLVING A NONLINEARITY WITH EXPONENTIAL GROWTH

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Abstract. In the present work, we are concerned with the Choquard Logarithmic equation $-\Delta u + au + \lambda(\ln|\cdot| * |u|^2)u = f(u)$ in $\mathbb{R}^2$, for $a > 0$, $\lambda > 0$ and a nonlinearity f with exponential critical growth. We prove the existence of a nontrivial solution at the mountain pass level and a nontrivial ground state solution. Also, we provide these results under a symmetric setting, taking into account subgroups of $O(2)$.

1. Introduction

In this paper, we are interested in studying standing wave solutions for the following Schrödinger–Poisson system:

$$\begin{cases} i\psi_t - \Delta\psi + \widetilde{V}(x)\psi + \gamma\omega\psi = 0 & \text{in } \mathbb{R}^N \times \mathbb{R}, \\ \Delta\omega = |\psi|^2 & \text{in } \mathbb{R}^N, \end{cases} \tag{1.1}$$

where $\psi\colon \mathbb{R}^N \times \mathbb{R} \to \mathbb{C}$ is the time-dependent wave function, $\widetilde{V}\colon \mathbb{R}^N \to \mathbb{R}$ is a real external potential and $\gamma > 0$ is a parameter. The function ω represents

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