Topological **M**ethods in **N**onlinear **A**nalysis
Volume 60, No. 1, 2022, 343–361
DOI: 10.12775/TMNA.2021.060

THE REGULARIZED FREE FALL
II. HOMOLOGY COMPUTATION VIA HEAT FLOW

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Abstract. In [1] Barutello, Ortega, and Verzini introduced a non-local functional which regularizes the free fall. This functional has a critical point at infinity and therefore does not satisfy the Palais–Smale condition. In this article we study the L^2 gradient flow which gives rise to a non-local heat flow. We construct a rich cascade Morse chain complex which has one generator in each degree $k \geq 1$. Calculation reveals a rather poor Morse homology having just one generator. In particular, there must be a wealth of solutions of the heat flow equation. These can be interpreted as solutions of the Schrödinger equation after a Wick rotation.

1. Introduction

The free fall describes the motion of a particle on a line in the gravitational field of a heavy body. The particle will after some time collide with the heavy body. However, collisions can be regularized so that after collision the particle bounces back. An interesting new approach for regularizing collisions was discovered in the recent paper [1] by Barutello, Ortega, and Verzini. Change of time gives rise to a *delayed*, that is non-local, regularized functional $\mathcal{B}$ with an intriguing mathematical structure.

In fact, there are two non-local functionals describing the free fall, namely, a Lagrangian version $\mathcal{B}$, defined in (2.1) below, and a Hamiltonian version $\mathcal{A}_{\mathcal{H}}$. The two functionals are Morse–Bott and related to each other by a non-local

2020 *Mathematics Subject Classification.* 53Dxx.
Key words and phrases. Heat flow; Morse complex.
U. Frauenfelder acknowledges support by DFG grant FR 2637/2-2.