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REALIZATION OF ROTATION VECTORS FOR VOLUME PRESERVING HOMEOMORPHISMS OF THE TORUS

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Abstract. In this note we study the level sets of rotation vectors for C^0-generic homeomorphisms in the space $\mathrm{Homeo}_{0,\lambda}(\mathbb{T}^m)$ ($m \geq 3$) of volume preserving homeomorphisms isotopic to the identity, and contribute to the ergodic optimization of vector valued observables. It is known that such homeomorphisms satisfy the specification property and their rotation sets are polyhedrons with rational vertices and non-empty interior, and stable [3], [11], [16]. For a C^0-generic homeomorphism we prove uniform bounded deviations for the displacement of points in $\mathbb{T}^m$ in the support of any ergodic probability that generates a rotation vector in the boundary of the rotation set. As consequences, we show: (i) the support of ergodic probabilities generating rotation vectors in the boundary of rotation sets has empty interior, and (ii) weak version of Boyland's conjecture: the rotation vector of the Lebesgue measure lies in the interior of the rotation sets for a C^0-open and dense subset of homeomorphisms in $\mathrm{Homeo}_{0,\lambda}(\mathbb{T}^m)$.

1. Introduction and statement of the main results

1.1. Rotation sets. Inspired by the rotation theory initiated by Poincaré for circle homeomorphisms, Misiurewicz and Ziemian [18] introduced the concept of rotation sets for continuous maps homotopic to the identity on the torus $\mathbb{T}^m$,

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