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SIGN-CHANGING MULTI-BUMP SOLUTIONS FOR CHOQUARD EQUATION WITH DEEPENING POTENTIAL WELL

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Abstract. In this paper, we are concerned with the existence of sign-changing multi-bump solutions for the following nonlinear Choquard equation

$$-\Delta u + (\lambda V(x) + 1)u = (I_\alpha * |u|^p)|u|^{p-2}u \quad \text{in } \mathbb{R}^N, \tag{0.1}$$

where I_α is the Riesz potential, $\lambda \in \mathbb{R}^+$, $(N-4)^+ < \alpha < N$, $2 \le p < (N+\alpha)/(N-2)$, and $V(x)$ is a nonnegative continuous function with a potential well $\Omega := \operatorname{int}(V^{-1}(0))$ which possesses k disjoint bounded components $\Omega_1, \dots, \Omega_k$. We prove the existence of sign-changing multi-bump solutions for (0.1) if λ is large enough.

1. Introduction

We study the following nonlinear Choquard equation

$$-\Delta u + (\lambda V(x) + 1)u = (I_\alpha * |u|^p)|u|^{p-2}u \quad \text{in } \mathbb{R}^N, \tag{1.1}$$

where $\lambda \in \mathbb{R}^+$, $V(x) \in \mathcal{C}\big(\mathbb{R}^N, \mathbb{R}\big)$ is a potential function, $2 \le p < (N+\alpha)/(N-2)$, $I_\alpha \colon \mathbb{R}^N \to \mathbb{R}$ is the Riesz potential defined at each point $x \in \mathbb{R}^N \setminus \{0\}$ by

$$I_\alpha(x) = \frac{A_\alpha}{|x|^{N-\alpha}}, \qquad A_\alpha = \frac{\Gamma((N-\alpha)/2)}{\Gamma(\alpha/2)\pi^{N/2}\,2^\alpha},$$

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