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ASYMPTOTIC BEHAVIOR OF BIFURCATION CURVE OF NONLINEAR EIGENVALUE PROBLEM WITH LOGARITHMIC NONLINEARITY

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Abstract. We study the following nonlinear eigenvalue problem

$$-u''(t) = \lambda u(t)^p \log(1 + u(t)), \quad u(t) > 0, \quad t \in I := (-1, 1), \quad u(\pm 1) = 0,$$

where $p \geq 0$ is a given constant and $\lambda > 0$ is a parameter. It is known that, for any given $\alpha > 0$, there exists a unique classical solution pair $(\lambda(\alpha), u_\alpha)$ with $\alpha = \|u_\alpha\|_\infty$. We establish the asymptotic formulas for the bifurcation curves $\lambda(\alpha)$ and the shape of solution u_α as $\alpha \to \infty$ and $\alpha \to 0$.

1. Introduction

We consider the following nonlinear eigenvalue problems

$$-u''(t) = \lambda u(t)^p \log(1 + u(t)), \quad t \in I := (-1, 1), \tag{1.1}$$

$$u(t) > 0, \quad t \in I, \tag{1.2}$$

$$u(-1) = u(1) = 0, \tag{1.3}$$

where $p \geq 0$ is a given constant and $\lambda > 0$ is a bifurcation parameter.

The problem (1.1)–(1.3) is motivated by one-dimensional stationary logarithmic Schrödinger equation which was introduced by Białynicki-Birula and Mycielski [2]. The equations with logarithmic nonlinearity have been studied by many authors to investigate the phenomena in the branches of physics. We

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