

POHOŽAEV-TYPE GROUND STATE SOLUTIONS FOR CHOQUARD EQUATION WITH SINGULAR POTENTIAL AND CRITICAL EXPONENT

SENLI LIU — HAIBO CHEN

ABSTRACT. In this paper, we are concerned with the existence of Pohožaev-type ground state solutions for the Choquard equation with a singular potential and a critical exponent. By virtue of a generalized version of the Lions-type theorem and the Pohožaev manifold, we obtain the existence of a Pohožaev-type ground state solution for the above problem. Some recent results from the literature are improved and extended.

1. Introduction

In this paper, we study the following Choquard equation:

$$(1.1) \quad -\Delta u + \frac{A}{|x|^\theta} u = (I_\alpha * F(u))f(u), \quad x \in \mathbb{R}^3,$$

where $\theta \in (1, 2)$, $\alpha \in (0, 3 - 3\theta/2)$ and $I_\alpha: \mathbb{R}^3 \rightarrow \mathbb{R}$ is the Riesz potential given for each $x \in \mathbb{R}^3 \setminus \{0\}$ by

$$I_\alpha(x) = \frac{\Gamma((3-\alpha)/2)}{2^\alpha \pi^{3/2} \Gamma(\alpha/2) |x|^{3-\alpha}}.$$

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Key words and phrases. Choquard equation; Lions-type theorem; Singular potential; Pohožaev-type ground state solution; Critical exponent.

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