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FIXED POINT THEOREM FOR GENERIC 2-GENERALIZED HYBRID MAPPINGS IN HILBERT SPACES

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Abstract. We establish a fixed point theorem for a class of mappings called generic 2-generalized hybrid mappings in the setting of a real Hilbert space. Two examples of that class of mappings are presented herein. The mappings are not quasi-nonexpansive even though they have fixed points. One of these maps is even not continuous. The fixed point theorem proved in this article improves many previous works in the literature.

1. Introduction

Let E be a Banach space with a norm $\|\cdot\|$. For a mapping $T\colon C \to E$, the set of fixed points is denoted as

$$F(T) = \{x \in C : Tx = x\},$$

where C is a nonempty subset of E. The Schauder fixed point theorem [21] asserts that any continuous mapping defined on a compact and convex set has a fixed point. A mapping $T\colon C \to E$ is called *nonexpansive* if

$$\|Tx - Ty\| \leq \|x - y\| \quad \text{for all } x, y \in C.$$

Obviously, a nonexpansive mapping is continuous. Under the setting of a reflexive Banach space, Kirk [12] proved the existence of fixed points for nonexpansive

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