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KIRCHHOFF TYPE ELLIPTIC SYSTEMS WITH EXPONENTIAL GROWTH NONLINEARITIES

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Abstract. In this paper we are interested in the existence of solutions for the following Kirchhoff type elliptic systems

$$\begin{cases} -M\left(\sum_{j=1}^{m}\|u_j\|^2\right)\Delta u_i = f_i(x,u_1,\dots,u_m) & \text{in } \Omega,\\ u_1=\dots=u_m=0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^2$, M is a Kirchhoff type function, $\|u_i\|^2 := \int_\Omega |\nabla u_i|^2\,dx$, f_i behaves like $\exp(\beta s^2)$ when $|s|\to\infty$ for some $\beta>0$, $i=1,\dots,m$. By variational methods with the Trudinger–Moser inequality, we obtain the existence of solutions for the above systems.

1. Introduction

In the last decades, a great attention has been focused on the study of problems involving exponential growth nonlinearities, which is related to the famous Trudinger–Moser inequality. Let Ω be a bounded domain in $\mathbb{R}^2$, and denote with $H_0^1(\Omega)$ the standard first order Sobolev space given by

$$H_0^1(\Omega)=\mathrm{cl}\left\{u\in C_0^\infty(\Omega):\int_\Omega |\nabla u|^2\,dx<\infty\right\},\qquad \|u\|=\left(\int_\Omega |\nabla u|^2\,dx\right)^{1/2}.$$

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