

## LIOUVILLE-TYPE THEOREMS FOR GENERALIZED HÉNON–LANE–EMDEN SCHRÖDINGER SYSTEMS IN $\mathbb{R}^2$ AND $\mathbb{R}^3$

XIYOU CHENG — KUI LI — ZHITAO ZHANG

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ABSTRACT. In the paper we study the Liouville-type theorems for generalized Hénon–Lane–Emden elliptic system in  $\mathbb{R}^N$ . By the methods of spherical averages, Rellich–Pohozaev type identities, Sobolev inequalities on  $S^{N-1}$ , feedback and measure arguments, and scale invariance of the solutions, we show that if the pair of exponents is subcritical, then this system has no positive solutions for  $N = 2$  and no bounded positive solutions for  $N = 3$ .

### 1. Introduction

In this paper we consider the following generalized Hénon–Lane–Emden–Schrödinger system

$$(1.1) \quad \begin{cases} -\operatorname{div}(|x|^\vartheta Du) = |x|^a v^p, & x \in \mathbb{R}^N, \\ -\operatorname{div}(|x|^\vartheta Dv) = |x|^b u^q, & x \in \mathbb{R}^N, \end{cases}$$

where the dimension  $N \geq 2$ ,  $p, q > 0$ ,  $\vartheta \geq 2 - N$  and  $a, b \in \mathbb{R}$ . This system is a natural extension of the weighted Lane–Emden equation (see [7], [11])

$$(1.2) \quad -\operatorname{div}(|x|^\vartheta Du) = |x|^a u^p, \quad x \in \mathbb{R}^N.$$

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