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UNBALANCED FRACTIONAL ELLIPTIC PROBLEMS WITH EXPONENTIAL NONLINEARITY: SUBCRITICAL AND CRITICAL CASES

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Abstract. This paper deals with the qualitative analysis of solutions to the following (p,q)-fractional equation:

$$(-\Delta)_p^{s_1} u + (-\Delta)_q^{s_2} u + V(x)\big(|u|^{p-2}u + |u|^{q-2}u\big) = K(x)\frac{f(u)}{|x|^\beta} \quad \text{in } \mathbb{R}^N,$$

where $1 < q < p$, $0 < s_2 \le s_1 < 1$, $ps_1 = N$, $\beta \in [0, N)$, and $V, K\colon \mathbb{R}^N \to \mathbb{R}$, $f\colon \mathbb{R} \to \mathbb{R}$ are continuous functions satisfying some natural hypotheses. We are concerned both with the case when f has a subcritical growth and with the critical framework with respect to the exponential nonlinearity. By combining a Moser–Trudinger type inequality for fractional Sobolev spaces with Schwarz symmetrization techniques and related variational and topological methods, we prove the existence of nonnegative solutions.

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