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COMPACTLY GENERATED SHAPE INDEX THEORY AND ITS APPLICATION TO A RETARDED NONAUTONOMOUS PARABOLIC EQUATION

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Abstract. We establish the compactly generated shape (H-shape) index theory for local semiflows on complete metric spaces via more general shape index pairs, which allows the phase space to be not separable. The main advantages are that the quotient space N/E is not necessarily metrizable for the shape index pair (N, E) and $N \setminus E$ need not to be a neighbourhood of the compact invariant set. Moreover, we define H-shape cohomology groups and the H-shape cohomology index that is used to develop the Morse equations. Particularly, we apply H-shape index theory to an abstract retarded nonautonomous parabolic equation to illustrate these advantages for the new shape index theory, in which we obtain a special existence property of bounded full solutions.

1. Introduction

Index theory plays a significant role in the development of dynamical systems, including Leray–Schauder degree, Morse index, Conley index, Maslov index and shape index. Among these indices, Conley and shape indices are used to describe the topological property of invariant sets and have extensive applications ([5], [23], [30], [32]). Conley index was introduced by Conley [4] and later extended by Rybakowski [23] to local semiflows on complete metric spaces.

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