Topological **M**ethods in **N**onlinear **A**nalysis
Volume 59, No. 1, 2022, 221–260
DOI: 10.12775/TMNA.2021.025

MULTIPLE SOLUTIONS FOR BIHARMONIC CRITICAL CHOQUARD EQUATION INVOLVING SIGN-CHANGING WEIGHT FUNCTIONS

Anu Rani — Sarika Goyal

Abstract. The purpose of this article is to deal with the following biharmonic critical Choquard equation

$$\begin{cases} \Delta^2 u = \lambda f(x)|u|^{q-2}u + g(x)\bigg(\displaystyle\int_\Omega \frac{g(y)|u(y)|^{2^*_\alpha}}{|x-y|^\alpha}\,dy\bigg)|u|^{2^*_\alpha-2}u & \text{in } \Omega, \\ u,\ \nabla u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, $N \geq 5$, $1 < q < 2$, $0 < \alpha < N$, $2^*_\alpha = (2N-\alpha)/(N-4)$ is the critical exponent in the sense of Hardy–Littlewood–Sobolev inequality and $\lambda > 0$ is a parameter. The functions $f, g\colon \overline{\Omega} \to \mathbb{R}$ are continuous sign-changing weight functions. Using the Nehari manifold and fibering map analysis, we prove the existence of two nontrivial solutions of the problem with respect to parameter λ.

1. Introduction

In this article, we are concerned with the existence of two nontrivial solutions for the following biharmonic critical Choquard equation

$$(\mathrm{E}_\lambda) \qquad \begin{cases} \Delta^2 u = \lambda f(x)|u|^{q-2}u + g(x)\bigg(\displaystyle\int_\Omega \frac{g(y)|u(y)|^{2^*_\alpha}}{|x-y|^\alpha}\,dy\bigg)|u|^{2^*_\alpha-2}u & \text{in } \Omega, \\ u,\ \nabla u = 0 & \text{on } \partial\Omega, \end{cases}$$

2020 *Mathematics Subject Classification.* 35A15, 35B33, 35J91.

Key words and phrases. Biharmonic Choquard equation; critical exponent; sign-changing weight functions; Nehari manifold; concave-convex nonlinearities.

The second author is supported by Science and Engineering Research Board, Department of Science and Technology, Government of India, grant number ECR/2017/002651.