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## SIGN-PRESERVING SOLUTIONS FOR A CLASS OF ASYMPTOTICALLY LINEAR SYSTEMS OF SECOND-ORDER ORDINARY DIFFERENTIAL EQUATIONS

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ABSTRACT. We study multiplicity of solutions to an asymptotically linear Dirichlet problem associated with a planar system of second order ordinary differential equations. The existence of two sign-preserving component-wise solutions is guaranteed when the Morse indexes of the linearizations at zero and at infinity do not coincide, and one of the asymptotic problems has zero-index. The proof is developed in the framework of topological and shooting methods and it is based on a detailed analysis and characterization of the phase angles in a two-dimensional setting.

## 1. Introduction

This paper is devoted to the study of existence of a pair of sign-preserving component-wise solutions to the planar Dirichlet problem

(1.1) 
$$\begin{cases} u''(t) + A(t, u(t))u(t) = 0 & \text{for } t \in [0, \pi], \\ u(0) = u(\pi) = 0, \end{cases}$$

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