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ISOLATING NEIGHBOURHOODS AND THEIR STABILITY FOR DIFFERENTIAL INCLUSIONS AND FILIPPOV SYSTEMS

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Abstract. Conley index theory is a powerful topological tool for obtaining information about invariant sets in dynamical systems. A key feature of Conley theory is that the index is robust under perturbation; given a continuous family of flows $\{\varphi_\lambda\}$, the index remains constant over a range of parameter values, avoiding many of the complications associated with bifurcations. This theory is well-developed for flows and homeomorphisms, and has even been extended to certain classes of semiflows. However, in recent years mathematicians and scientists have become interested in differential inclusions. Here the theory has also been studied for inclusions which satisfy certain bounding properties. In this paper we extend some of these results–in particular, the stability of isolating neighbourhoods under perturbation–to inclusions which do not satisfy these bounding properties. We do so by utilizing a novel approach to the solution set of differential inclusions which results in an object called a multiflow. This perspective allows us to relax the assumptions of the earlier work and also to develop tools needed to extend the continuation of Conley's attractor-repeller decomposition to differential inclusions, a result which is addressed in subsequent work. Our interest in these results is in the study of piecewise-continuous differential equations–which are typically reframed as a certain type of differential inclusion called Filippov systems–and how these discontinuous equations relate to families of smooth systems which limit to them. Therefore this paper also discusses in some detail how the generalization of Conley index theory applies to Filippov systems.

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