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## POSITIVE SOLUTIONS OF NEUMANN BOUNDARY VALUE PROBLEMS AND APPLICATIONS TO LOGISTIC TYPE POPULATION MODELS

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ABSTRACT. We study the existence of nonzero nonnegative or strictly positive solutions of second order Neumann boundary value problems with nonlinearities which are allowed to take negative values via a recently established fixed point theorem for r-nowhere normal-outward maps in Banach spaces. As applications, we obtain results on the existence of strictly positive solutions for some models of population inhabiting one dimensional heterogeneous environments with perfect barriers, where the local rate of change in the population density changes sign.

## 1. Introduction

One of important topics in the population dynamics is to study the existence of nonzero nonnegative or strictly positive steady-state solutions of reactiondiffusion equations which model dynamics of population inhabiting one or higher dimensional heterogeneous environments with perfect barriers, for example see [3]-[6], [18], [20], [22]). One of these models is the one dimensional population

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