Topological **M**ethods in **N**onlinear **A**nalysis
Volume 59, No. 2A, 2022, 585–621
DOI: 10.12775/TMNA.2020.068

THE HOMOTOPY CLASSIFICATION OF PROPER FREDHOLM MAPS OF INDEX ONE

Alberto Abbondandolo — Thomas Rot

In memory of Andrzej Granas

Abstract. In a previous paper, we classified the homotopy classes of proper Fredholm maps from an infinite dimensional Hilbert manifold to its model space in terms of a suitable version of framed cobordism. We explicitly computed these homotopy classes for non-positive index. In this paper, we compute the homotopy classes of proper Fredholm maps of index one from a simply connected Hilbert manifold to its model space. This classification uses a new numerical invariant for proper Fredholm maps of index one.

1. Introduction

Consider a real separable and infinite dimensional Hilbert space $\mathbb{H}$. The space of Fredholm operators on $\mathbb{H}$ of index n is denoted by $\Phi_n(\mathbb{H})$.

By Hilbert manifold we mean here a connected paracompact smooth manifold modeled on $\mathbb{H}$. A smooth map $f\colon M \to N$ between Hilbert manifolds is called Fredholm of index n if its differential at every point is a Fredholm operator of index n. All Fredholm maps we consider in this paper are tacitly assumed to be smooth. By Kuiper's theorem [11], the general linear group of $\mathbb{H}$ is contractible

2020 *Mathematics Subject Classification.* 58B05, 58B15, 57R90, 47A53, 47H11.
Key words and phrases. Fredholm maps; Pontryagin–Thom construction; framed cobordism.
The research of A. Abbondandolo is supported by the DFG-Project 380257369 "Morse theoretical methods in Hamiltonian dynamics".
The research of T.O. Rot is supported by NWO-NWA Startimpuls – 400.17.608.