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LIPSCHITZ PERTURBATION TO EVOLUTION INCLUSION DRIVEN BY TIME-DEPENDENT MAXIMAL MONOTONE OPERATORS

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Abstract. An evolution inclusion driven by a time-dependent maximal monotone operator and a Lipschitz closed valued perturbation, in a separable Hilbert space is considered. The inclusion with a convexified perturbation term is also studied. Then, the existence of solutions and the relaxation property between these evolution inclusions are proved. Applications to dynamical systems governed by a couple of a fractional equation and an evolution inclusion involving time-dependent maximal monotone operators with a Lipschitz perturbation are presented.

1. Introduction

Let H be a separable Hilbert space and $I := [0,1]$. In this paper, we establish existence of solutions for both problems:

$$(\mathcal{P}_F) \qquad \begin{cases} -\dot{u}(t) \in A(t)u(t) + F(t,u(t)) & \text{for a.e. } t \in I, \\ u(t) \in \mathrm{D}(A(t)) & \text{for a.e. } t \in I, \\ u(0) = u_0 \in \mathrm{D}(A(0)), \end{cases}$$

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