COMPONENTWISE LOCALIZATION OF CRITICAL POINTS FOR FUNCTIONALS DEFINED ON PRODUCT SPACES

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ABSTRACT. A new notion of linking is introduced to treat minima as minimax points in a unitary way. Critical points are located in conical annuli making possible to obtain multiplicity. For functionals defined on a Cartesian product, the localization of critical points is given on components and the variational properties of the components can differ, part of them being of minimum type, others of mountain pass type.

1. Introduction

For many problems arising from physics and other sciences, the solution appears to be the minimum or the maximum of some functional usually called the energy functional. Thus solving such kind of problems reduces to finding critical points (minima, maxima and minimax points) of the associated functionals, and lot of effort in the calculus of variations has been devoted to this goal. One can distinguish between direct methods in the calculus of variations, which are directed to the finding of extrema, and minimax methods wishing to produce critical points that are neither maxima nor minima (see, e.g. [1], [4], [5], [12]–[14], [16]).

First in this paper, we introduce a new notion of linking which allows to see minima as minimax points associated to a limit case of linking, and consequently, to treat extrema and minimax points as one, in a uniform way.