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LINEARIZATION OF TOPOLOGICALLY ANOSOV HOMEOMORPHISMS OF NON COMPACT SURFACES OF GENUS ZERO AND FINITE TYPE

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Abstract. We study the dynamics of *topologically Anosov* homeomorphisms of non-compact surfaces. In the case of surfaces of genus zero and finite type, we classify them. We prove that if $f\colon S\to S$, is a Topologically Anosov homeomorphism where S is a non-compact surface of genus zero and finite type, then $S=\mathbb{R}^2$ and f is conjugate to a homothety or reverse homothety (depending on wether f preserves or reverses orientation). A weaker version of this result was conjectured in [6].

1. Introduction

A homeomorphism $f\colon M\to M$ of the metric space M to itself is called *expansive* if there exists $\alpha>0$ such that given $x,y\in M$, $x\neq y$, then $d(f^n(x),f^n(y))>\alpha$ for some $n\in\mathbb{Z}$. If $\delta>0$, a δ-pseudo-orbit for f is a sequence $(x_n)_{n\in\mathbb{Z}}$ such that $d(f(x_n),x_{n+1})<\delta$ for all $n\in\mathbb{Z}$. If $\varepsilon>0$, we say that the orbit of x ε-shadows a given pseudo-orbit $(x_n)_{n\in\mathbb{Z}}$ if $d(x_n,f^n(x))<\varepsilon$ for all $n\in\mathbb{Z}$. Finally, we say that f has the shadowing property if for each $\varepsilon>0$ there exists $\delta>0$ such that every δ-pseudo-orbit is ε-shadowed by an orbit of f. Roughly speaking, expansivity means that every point has a distinctive dynamical behavior, and systems

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