

A SINGULAR PERTURBED PROBLEM WITH CRITICAL SOBOLEV EXPONENT

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ABSTRACT. This paper deals with the following nonlinear elliptic problem

$$(0.1) \quad -\varepsilon^2 \Delta u + \omega V(x)u = u^p + u^{2^*-1}, \quad u > 0 \quad \text{in } \mathbb{R}^N,$$

where $\omega \in \mathbb{R}^+$, $N \geq 3$, $p \in (1, 2^* - 1)$ with $2^* = 2N/(N - 2)$, $\varepsilon > 0$ is a small parameter and $V(x)$ is a given function. Under suitable assumptions, we prove that problem (0.1) has multi-peak solutions by the Lyapunov–Schmidt reduction method for sufficiently small ε , which concentrate at local minimum points of potential function $V(x)$. Moreover, we show the local uniqueness of positive multi-peak solutions by using the local Pohozaev identity.

1. Introduction

In this paper, we consider the following nonlinear elliptic problem

$$(1.1) \quad -\varepsilon^2 \Delta u + \omega V(x)u = |u|^{p-1}u + |u|^{2^*-2}u, \quad \text{in } \mathbb{R}^N,$$

where $N \geq 3$, $\omega \in \mathbb{R}^+$, $p \in (1, 2^* - 1)$ with $2^* = 2N/(N - 2)$, $\varepsilon > 0$ is a small parameter and $V: \mathbb{R}^N \rightarrow \mathbb{R}$ is a continuous function. Problem (1.1) is related to

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