Topological Methods in Nonlinear Analysis Volume 58, No. 1, 2021, 135–160 DOI: 10.12775/TMNA.2020.065

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NONLINEAR VOLTERRA DELAY EVOLUTION INCLUSIONS SUBJECTED TO NONLOCAL INITIAL CONDITIONS

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ABSTRACT. This paper deals with a nonlinear Volterra delay evolution inclusion subjected to a nonlocal implicit initial condition. The evolution inclusion involves an m-dissipative operator (possibly multivalued and/or nonlinear) and a noncompact interval. We first consider the evolution inclusion subjected to a local initial condition and prove an existence result for bounded C^0 -solutions. Then, using a fixed point theorem for upper semicontinuous multifunctions with contractible values, we obtain a global solvability result for the original problem. Finally, we present an example to illustrate the abstract result.

1. Introduction

Nonlinear evolution inclusions subjected to nonlocal implicit initial conditions in infinite-dimensional spaces have become an object of extensive study in recent years. Please see, e.g. Benedetti et al. [3], [4], Burlică et al. [7], Chen et al. [8], Malaguti and Rubbioni [16], Vrabie [18]–[20], Wang et al. [22], [23] and references therein.

Assume that $(X, \|\cdot\|)$ is a Banach space. For $a \in \mathbb{R}$, we denote $\mathbb{R}_a^+ = [a, +\infty)$ for simplicity. Let $\tau \in \mathbb{R}_0^+$. In the present paper we consider a class

 $^{2020\} Mathematics\ Subject\ Classification.$ Primary: 34G25, 47H20; Secondary: 45D05, 45K05.

Key words and phrases. Volterra delay evolution inclusion; nonlocal implicit initial condition; C^0 -solutions; equicontinuous semigroup; global solvability.

The research leading to the results of this paper has received funding from the NSFC (Nos. 11971317, 11471083).