Topological **M**ethods in **N**onlinear **A**nalysis
Volume 58, No. 1, 2021, 97–103
DOI: 10.12775/TMNA.2020.063

EXTENDING AND PARALLELING STECHKIN'S CATEGORY THEOREM

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Abstract. We strengthen one of Stechkin's theorems. We also obtain results in the same spirit regarding the farthest point mapping. We work in length spaces, sometimes without bifurcating geodesics, sometimes with geodesic extendability.

Let $K \subset \mathbb{R}^d$ be a closed set and p_K the *nearest point mapping*, which associates to every point $x \in \mathbb{R}^d$ the set of all points in K closest to x. Asplund and Stechkin have been the pioneers and founders of the smallness theory for the set of points with unique nearest points from a given compact set. It was already well-known that p_K is single-valued almost everywhere, when Stechkin [14] proved in 1963 that, from the point of view of Baire categories, too, p_K is single-valued at most points of $\mathbb{R}^d$. See also Cobzaş [6] and the surveys of Konyagin [13] and Vlasov [15].

We always say that *most* elements of a Baire space have property $\mathscr{P}$, if those not enjoying $\mathscr{P}$ form a first category set, i.e. a countable union of nowhere dense sets.

We showed in [18] that, in any Alexandrov space with curvature bounded below, p_K is properly multivalued on a σ-porous set (which is in general "smaller" than a set of first Baire category). We also extended Stechkin's result to more general metric spaces in [19]. Theorem 3 from [18] is a generalization of an

2020 *Mathematics Subject Classification.* 54E50, 54E52.

Key words and phrases. Complete metric space; nearest point; farthest point; Baire category.