

## PERIODIC SOLUTIONS TO REVERSIBLE SECOND ORDER AUTONOMOUS SYSTEMS WITH COMMENSURATE DELAYS

ZALMAN BALANOV — FULAI CHEN  
JING GUO — WIESLAW KRAWCEWICZ

---

*This work is dedicated to the memory of Professor Andrzej Granas*

ABSTRACT. Existence and spatio-temporal patterns of periodic solutions to second order reversible equivariant autonomous systems with commensurate delays are studied using the Brouwer  $O(2) \times \Gamma \times \mathbb{Z}_2$ -equivariant degree theory, where  $O(2)$  is related to the reversing symmetry,  $\Gamma$  reflects the symmetric character of the coupling in the corresponding network and  $\mathbb{Z}_2$  is related to the oddness of the right-hand side. Abstract results are supported by a concrete example with  $\Gamma = D_6$  – the dihedral group of order 12.

### 1. Introduction

**1.1. Subject and goal.** Existence of periodic solutions to equivariant dynamical systems together with describing their spatio-temporal symmetries constitute an important problem of equivariant dynamics (see, for example, [10], [11])

---

2020 *Mathematics Subject Classification.* Primary: 34K13, 37J45, 37C80, 47H11; Secondary: 39A23.

*Key words and phrases.* Second order delay-differential equations; periodic solutions; commensurate delays; Brouwer equivariant degree; Burnside ring; reversible systems; equivariant systems.

Zalman Balanov was supported in part by National Natural Science Foundation of China (no. 12071395), the Applied Characteristic Discipline in Xiangnan University and Furong Scholars Award Program in Hunan Province.

Fulai Chen was supported in part by the NNSF-China (Grant No. 11926309), and the Applied Characteristic Discipline in Xiangnan University (China).

Wiesław Krawcewicz was supported by the NNSF-China (Grant No. 11871171).