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EXISTENCE AND NONEXISTENCE OF LEAST ENERGY NODAL SOLUTIONS FOR A CLASS OF ELLIPTIC PROBLEM IN $\mathbb{R}^2$

Claudianor O. Alves — Denilson S. Pereira

Abstract. In this work, we prove the existence of least energy nodal solutions for a class of elliptic problem in both cases, bounded and unbounded domain, when the nonlinearity has exponential critical growth in $\mathbb{R}^2$. Moreover, we also prove a nonexistence result of least energy nodal solution for the autonomous case in whole $\mathbb{R}^2$.

1. Introduction

This paper concerns with the existence of least energy nodal solutions for the following class of elliptic problem

$$\text{(P)}\qquad \begin{cases} -\Delta u + V(x)u = f(u) & \text{in } \Omega, \\ u \in H_0^1(\Omega), \end{cases}$$

where $\Omega \subset \mathbb{R}^2$ is a smooth bounded domain or $\Omega = \mathbb{R}^2$, $V : \overline{\Omega} \to \mathbb{R}$ is a continuous function verifying some hypotheses which will be fix later on. Concerning the nonlinearity, we assume that $f : \mathbb{R} \to \mathbb{R}$ is a C^1-function, which can have an exponential critical growth at both $+\infty$ and $-\infty$, that is, it behaves like $e^{\alpha_0 s^2}$,

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as $|s| \to \infty$, for some $\alpha_0 > 0$. More precisely,

$$\begin{aligned} &\lim_{|s|\to\infty} \frac{f(s)}{e^{\alpha|s|^2}} = 0 \quad \text{for all } \alpha > \alpha_0, \\ &\lim_{|s|\to\infty} \frac{f(s)}{e^{\alpha|s|^2}} = \infty \quad \text{for all } \alpha < \alpha_0, \end{aligned} \tag{1.1}$$

(see [20]). In the last years, we have observed that the existence of nodal solution has received a special attention of a lot of researches. In Cerami, Solimini and Struwe [19], the authors showed the existence of multiple nodal solutions for the following class of elliptic problem with critical growth

$$\begin{cases} -\Delta u - \lambda u = |u|^{2^*-2}u & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{P$_1$}$$

where $\Omega = B_R(0) \subset \mathbb{R}^N$, $N \geq 7$, $2^* = 2N/(N-2)$ and $\lambda \in [0, \lambda_1]$, with λ_1 being the first eigenvalue of $(-\Delta, H_0^1(\Omega))$. In Bartsch and Willem [12], the existence of infinitely many radial nodal solutions was proved for the problem

$$\begin{cases} -\Delta u + u = f(|x|, u) & \text{in } \mathbb{R}^N, \\ u \in H^1(\mathbb{R}^N), \end{cases} \tag{P$_2$}$$

where f is a continuous function with subcritical growth and verifying some hypotheses. In Cao and Noussair [17], the authors studied the existence and multiplicity of positive and nodal solutions for the following class of problems

$$\begin{cases} -\Delta u + u = Q(x)|u|^{p-2}u & \text{in } \mathbb{R}^N, \\ u \in H^1(\mathbb{R}^N) \end{cases} \tag{P$_2$}$$

by supposing $2 < p < (N+2)/(N-2)$, $N \geq 3$ and some technical conditions on Q. In that paper, the main result connects the number of positive and nodal solutions with the number of maximum points of function Q.

In Castro, Cossio and Neuberger [18] and Bartsch and Wang [13], the authors studied the existence of nodal solution for a problem like

$$\begin{cases} -\Delta u = f(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{P$_3$}$$

where Ω is a smooth bounded domain and f verifies some hypotheses. In [18], it was assumed that f is superlinear, while in [10] that f is asymptotically linear at infinity. In Bartsch and Weth [10], existence of multiple nodal solutions was also considered for problem (P$_3$).