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Copernicus and the Hypothetico-Deductive Method of Correspondence Thinking[1]

An Introduction

1. Problem

The fundamental task of the history and philosophy of the so-called exact sciences there is the proper understanding of a genesis, contents and reception of certain considered episodes which occurred at the History of the so-called exact sciences. However this task is not easy, at all, as showed the hitherto development of these branches. On the one hand, many historians of science, believing still that pure historical facts exist, searched their branch as if science would be nonphilosophical. On the other hand, many philosophers of science, neglecting historical dimension of science, analysed their branch as if science would be nonhistorical. Thus, such generated the "nonhistorical philosophy" and "nonphilosophical history" of the so-called exact sciences, did not pay sufficient notice, on

[1] This article is a development of my earlier papers entitled "Copernicus' correspondence thinking" and "Copernicus' astronomical works — A remarkable case of applying the methodological idea of correspondence" which were delivered at the XIXth International Congress of History of Science (22-29.08.1993, Zaragoza, Spain) and 10th International Congress of Logic, Methodology and Philosophy of Science (19-25.08.1995 Florence, Italy).

To avoid misunderstandings at the very beginning of this article, let us explain here, that the term the 'correspondence thinking' has for us a special, methodological character that is closely connected, among other things, with the process of framing by theorists the theories which are linked by the correspondence principle. Regarding the linguistic form, the term 'correspondence thinking" is comprehended by us as equivalent to Feyerabend's German term "Korrespondenzdenken" that denotes the sort of thinking of Bohr, Kramers and Heisenberg, which was centered on Bohr's correspondence principle (see Feyerabend [1962a] p. 203, p. 254 fn. 2). However, the term the 'correspondence thinking' will be comprehended by us in the broader sense on which we will say more below.

the one hand, to the historical scientific questions, and on the other, to their specific philosophical character. The "nonhistorical philosophy" did not consider a real philosophical-scientific problematic situation that is met by scientist trying to make a scientific discovery at a definite moment in history. On the other hand, the "nonphilosophical history" made no attempt to understand the complicated structure of theory, for instance, the issue of the empirical or theoretical sense of some elements of this structure. In consequence such history was almost blind to the complexity of scientific research programme, which guided scientist looking for the new theory.

Moreover, "the nonphilosophical history" and, especially, "the nonhistorical philosophy" of the so-called exact sciences often forgot, among other things, that the so-called exact sciences concentrate around the specific deeply mathematical questions, what, for example, Galileo's famous remark states:

> Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth.[2]

In consequence, such understood the history and philosophy of the so-called exact sciences are very often, in some degree, inconsistent with the spirit and the ethos penetrating the subject of their interests. And, as one knows, these spirit and ethos manifest especially during the so-called scientific revolutions or while searching for the new and more general theory than the hitherto existing old one.

The above mentioned inconsistency is particularly obvious in the case of a genesis, contents and reception of Copernicus' achievements. On the one hand, many eminent twentieth-century historians and philosophers of the so-called exact sciences criticized severely Copernicus' results and his scientific method. Therefore the following conclusion seemed sound:

> If there was a revolution in astronomy, that revolution was Keplerian and Newtonian, and not in any simple or valid sense Copernican. (Cohen, I.B. [1985], p. 125)

On the other hand, every student of physics and astronomy knows that the eminent modern scientists as Kepler, Galileo, Newton, in prin-

[2] The Assayer (1623), in Stillman Drake, Discoveries and Opinions of Galileo (New York, 1957), p. 237-238.

cipie, appreciated Copernicus' conceptions. Moreover, the famous scientists of the 19th and the 20th century, for instance Poincare, also appreciated them. And they all did it for the same reason. Without, among other things, the heliocentric reference frame, the hypotheses of terrestrial diurnal and annual motions, it would be impossible to formulate three Keplerian laws, and later the Newtonian theory of gravitation based on the Newtonian dynamics. Thanks to the later, which uses Copernicus hypothesis of movable earth with its daily and annual motions, many different phenomena as e.g. winds, tides and precession of the equinoxes were unified, mathematically described and physically explained. And such an unification was impossible in the geocentric cosmology.'[3] But, in the context of the discussion about the Theory of Relativity, majority of physicists accepted that Copernicus' and Ptolemy's reference frames are equivalent. Thus Copernicus' achievement could not be so big as assumed in the 18th and the 19th centuries.[4], Next, however, thanks to more detailed considerations, this opinion was finally rejected by physicists.[5]

Moreover, many scientists thought that Copernicus lied the foundation of scientific method which was next developed by Galileo and Newton.

But, after the works of historians of science, it is also clear, that the above mentioned physicists, not been professionally interested in the

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[3] See, for instance, H. Poincare [1906]: *La Valeur de la Science*, Paris: Flammarion, (ed. 1932, p. 271-274) English translation, reprint, Value of Science, Dover [1958] p. 140-141.

[4] See E. Mach [1883]: *Die Mechanik In Irhrer Entwicklung Historirisch-Kritisch Dargestellt*, Leipzig: Brockhaus; M. Born [1922]: *Die Relativitatstheorie Einstein und ihre physikalische Grundlagen*, Berlin; F. Frank [1941]: *Between Physics and Philosophy,* A. Einstein, L. Infeld [1942]: *Evolution of Physics,* New York; H. Reinchenbach [1942]: *From Copernicus to Einstein*. New York.

[5] See, for instance, A. Eddington [1922]: *Mathematical Theory of Gravitation',* B. A. ©ок [1943]: *CucmeMa Konepnuxa u cucmeMa tlmonoen e caeme o6w,eu meopuu omiocumemocmu*, in MHxaftjioB, A A (ed) [1943] p. 180-186; A. Ujemow [1953]: "System heliocentryczny Kopernika a teoria względności", *Myśl filozoficzna,* 1953, 1 (7) p. 144—175; L. Infeld [1955]: "Od Kopernika do Einsteina", *Kosmos* Seria B 1955, R I, z. 3, p. 209-226.

Infeld [1955], knowing works of ©ок [1943] and Ujemow [1953] arguments as follows. (1) the Newtonian laws of dynamics are defined within the inertial reference frame; (2) to solve the problem of planet, within the context of the Newtonian theory, it is necessary at first to find the sufficiently inertial reference frame; in this case it is the reference frame of the center of mass of the solar system; the heliocentric frame is less inertial, and the least, geocentric; (3) though the General theory of relativity does not favour any framework, nevertheless to describe the planetary motions, within the context of this theory, it is necessary to choose exactly the same reference frame as in the case of the Newtonian theory of gravitation. Hence, Copernicus decision to move the earth from the center of the geocentric universe was also sound from the point of view of the General theory of relativity.

history of science, were misinformed in many details of Copernicus' theory.[6]

These diametrically different interpretations of the Copernican revolution, Copernicus' achievements and his scientific method made by historians and philosophers of science, on the one hand, and scientists, on the other, are, as we think, worthy of notice. Thus the below questions arise. Was the Copernican revolution a revolution in astronomy and, generally, in science, in any simple or valid sense? and was Copernicus' scientific method correct and original?

2. Plan

Trying to avoid weak points of "the nonhistorical philosophy" of the so-called exact sciences, we fix our attention purely on the above mentioned Copernicus' issue. But trying to escape weak sides of the "non-phil- osophical history" of the so-called exact sciences we will at first briefly discuss the method of carrying out creative research in the field of the so-called exact sciences during **modem times.** Such an introduction to the analysis of Copernicus' achievement is done wittingly, in order **to sensitize us initially** for the fundamental issues of the method of mathematicization of phenomena.[7] Taking this into account, at the following step, we will discuss Copernicus' method of search for the new, more general astronomical theory than Ptolemy's. To close this article, some opinions, taken by the way of example, regarding the genesis, contents and reception of Copernicus' theory, those we do not share, will be listed. Finally, some general conclusions for creative working in the field of history and philosophy of the so-called exact sciences will be presented.

3. Looking for the method of the so-called exact sciences

My **foreunderstanding** of the methodology of the so-called exact sciences is determined by, among other things, the following sources:

[6] Note, for instance, that all of them erroneously thought that the center of the real sun, and not the abstract point called the mean sun, that is the centre of the *Orbis Magnus* (the centre earth's orbit around the sun, using modern terminology), governs the motions of the planets in Copernicus' theory.

[7] Though such initial and, at first sight, "ihistorical" procedure may seem to humanists and historians strange, nevertheless, it is quite natural from the methodological point of view. Copernicus, in his astronomical works, similarly like Newton, Fresnel, Planck, Bohr, Einstein or Heisenberg, mathematicized, or to be exact, geometrized phenomena.

(1) the trend called the philosophy in science; it is strictly connected with the methodological awareness of the scientific problems that have the creative modem theorists, such as Planck, Bohr, Einstein, Schrodinger, Heisenberg; especially, it is the case of Bohr's correspondence principle;
(2) the standard hypothetico-deductive method of the exact sciences, on the one hand, and the conception of scientific hypothesis, deduction, induction, Peirce-Hanson's abduction or retroduction, on the other;
(3) the conception of the observational and formal, mathematical equivalence and non-equivalence of theories;
(4) the idea of a theory understood as a complicated multi-level model and a structure;
(5) the principle of undertermination of theory by data and the issue of the theory-ladenness of facts;
(6) Kuhn's and Fayerabend's criticism of Nagel's deductive model of reduction and Hempel-Oppenheim's deductive model of explanation, and the idea of the incommensurability of theories;
(7) Lakatos' conception of methodology of the research programme;
(8) the issue of scientific, conceptual or theoretical change and the philosophy of scientific discovery;
(9) the methodological analyses of Bohr's correspondence principle and its generalization and their importance for the progress of science, especially made by Кузнецов [1948], Krajewski, ... (eds.) [1974].°

To agree with the creative scientists as Einstein, Heisenberg, Bohr, etc, it is necessary to emphasize one very important fact. The sciences as (modem) physics, astronomy and cosmology, being grounded on mathematics, on the one hand, and on the observations and measurements, on the other, are philosophical to the core. It manifests especially while searching for new fundamental theories. It is in such situations that the theorists are forced to touch the epistemological, methodological, quasi- ontological and axiological issues. And though they are less exact than the questions of calculations and measurements, nevertheless, they are not less important at all. To remember about such philosophical character of sciences commonly called, the exact ones, let us name them the so-called exact sciences.[8] [9]

[8] We restrict ourselves to giving only the selected bibliography concerning methodological issues. Since the point 9, being one of my key issues, is the least known still, we emphasize it by giving more extensive bibliography. See pp. 99-101.

[9] The scientists very well know that a creative work in the field of the so-called exact sciences is very often the negation of the logical, abstract exactness that was the object of admiration of neopositivism. On the other hand, it doesn't mean at all, that this creative work is irrational.

Next, since the so-called exact sciences are philosophical to the core, the method thanks to which these sciences are created must touch the philosophical bases of these sciences.

Taking the above mentioned issues into account, we think, that scientists who carry out creative research in the field of the so-called exact sciences, looking for and finding the mathematical models of a class of phenomena consequently and constantly make use of the same method. We call it:

3.1. The Hypothetico-Deductive method of Correspondence Thinking (HDMCT)

The *HDMCT* consists of two parts; namely, *the hypothetico-deductive method* (HDM) and *the method of correspondence thinking* (MCT). However, this division is not sharp. The HDM and the MCT interpenetrate and it is that interpenetration that decides about the specific of the so-called exact sciences.

The HDM concentrates on the idea of the scientific hypothesis and deduction. The scientific hypotheses and deductions are always used within the broad context of the accepted epistemological, methodological assumptions, principles (for instance, the principle of idealization), assumed mathematical languages, etc. It is standard, that such assumptions and principles are never enlightened and verbalized by scientists fully. In consequence, scientific deductions, considered from the formal, logical point of view, as a rule, are incomplete and informal and therefore are called the enthymematic deductions. Moreover, that incompleteness and informality is not banal, at all. This fact is true not only for the phase of framing mathematical models of phenomena, that is, among others things, for the phase of creating special mathematical deductions by theorists, but also for a completed product of such searches, that is, for deductions made in the context of created mathematical models of phenomena.[10]

[10] The notion *deduction* is used here in two meanings. In the first and the classical one, *deduction* is the name of a procedure of inferring of sound, logical, mathematical conclusions from accepted and fixed assumptions, hypotheses, laws and theories. This type of *deduction* is always enthymematic in the field of the socalled exact sciences. In the latter, *deduction* is the name of a procedure of inferring new assumptions, hypotheses, laws and theories within the context of given measuring data, old laws, old theories and the methodological and epistemological principles of the so-called exact sciences. In our opinion, such broad understanding of the notion of *deduction* let us effectively to say as well on the

The HDM is used as well while searching for new theories as analysing many consequences which result from formulated old theories. But, the HDM defines the method used in the so-called exact sciences only initially.

The MCT supplements the HDM in the following important way. Without the MCT, the so-called exact sciences, for instance, modem physics and cosmology, could not come into being, because they would remain only philosophical speculations or a branches of pure mathematics without any connections with natural phenomena. The MCT concentrates on the methodological idea of correspondence of: (a) theoretical and measurable magnitudes, (b) old and new laws and (c) old and new theories. This subject will be discussed later. Now let us say something more about the following.

3.1.1. The Hypothetico-Deductive Method (HDM)

The HDMCT's aim is to find a certain set of 'observables', that is, a certain set of measurable magnitudes, which characterize mathematically the course of a certain chosen group of phenomena. It is impossible to do that without building up a certain deductive and mathematical theory (model) of the above mentioned group of phenomena and without coupling this model with numerical data given by certain measurement equipments.

To construct such theories it is necessary to realize, among other things, two fundamental postulates. On the one hand, theories should be internally perfect (Einstein). On the other, they must agree with the measurements of the observed phenomena: They should be able to describe already known phenomena and, if it is possible, to predict some new their kinds.

However, the above-mentioned postulate of the inner perfection of a theory as well the mathematical as deductive character of a theory, are not model examples of mathematical or logical perfection and exactness. Within the context of the so called exact sciences, they have an utilitarian

context of the justification as on the context of the discovery of certain new hypotheses, laws, and theories without falling into errors of psychologism, sociologism, historicism, on the one hand, and logicism, on the other.

Of course, such understanding of the conception of scientific deduction is far from the formal exactness that was the thing of admiration of neopositivists and their inheritors. But formal reconstructions of theories belonging to the so-called exact sciences in the shape of deductive, axiomatic systems stand, in principle, beyond the limit of the so-called exact sciences. Such a task belongs to logic.

and relative character. Therefore, the created theories should be only precise enough to be able to predict phenomena, but formal exactness of such theories needn't meet with approbation of mathematicians and logicians. This thesis is splendidly corroborated by historical events. For instance, as one knows, the Newtonian physics was progressed dynamically since the date of the publication of Newton's Principia in 1687 to the end of the 19th century. Nevertheless, as Berkeley showed in *The Analyst*[11] in 1734, the mathematical foundation of the Newtonian physics, that is, the infinitesimal calculus, based on the method of fluxious and fluents, was internally contradictory. It used, since, among other things, the infinitesimal treated sometimes as equal to zero and sometimes as differ than zero. The logical difficulties of this sort were not eliminatedtill the works of Couchy in 1812, who defined precisely notion of the limit, and next the works of Weierstrass in 1872, who using the notion of the limit and the axiomatic, Euclidian approach, eschewed infinitesimals. But it was not the end of this story because Abraham Robinson (1918-1974), using formal logic framed c. 1966 the nonstandard analysis, which yet without any logical discrepancy let use the infinitesimals.[12] Thus the conclusion suggests itself: if the scientists who are engaged in the so-called exact sciences had minds of logicians, the Newtonian physics might not be progressed or even be created for circa 300 years since the factual moment of its origin.[13]

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[11] The whole title of Bishop George Berkeley's work reads as follows "The Analyst: Or a Discourse Addressed to an Infidel Mathematicians [referring to Newton's friend Halley). Wherein it is Examined Whether the Object, Principles, and Inferences of the Modem Analysis Are More Distinctly Conceived, or More Evidently Deduced than Religious Mysteries and Points of Faith."

[12] On this subject see, for instance: M. Kline, *The History of the Calculus and its Conceptual Development (The Concepts of Calculus),* Dover Publications, Inc. 1949; the chapter 16.9 "Early Ordeal of the Calculus", in A. B. Arons, *Concepts of Physics,* Reading, Mass.: Addison-Wesley, 1965; A. Robinson, *Nonstandard Analysis,* North-Holland, Amsterdam, 1966; P. Juszkiewicz (ed.), *Historia matematyki;* przekład Stanisław Dobrzycki, Warszawa: PWN 1976, t. 2-3; the chapter "Nonstandard Analysis", in P. J. Davis, R. Hersh *The Mathematical Experience,* Birkhauser Boston, 1981.

[13] There are many examples of this kind and similar histories are related with, for instance, the so-called Dirac's function, the infinite vectors in the standard Hilbert space (of the finite vectors) and the divergent integrates in the theories of fields. Moreover, the above mentioned "imperfect utilitarianism" explains, in my opinion, why the discovery or the formulating of the limitation theorems by logicians has got (at least up to now) an inconsiderable influence on the way of understanding of the theories belonging to the so-called exact sciences. It happened, since, the difficulties discovered by logicians are very little in fact, in comparison to the quandaries of astronomers, physicians, etc. For instance, the difficulties connected with the choosing adequate hypotheses of a theory and with the assigning to a used mathematical language a measurement interpretation.

But, though, physicists have not minds of logicians, they would like to frame theories that are internally perfect. And to make a theory internally perfect, formulating and developing it within the context of a certain research programme, it is necessary to avoid any accidental and incoherent hypotheses, assumptions or axioms, which are called ideas *ad hoc.*[14] To do so, a good theory must be based on mathematically, physically clear-cut, simple and therefore beautiful ideas.[15] Their choice may not be prejudged only by wish to coordinate the predictions of the new theory with observations and measurements. It happens for the fundamental reason. On the ground of observational, measurement data it is impossible to determine mathematical and ontological dimensions of a theory in a univocal way. For the some reason, that it is possible to explain the same facts by different non-equivalent formally theories. Moreover, the bare facts do not exist and the theories do decide what is observed (Einstein)[16] — this is the important issue of theory-ladenness of facts. But on the other hand, this supremacy of theoretical thinking does not mean that the observed facts are only the products of theories: the phenomena as falling of stones or cannon balls, flashes of lightnings and nuclear explosions never reduce only to a theoretical, linguistic dimension. Such regularities do exist in the Universe. Creating a mathematical model of a certain chosen group of phenomena, it is necessary to select an appropriate mathematical language and basic entities[17] of a theory to the

[14] However, it must be emphasized, that what is or seems to be "ad hoc" for logicians or philosophers of science, need not be the same character, for instance, for astronomers and (modern) physicists. The case of Bohr's model of atom and his correspondence principle is the best proof for this thesis.

[15] It is proper to emphasize that, within the domain of the so-called exact sciences, the ideas of beauty and aesthetics are closely connected with the ideas of symmetry and harmony. On that subject see Heisenberg [1971] p. 288-305.

[16] On what, as Werner Heisenberg wrote, Albert Einstein had instructed the young author of the matrix mechanics: "It is generally believed that our science is empirical, and that we draw our concepts and our mathematical constructs from empirical data. If this were the whole truth, when entering into a new field, we should introduce only such quantities that can directly be observed and formulate natural laws only by means of these quantities. When I was a young man I believed that this was just the philosophy that Einstein followed in his theory of relativity. Therefore, I tried to take a corresponding step in quantum theory by introducing the matrices. But when I later asked Einstein about it, he answered: "This may have been my philosophy, but it is nonsense all the same. **It is never possible to introduce only observable quantities in a theory. It is the theory which decides what can be observed."** W. Heisenberg, Tradition in Science, in Gingerich (ed.) [1975] p. 219-236, p. 228. The same story is described more precisely in "Die Quantenmechanik und ein Gesprach mit Einstein (1925-1926)" in Heisenberg [1971]: p. 58-69.

[17] Using here the term *entity (-ies),* I remember Saint Thomas Aquinas's distinction:

problem that is being solved. So used the mathematical language and postulated entities have an absolute, ontological character within the limits of the theory. As one seems, this thesis is confirmed finally by the consistence of the predictions of theory with measurements.[18] However, as a matter of fact, these ontological entities and this ultimate confirmation, are only the quasi-entities and only the quasi-confirmation. It results from the fact, that the mathematical language, used at a basis of every theory, is always an imperfect, restricted net of relations. Just therefore, though mathematics is very effective in the field of the so-called exact sciences[19], phenomena of Nature still go beyond a scheme of our theories. On the other hand, step by step, together with the complicated progress of science we see more precisely the course of phenomena and the relationships between them.

3.1.2. The Method of Correspondence Thinking (MCT)

To model a course of phenomena, it is insufficient to have only a certain mathematical language and certain postulated quasi-entities. To reahze this task, it is necessary also to have a special interpretative mechanism by help of which a pure theory (that is, a certain chosen mathematical language and postulated quasi-entities) obtains a certain measuring interpretation. It is the idea of correspondence that accomplishes this task in the Hypothetico-Deductive Method of Correspondence Thinking. In order that certain phenomena might be predicted by new theories, the

ens = essentia + existentia (entity = essence + existence). For instance, from the point of view of the Newtonian theory of gravitation, the gravitation exists in the Universe, and similarly, from the point of view of chromodynamics, the quarks exist in the Universe. The essences of these entities are defined by the context of the whole theories in which these entities exist.

[18] Let us add here that the incompatibility of observations and measurements of phenomena with predictions of fully developed (and the more just being created) theories need not prejudge about the rejection of such theories and their entities at all. These imperfect theories and their entities are accepted if there are not any better ones or there are more or less justifiable hopes that it is possible to harmonize them with measurements in an undefined future. But as taught us history of science, such kind of hopes may be still reasonable even if it is impossible to realize them during tens of years. Sometimes it happens only after hundreds of years. It is the so-called the Copernican revolution which gives us the best example of this sort. We mean the trials of determination the stellar parallax measured not before 1837-1838, 1838 and 1840 by T. Henderson, F. Bessel and F. B. Struve respectively.

[19] See Wigner, E. P., “The Unreasonable Effectiveness of Mathematics in the Natural Sciences”, *Communications in Pure and Applied Mathematics,* 13, 1-14 (1960).

following conditions must be realized. In the first place, it is necessary to choose such theoretical magnitudes (defined within the context of new theory) which have an operational sense. It means, that defining theoretical magnitudes, we must also show the methods by which such magnitudes are measured. And every measurement consists, in principle, in finding a numerical measure that is a multiple of a selected standard unit. Next, note, that not all theoretical magnitudes must (and might) have an operational sense.

Moreover, in order to construct a certain measuring interpretation of a pure theory, scientists do not look only for numbers. They also search consequently for certain functions modelling the course of a select group of phenomena. Therefore, if there exists already a certain old theory that functions are quantitatively and qualitatively consistent with measuring data a in certain range of considered phenomena, but, inconsistent beyond this range, theorists must do what follows. They have to search for such new functions (defined within the context of the new theory) which, should as best imitate the old functions (defined within the context of the old theory) in a certain region of phenomena. Thus, it is necessary to search for new functions which should "overlap" or "fuse" with the old functions in certain region of the modelled phenomena. Seeing globally, one must look for new theories which mathematical structures should "overlap" or "fuse" with the mathematical structures of the old theories in a certain region. Thus, the new and old functions, and the new and old mathematical structures should be approximately or exactly "tangent" in a certain region. When this task is realized a certain correspondence relation (or relations) of the new and the old laws and a certain correspondence relation (or relations) of the new and the old theories are found. Of course, such result is not accidental at all. We say, it is governed by the postulate of correspondence of the new and the old laws and the postulate of correspondence of the new and the old theories. Let us repeat once again. To construct a more general theory than the old one, but which already adequately describes numerically a certain group of phenomena in their certain range, scientists try to pick out new quasi-entities and, closely connected with them, a mathematical language, which, however, needn't be new. Next, to save a group of valuable predictions of the old theory, they want to find a certain kind "bridge" linking the new theory and the old one. It is the correspondence postulate and the correspondence relation which realizes this task. The postulate of correspondence is a methodological principle which points out the aim of looking for the new theory, and the correspondence relation is a concrete realization of the correspondence postulate.

It seems that the above mentioned methodological idea of correspondence is very simple. Nevertheless, it is not trite at all. It always creates the fundamental problems to all theorists who attempt to find the new "observables", that is, the new observational magnitudes, and the new and more general theories than the old ones.

Let us now fix our attention on the following subject.

Correspondence relation of laws or theories

In order to realize the correspondence relation of laws (or theories), there must exist such a region of the new law (theory), called the limit region, that is defined by a certain mathematical limit. In this limit space of the new law (theory), for a definite value of a certain parameter (or parameters), called the correspondence parameter(-s), the quantitative and qualitative predictions of the new and the old laws (theories) should not be different. In addition, this condition is diminished considering the restricted precision of measurements. Just therefore it is required, that in the limit region the difference between the predictions of the new law (theory) and the old one, must be less than the accuracy of measurements. In such an asymptotic case, the new and the old laws (theories) are equivalent in the observational but not numerical and formal sense. Moreover, in the case of theories, when the incommensurability of mathematical structures takes place, the correspondence between theories needn't be full. Then the predictions of the old theory and the hmit case of the new one, may already differ quantitatively, even considerably, for a certain group of phenomena. In such a case, the new and the old theories, being observationally equivalent, and formally, and numerically non-equivalent for a certain group of phenomena, are observationally, and numerically, and formally non-equivalent for the others.

Next, it must be emphasized, that the above correspondence relation (between two laws or/and theories) regards only numerical, mathematical questions. Note, the above correspondence relation doesn't destroy in any way the fundamental differences between the quasi-entities defined within the context of new and old laws or/and theories.[20]

[20] Therefore, Bohr states "the asymptotic connection [between the quantum theory and classical mechanics and electrodynamics] as it is assumed in the principle of correspondence ... does not at all entail a gradual disappearance of the difference between the quantum theoretical treatment of radiation phenomena and the ideas off classical electrodynamics; all that is asserted is an asymptotic agreement of numerical statistical results." (Zs. Physik, Vol. 13, 1922, p. 144, translated by Feyerabend [1975] p. 276 fn. 132) *(verte)*

Now, let us express schematically the correspondence relation between the new and old laws of nature, and next the correspondence relation between the new and old theories.

Let, according to the old law defined within the context of the old theory, the old parametric function modelling the course of certain phenomena be defined. Similarly, let, according to the new law defined within the context of the new theory, the new parametric function modelling the course of certain phenomena more precisely than the old parametric function be defined. Then,

(1)
$$F_n(x_n, chp_n) \rightarrow F_{nl}(x_n, lchp_n) = F_o(x_o, chp_o)$$
$$chp_n \rightarrow lchp_n$$

indication n, o denotes the new and the old theories (read: defined within the context of the new, of the old theory), respectively:

x_n, x_o – corresponding variables;

chp_n – characteristic parameter which is explicitly defined within the context of the new theory;

chp_o – characteristic constant parameter which is hidden within the context of the old theory;

$lchp_n$ – limit value of a characteristic parameter of the new theory; that magnitude modells corresponding to it the characteristic constant parameter of the old theory;

$F_n(x_n, chp_n)$, $F_o(x_o, chp_o)$ – corresponding parametric functions;

$F_{nl}(x_n, lchp_n)$ – limit parametric function defined within the context of the new theory;

$chp_n \rightarrow lchp_n$ – limiting condition defined within the context of the new theory.

"The [asymptotic connection of Bohr's theory of quantum radiation with classical electrodynamics demanded by the correspondence principle] means that in the limit of large quantum numbers, where the relative difference between adjacent stationary states vanishes asymptotically, mechanical pictures of electronic motions may be rationally utilized. It must be emphasized, however, that this connection cannot be regarded as a gradual transition towards classical theory in the sense that the quantum postulate would lose its significance for high quantum numbers." (N. Bohr, *Atomic Theory and the Description of Nature*, Cambridge, 1932, p. 70)

In other words: "the principle of correspondence [between the quantum theory and classical mechanics and electrodynamics] must be regarded as a purely quantum theoretical law which cannot in any way diminish the contrast between the postulates [of Bohr's theory of quantum radiation] and electro-magnetic theory." (N. Bohr, *Atomic Theory and the Description of Nature,* Cambridge, 1932, p. 142, footnote)

Moreover, let us call the condition:

(2) $$F_{nl}(x_n, \mathrm{lchp}_n) = F_o(x_o, \mathrm{chp}_o)$$

the limit equivalence of the new and the old laws or the limit equivalence of the correspondence principle of the new and the old laws.

Let us schematically denote the above considered relation between laws in the short form:

(3) $$\langle F_n, F_o; \mathrm{chp}_n \rightarrow \mathrm{chp}_o \rangle.$$

In consequence, the pair of new and old theory, which functions are linked by the above mentioned correspondence relation of laws, are joined by the correspondence relation of theories. Let us denote this correspondence relation by the following expression:

(4) $$\langle T_n, T_o; \mathrm{chp}_n \rightarrow \mathrm{chp}_o \rangle,$$

T_n, T_o - new and old theories.

Laws, theories which are linked by a certain correspondence relation, we called ‘correspondence laws’ and ‘correspondence theories’, respectively.

Now, let us list some important features of new and old laws or theories linked by certain correspondence principle. Firstly, a new law or theory is more general then an old one. Secondly, the mathematical form of a new law, or theory is chosen in such a way to imitate for certain limiting values of certain characteristic parameters, called the correspondence parameters, a part of fundamental and of characteristic features of an old law or theory expressed in the language of mathematical functions. Thirdly, for the above mentioned limit values of correspondence parameters new and old laws or theories are observationally equivalent, and for non-limit values, they are observationally nonequivalent. But let us notice, in the case of theories, such observational equivalence needn’t take place for all modelled phenomena. Fourthly, laws, theories linked by certain correspondence principle are ontologically and notionally incommensurable. However these laws, theories are not uncomparable, but on the other hand, they are mutually irreducible.[21]

[21] In other words, Kuhn and Feyerabend are right when emphasize: (1) the ontological and notional incommensurability of pairs of theories such as the special theory of relativity and classical mechanics, and (2) the mutual irreducibility of such pairs of theories. On the other hand, they are fault when, for such reasons, reject the correspondence principle of theories leaving out the issue of the limit correspondence of functions defined within the context of the old and new theories.

The **typical examples** of the correspondence relations between theories or laws there are, among others, relations linking:

(i) Newton's dynamics and theory of gravitation and Kepler's laws; Newton's dynamics and theory of gravitation and Galileo's kinematics [Newton 16871;

(ii) the kinetic theory and thermodynamics ($N \to \infty$, where: N – number of atoms or molecules), [Bernoulli 1748, Claussius 1848, Maxwell 1856];

(iii) the wave and geometric optics ($d/\lambda \to \infty$, where d – linear dimension, λ – length of wave) [Young, Fresnel 1804–1818];

(iv) Planck's law of radiation and Wien's law ($h\nu/kT \to \infty$, h, k – Planck's and Boltzmann's constants, ν – frequency, T – temperature) [Planck 1900];

(v) the special theory of relativity and classical mechanics ($v/c \to 0$, where v – speed, c – speed of light), [Einstein 1905–1907];

(vi) the general and special theory of relativity ($g_{\mu\nu} \to 0$ for $\mu \neq \nu$ and $g_{\mu\nu} \to 1$ $\mu = \nu$, where $g_{\mu\nu}$ the so-called gravitational potential); general theory of relativity and classical mechanics ($g_{\mu\nu} \to 0$ for $\mu \neq \nu$ and $g_{\mu\nu} \to 1$ $\mu = \nu$ and $v/c \to 0$) [Einstein 1907–1919].[22]

(vii) the law of radiation of Bohr's model of atom and the classical law of radiation ($n \to \infty$, where n-quantum number) [Bohr 1913–1923];

(viii) the quantum and classical mechanics ($h \to 0$, h – Planck's constant), [Heisenberg, Schrödinger 1924–1927];

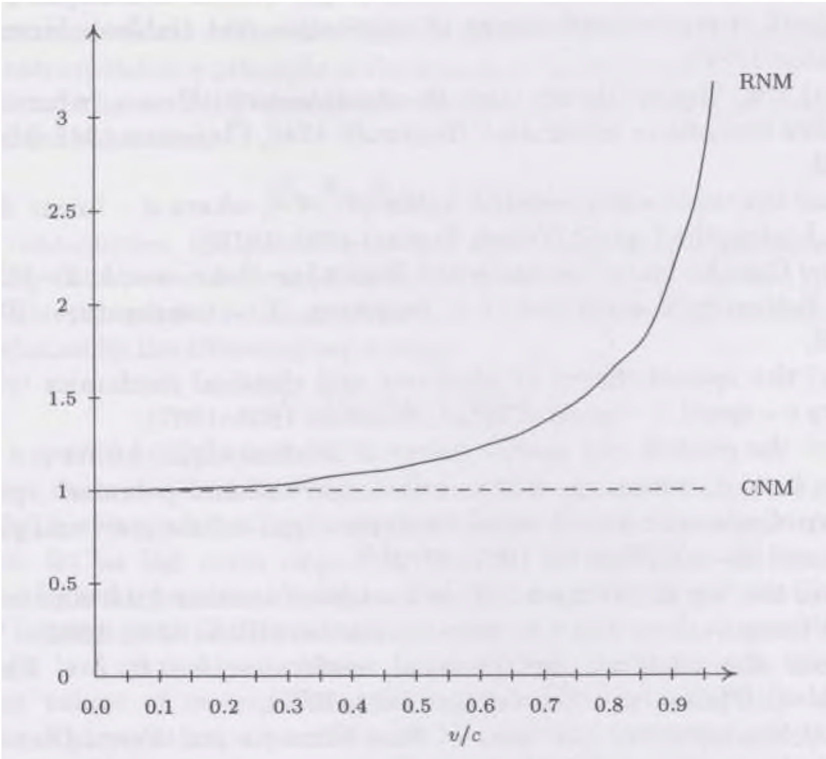
(ix) the quantum statistics of Bose-Einstein and Fermi-Dirac, and classical statistic of Boltzmann ($h\nu/kT \to \infty$, h, k – Planck's and Boltzmann's constants, v - frequency, T - temperature) [Einstein 1907], [Bose, Einstein 1924, 1925] and [Fermi, Dirac 1926].

Let us now illustrate on the below graph a correspondence relation of laws or theory in the special case of the relation linking two functions: the relativistic normalized mass (RNM) defined within the context of the Special Theory of Relativity (SR) and the classical normahzed mass (CNM) defined within the context of Classical Mechanics (CM):

$$\text{RNM} \equiv m_{\text{SR}}/m_0 = (1 - (v/c)^2)^{-1/2} \quad \text{and} \quad \text{CNM} \equiv m_{\text{CM}}/m_0 = 1. \tag{5}$$

[22] The names of authors and dates given above are listed without care on historical details. In fact, the detailed historical genesis of every mentioned correspondence relation was very complicated and there were some other co-authors of these methodological relations.

Relativistic and classical functions of normalized mass: RNM and CNM



The Real and Imaginary Correspondence Principles

Note, now, that the correspondence relation (of laws or theories) is interpretated by scientists in a real way or only in a maginery one. Therefore it is proper to differentiate between two types of correspondence relations according as the characteristic parameter of the new theory
is interpreted as a physical variable – v_n or a physical constant – c_n.

Let us call initially the first case, the correspondence relation $v_n - c_o$. In this case, the limiting condition:

(6) $$v_n \to c_o$$

and the correspondence relation:

(7) $$\langle F_n, F_o; v_n \to c_o \rangle$$

do model physical processes occurring in the world seen from the point of view of the new theory. Therefore we may say, that this relation occurs in the **real world.** Of course, saying this, we remember, that for physicists the "real world" is not the ultimate reality. Taking into account that after Bohr such type of correspondence relation is called the correspondence principle, we named it the **real correspondence principle.**

Let us call initially the second case, the correspondence relation c_n - c_0. In this case, the limiting condition:

(8) $$C_n \rightarrow C_o$$

and the correspondence relation:

(9) $$<F_n, F_o; c_n \rightarrow c_0>$$

do not model any physical processes occurring in the world seen from the point of view of the new theory. Therefore one may say, that this relation occurs in the **imaginary world.** Hence this kind of correspondence relation may be named the **imaginary correspondence principle.**[23]

In addition, note, that when the correspondence parameter of the new theory is chosen in the form of a fraction: $^{u}{}_{n}/c_n$, then the limiting condition may be, and, in fact, is, interpreted by scientists in two different ways, namely, as acting in the real world and or as acting in the imaginary world. It is, for instance, the case of the correspondence relation between the STR (Special Theory of Relativity) and the CM (Classical Mechanics).

Scientific Revolutions and the HDMCT

Within the context of the HDMCT, there is defined the simple condition of a scientific revolution. If a certain correspondence principle of theories is realised (and theories are based on well measured data), a scientific revolution occurs in the so-called exact sciences.[24] This revolution, accord-

[23] In this place, let us refer to two delightful books by G. Gamow: *MR Tompkins in Wonderland or stories of c, G, h* and *MR Tompkins explores the atom,* Cambridge England, University Press, 1953. In addition, let us note, that Gamow's intellectual experiments with the alteration of the values of the characteristic constants it is the standard and necessary thing for every theorist. Since, though, the concrete values of the constants of any theory are determined by measurements, nevertheless, testing the power of a framed theory, the nonrealistic values of the characteristic parameters are used. Scientists do it especially to see clearly the qualitative and quantitative consequences which result from an analysed theory.

[24] In this important point, I agree both with physicists as, for instance, Heisenberg [1969] p. 230–232, and historians of science as Кудрявцев [1973] p. 79–81, and philosophers of science as Krajewski [1974] p. 135–136, [1977] p. 88–91, [1985] p. 294–298.

ing to the importance of these theories, is global (the macro-revolution) or local (the micro-revolution). Moreover, the more mathematical languages of the new and the old theories will differ the deeper will be this revolution.

But, it should be emphasized, that this last statement is not a necessary condition of a scientific revolution. We think so, since, for instance, the special theory of relativity, which is linked with the classical mechanics by the known correspondence relation, is the classical example of the scientific revolution. Though, these theories do use, in principle, the same mathematical language, that is, the differential calculus.

Moreover, note that, within the context of the HDMCT, "a scientific revolution" is, in principle, very conservative on the level of regularities of phenomena. On the other hand, it may be very revolutionary on the levels of quasi-entities and assumed mathematical language.[20]

The HDMCT - the method of creating and developing of the so-called exact sciences

Finishing this introduction to *the Hypothetico-Deductv. e Method of Correspondence Thinking* (HDMCT), it is proper to notice, that the importance of this method did not pass with Galileo, Newton. Fresnel, Young, Bohr,

[25] Thus Heisenberg states: "If we discussing revolutions in science, we ought to look more closely at what has been happening. Take Planck's quantum theory No doubt, you know that when Planck first tackled the subject he had no desire to change classical physics in any serious way. He simply wanted to solve a particular problem, namely, the distribution of energy in the spectrum of a black body. He tried to do so in conformity with all the established physical laws, and it took him many **years** to **realize** that this was impossible. Only at that stage did he out forward a hypothesis that did not fit into the framework of classical physics, and even then he tried to fill the breach he had made in the old physics with additional assumptions. That proved **impossible, and the** consequences of Planck's hypothesis finally led to a radical reconstruction of **all physics. But even** after that those realms of physics that can be described with the concepts of classical physics remained quite unchanged. In other words, only those revolutions in science will prove fruitful and beneficial whose instigators try to change as little as possible and limit themselves to the solution of a particular and clearly defined problem. Any attempts to make a clean sweep of everything or to change things quite arbitrary leads to utter confusion. In science only a crazed fanatic — for instance, the kind of man who maintains that he can invent a perpetual-motion machine - would **try to overthrow** everything, and, needless to say, all such attempts are completely abortive True. I don t know whether scientific revolutions can compared with social revolutions, but I suspect that even historically the most durable and beneficial revolutions have **been he ones** designed to serve clearly defined problems and which left the rest strictly alone." Heisenberg, [1971] p. 147-148)

Einstein or Heisenberg at all. Since, for instance, physicists and cosmologists, searching for a more general and more unified theory than the General Theory of Gravitation and Quantum Mechanics, still use this method.[26] Similarly, the specialists do, who investigate the theory of particles, the chaos and the other subjects which belong to the so-called exact sciences .

Therefore, it seems reasonable the following generalization. The HDMCT is the universal method by help of which the so-called exact sciences are created, and hence this method determines the progress of the so-called exact sciences.

But, this progress, seeing in the light of the HDMCT, is more subtle than described by the simplified vision of the cumulativism and anti-cu- mulativism, the dichotomous distinction: continuity and discontinuity, Kuhn's sociologism and psychologism, Feyerabend's methodological anarchism. It happens for the followin fundamental reason. Theories linked by a correspondence principle, though are mutually irreducible as onto- logically and notionally incommensurable (Kuhn, Feyerabend), nevertheless, are not uncomparable.

4. The Copemican issue

Keeping the above mentioned considerations in mind, let us now tackle an analysis of Copernicus' methodological thinking during his searches for

[26] See, for instance, the popular books: S. Weinberg, *The First Three Minutes: A Modem View of the Origin of the Universe,* Basic Books, Inc., New York 1977; Kaku M., Trainer J., *The cosmic quest for the theory of Universe,* 1987; S. W. Hawking, *A Brief History of Time: From the Big Bang to Black Holes,* Bantam Books, New York 1988; R. Penrose, *The Emperor's New Mind. Concerning Computers, Minds, and the Laws of Physics,* Oxford University Press, Oxford 1989; S. Weinberg, *Dreams of a Final Theory,* Pantheon Books, Newy York 1992; S. W. Hawking, *Black Holes and Baby Universes and Other Essays,* Bantam Books, New York 1993; P. Davies, *The Last Three Minutes. Conjectures about the Ultimate Fate of the Universe,* published in "Science Masters" Orion Publishing Group Ltd., "Science Masters", 1994; J. B. Barrow, *The Origin of the Universe,* published in "Science Masters", Orion Publishing Group Ltd., 1994. In addition, let us recommend the one text-book: B. F. Schutz, A *first Course in General Relativity*, Cambridge University Press, 1985; and the one scientific work: S. W. Hawking, W. Israel, (eds.), *Three Hundred Years of Gravitation,* Cambridge University Press, Cambridge, New York, New Rochelle, Melbourne, Sydney 1987.

[27] For instance, compare: R. F. Feynman, QED, *The Strange Theory of Light and Matter,* 1985; I. Prigogine, I. Stengers, *Order out of Chaos: Man's New Dialogue with Nature*, 1984; M. C. Gutzwiller, "Chaos kwantowy" ("The Quantum Chaos"), *Świat Nauki (the Polish edition of the Scientific American),* 1992, nr 3, p. 26-33; R. V. Jensen, "Quantum chaos", *Nature*, vol. 355, 23 Jan. 1992, p. 311—318.

the new and more general theory than Ptolemy's. But, it is needful to emphasize in this place, that my **foreunderstanding** of the different aspects of the Copernican issue is defined, firstly, by works of historians of astronomy especially: A. Birkenmajer, L. A. Birkenmajer, J. Dobrzycki, O. Neugebauer, J. R. Ravetz, G. Rosińska, E. Rosen, N. M. Swerdlow and R. S. Westman; secondly, by works of historians of science or/and philosophers of science especially: A. C. Crombie, P. Duhem, N. R. Hanson, A. Koyre, T. S. Kuhn and M. Markowski.

In order to realize our task, we will use Copernicus' *Commentariolus* (the latest 1 May 1514, but probably ca. 1507)[28], *The letter against Werner* (3 June 1524), *De revolutionibus* (1543) and Rheticus' *Narratio prima* (1540). When the above works are quoted in this article, the English translations made by Professor Edward Rosen will used:

Nicolaus Copernicus, Complete Works, vol. II, On The Revolutions, Edited by Jerzy Dobrzycki, Translated and Commentary Edward Rosen, Warsaw-Cracow, Polish Scientific Publishers 1978;

Nicolaus Copernicus, Complete Works, vol. Ill, Minor Works, Edited by Paweł Czartoryski, Translation and Commentary Edward Rosen with the assistance of Ema Hilfstein, Warsaw-Cracow, Polish Scientific Publishers 1978;

Georg Joachim Rheticus, "Narrratio prima" in Edward Rosen, *Three Copernican Treaties. ... The Narratio Prima of Rheticus,* translated with Introduction and Notes. Third edition, revised ... New York, Octagon Books 1971.

Before we will analyse Copernicus' methodology and some results obtained by help of its applying, let us notice the obvious fact for historians: Copernicus is the child of his own epoch. Therefore, let us consider at first the historical context of his knowledge.

4.1. The context of Copernicus' knowledge from astronomy and cosmology

As Nicolaus Copernicus (1473-1543)[29] studied in Cracow (1491-95), Bolonia (1496-1500), Padwa (1501-1503) and Ferrara (1503), and, for

[28] On the date of Commentariolus see L. A. Birkenmajer [1900] p. 17, 79-82, 387, 684; L. A. Birkenmajer [1924] p. 199-224, 352-355; Rosen [1971] p. 67 fa. 21; Swerdlow [1973] p. 429-431

[29] For Copernicus' biographies see Prowe [1883-1884], Birkenmajer, L. A. [1900], [1923], [1924], Rosen [1971] p. 313-408, Swerdlow, Neugebauer [1984] vol. I, p. 3-32, the popular biographies: Wasiutyński [1937], Kesten [1945], Koestler [1959].

instance, Albertus de Brudzewo (1445-1495) (Cracow) and, in some degree, Dominicus Maria de Novara (1454-1504) (Bologna)[30] were his teachers of astronomy,[31] it is not strange that his methodological consciousness of astronomical issues and their difficulties originate from the philosophical, astronomical, cosmological and physical tradition of those days.[32] Especially, Copernicus was acquainted with:

(1) the logical and methodological questions of astronomy — Copernicus knew the works of astronomers, especially of Claudius Ptolemy (90-168) and his commentators, and the works of Aristotle (384-322 B.C.) (not by chance called the Philosopher in the late Middle Ages and Renaissance);

(2) the cosmological and physical issues - he knew, firstly, the works of the Philosopher, secondly, the works of Plato (429-347 B.C.), some conceptions of Pythagoreans (6-4th C. B.C.) and Neo-Platonists and Neo-Pythagoreans, thirdly, the works of Averroes (1126-1198) (not by chance called the Commentator (of the Philosopher) in the late Middle Ages and Renaissance), fourthly, the works of the Buridan's school (John Buridan (c. 1300- after 1366), Nicole Oresme (c. 1320-1382), Albert of Saxony (1316-1390), Marsilius of Inghen (13 C.) and their commentators from the University of Cracow at fifteenth century;[33]

(3) the strictly astronomical questions - Copernicus knew the works by Ptolemy (90-168) and his commentators and successors, for instance, by Albertus de Brudzewo (1445-1495) (Cracow), Georg Puerbach (1423- 1461) and Regiomontanus (1436-1476) (Viena); especially, he knew about the trials of generalization of Ptolemy's primary system for the non-uni- form long-term alterations of values of Ptolemy's constants (the so-called issue of trepidation and the issue of the motions of the eighth sphere) made by: Thabit ibn-Qurra (836-901), al-Zarqal and the authors of the

[30] Copernicus, as known from Rheticus' *Narratio prima* was "non tarn discipulus, quam adiutor et testis observationum" (not so much the pupil as the asistant and witness of observations) of the learned Dominicus Maria Novara.

[31] On the Cracow fifteenth century astronomical center see Rosińska [1973], [1974]; for information on Copernicus' program of studies in Cracow, Copernicus' Cracow teachers and, especially, Albertus de Brudzewo, and next Domenico Novara see Birkenmajer [1924] p. 50-141 and Birkenmajer L. A. [1900] p. 424-48.

On Copernicus' studies in Italy and the Italian philosophical and scientific centres see: Vasoli [1975].

[32] About the list of books supposedly owned by Copernicus see Czartoryski [1978]. The list of the literature known by Copernicus was broader, of course. For instance, Copernicus knew the works of Aristotle and Averroes. On this subject see A. Birkenmajer [1963].

[33] See A. Birkenmajer [1963]; on the Cracow buridanism in fourteenth and fifteenth centuries see Markowski [1971].

Toledan Tables (c. 1080), the authors of the Toledian version of the *Alphonsine Tables* (c. 1272), John Linieres and John of Saxony — the authors of the Parisian version of the *Alphonsine Tables* (c. 1327), Georg Puerbach (1423-1461), Regiomontanus (1436-1476), Marcus Beneventa- nus (c. 1465-c. 1525), Jan Werner (1468-1528).[34] Moreover, theories of homocentric spheres which were developed by Eudoxus (ca.406-ca. 355 B.C.), Callipus (4th C.B.C.) and Aristotle (384-322 B.C.) were known by Copernicus in a some degree. And such a sort of theories revived in Italy during Copernicus' times, in works, for instance, of Girolamo Fracastoro (1478-1553) and Giovanni Baptisto Amico (1512-1538).[35]

Moreover, the works of the Maragha School's astronomers might be known by Copernicus by their conjectural extensive translations into latin, or by a conjectural diffusion of their fundamental conceptions in the Middle Ages or in Renaissance.[36]

Let us add here, that the Renaissance astronomy had four fundamental problems: (1) theory of the moon; (2) theory of the sun (3) planetary theory; (4) theory of the motion of the eighth sphere that is a theory of the long period phenomena.[37]

However, it must be emphasized, that the reform of the Julian calendar was the leading aim of the Renaissance astronomy.[38] That problem was especially connected with the second and forth issues.

[34] On this issue see L. A. Birkenmajer [1900], [901a], [1901b], Dobrzycki [1965],

[35] On this issue see L. A. Birkenmajer [1900], Swerdlow [1972], Bono [1995].

[36] On this subject see: Swerdlow, Neugebauer [1984] vol. I p. 41-48, 55, Bono [1995]. The exact examination of this issue is still one of the most important tasks of history of genesis of Copernicus' achievement.

[37] On these subjects see: *Theorica Lune, Solaris and Planetares, De Motu Octavae Sphaera* in: Puerbach, Regiomontanus, *The Epitome of Almagest* (1460-1463) printed in Venice 1496 in Georgius Puerbach, Theoreticae novae planetarum (Wien 1454), printed Venice 1499; Regiomontanus, *Disputationes inter Viennensem et Cracoviensem super Cremonensia in planetarum theoricas deliramenta* (1464) [where Vienniensis = Regiomontanus and Cracoviensis = Marcin Bylica (C.1433-C.1493)]; Albertus de Brudzewo, *Commen- tariolum Super Theoreticas Novas Planetarum Georgii Purbachii* (Cracow 1482), printed in Mediolan 1495.

On these subjects see, for instance: Birkenmajer [1900] p. 19—22, [1924] p. 88., Pedersen [1978] and Dobrzycki [1965].

[38] Such was the opinion of Regiomontanus (see Regiomontanus' letter to Bianchini in Swerdlow [1990] p. 170ff, 175ff), and of Rheticus whose *Narratio prima* (1540) begins from this subject and then considers the issue of theory of the motion of the moon and the theory of motions of planets (see Rheticus' *Narratio prima* in Rosen [1971] p. 111-133,136).

On this and related subjects see, for instance, L. A. Binkenajer [1924] p. 225-231, Dobrzycki [1964], Ravetz [1965], especially p. 47-55, Rosen [1971] p. 358-362, Felber [1973], O'Connell [1973], Wesolowskij, Belyj [1974] p. 190-197, Przypkowski [1975], Mercier [1977], Coyne, ... (eds.) [1983], Goldstein [1994],

4.2. Copernicus and the HDMCT

It is the undisputable truth, that contemporary history and philosophy of science, contributed extremely much to the understanding of Copernicus' astronomical works. Nevertheless, in my opinion, it is justifiable to maintain, that these history and philosophy created the over-simplified visions: the nonhistorical or/and nonphilosophical or/and nonmathemati- cal reading of these works. In consequence, it was impossible to pay **sufficient** attention to the fact, that Copernicus, searching for the new and more general theory than Ptolemy's, was sensitive to methodological issues of this branch.[39] In particular, the weak points of the "nonhistorical philosophy" and "nonphilosophical history" and "nonmathematical history and philosophy" of the so-called exact sciences did not allow to see, that Copernicus, while searching for the new and more general theory than Ptolemy's, had being using the idea of the correspondence postulate and the correspondence relation in a systematic way. Hence, the "nonhistorical philosophy", "nonphilosophical history", and "nonmathematical philosophy" of the so-called exact sciences could not see, that the Copernican methodology was exactly the same as the model method of modem times determined by Galileo, Newton, Einstein, Bohr, Heisenberg or Schrodinger's. Especially, Copernicus consequently and constantly used the method that we have called above the Hypothetico-Deductive Method of Correspondence Thinking.

Let us now prove that Copernicus really used the HDMCT. To simplify this task let us divide it into two stages. At first, we will want to demonstrate that Copernicus used the Hyphothetico-Deductive Method of the so-called exact sciences and next, that he also knew and used the Method of Correspondence Thinking. Trying to prove that Copernicus used the Hypothetico-Deductive Method, for brevity, we will fix our attention mostly on the issue of hypothesis.

4.2.1. Copernicus and the hypothetico-deductive method of the so-called exact sciences

Let us discuss some problems which are important for Copernicus.

[39] The works of: Birkenmajer [1901a], Birkenmajer [1936, 1953, 1963], Dobrzycki [1953] and Ravetz[1965] are the important exceptions. My conception is a continuation of such interests.

Copernicus’ awareness of the general methodological issues of mathematical astronomy and its fundamental difficulties

Sharing the Greeks’ persuasion about the existence of order and harmony of the Universe, Copernicus, in conformity with astronomical tradition, thinks, that there exist the laws, according to which the astronomical phenomena are governed. To discover these laws it is necessary: firstly, to make watchful observations and measurements of astronomical phenomena by means of special astronomical instruments, and secondly, using these data and the language of mathematics to formulate mathematical theories of astronomical phenomena. Hence, according to Copernicus and Greeks, astronomy is a mathematico-observational science. What is important, the astronomy cultivated in such a way, both in Copernicus’ and in the traditional (Plato’s and Ptolemy’s) opinion, is not only a part of mathematics but also “the consummation of mathematics” (On *the Revolutions,* B. I. Introduction, p. 7). But, though astronomy using mathematics is so perfect, this branch, in Copernicus’ opinion, is not free from some fundamental perplexities. Firstly, to be able to make numerically reliable observations and measurements of astronomical phenomena, which are indispensable to create correct mathematical theory of these phenomena, a long enough passage of time is necessary. Thus, such words we read in Rheticus’ *Narratio prima'.*

> In one respect, however, a burden greater than Ptolemy’s confronts my teacher (that is, Copernicus) [my addition]. For he must arrange in a certain and consistent scheme or harmony the series and order of all the motions and appearances, marshalled on the broad battlefield of astronomy by the observations of 2 000 years, as by famous generals. Ptolemy, on the other hand, had the observations of the ancients, to which he could safely entrust himself, for scarcely a quarter of this period. **Time the true god and teacher of the laws of the celestial state, discloses the errors of astronomy to us. For an imperceptible or unnoticed error at the foundation of astronomical hypotheses, principles, and tables is revealed or greatly increased by the passage of time** [my bold]. Therefore my teacher must not so much restore astronomy as build it anew. *{Narratio prima,* p. 131)

Moreover, formulating mathematical theories (= models) of astronomical phenomena, astronomers may choose in a different way the fundamental principles and assumptions of theories. Hence, mathematical astronomers by methodological necessity must analyse the following methodological issues:

The structure of the astronomical theory; cosmology and the hypothesis

In agreement with Greeks, Copernicus thinks that cosmology is the basis of astronomy. The cosmological principles and hypotheses constitute its parts.[40] The astronomical models, called by Copernicus astronomical hypotheses, are erected on this cosmological ground "as for the cornerstone". Let us emphasize at once, that not only astronomical models but also cosmology is deeply mathematical since they use a definite, mathematical language: Arithmetics, the Euclidian and Spherical Geometry, especially, the conception of the spheres, of the circles and of the uniform circular motion constitute its language.

Both to Copernicus and Ptolemy, cosmology is an ontology of astronomical theory, since it introduces fundamental entities of astronomical theory. By means of these entities, which are treated as the fundamental elements of the Universe, the astronomical phenomena are described and explained. It is demanded from cosmology firstly, to be non-contradictory or coherent; secondly, that the astronomical models based on this cosmology could give the simplest and the most economical description of astronomical phenomena.

According to Copernicus and the Greek astronomical tradition stemming from Plato's and Eudoxus' times, the most important principle of cosmology is the principle of absolute motion or the principle of regularity. This principle postulates that all celestial motions, that is all apparent changes of the position of stars on the firmament, should be represented by a certain combination of uniform revolutions of the circles or spheres around their proper centres.

But, according to astronomy of homocentric spheres (Plato, Eudoxus, Callippus and Aristotle, and the medieval Aristotelics, for instance: Averroes, Thomas of Aquino, Averoists and Thomists), all celestial spheres should turn uniformly around the same centre which is the centre of the Universe.

On the other hand, Copernicus, assumes in agreement with the Ptolemaic astronomy, that celestial spheres do not turn around the same centre. But, like Sosigenes, Siplikios, and AverroDs and their latin successors, he assumes that the proper centres cannot be the Ptolemaic points of equant.[41]

[40] Note that Copernicus, referring to Greeks, uses the term "hypothesis" only in the *De Revolutionibus*. Earlier, in the *Commentariolus*, he uses the synonymous terms "assumption" or "axiom".

[41] Sosigenes, Περι ανελι ττουσων (ca. 170 A. D.); Ibn al-Haytham, ει-*Shukuk 'ala Battlamyus – Daubts concerning Ptolemy* – (VIII c.); Averroës, *Commentum in quator libros*

The fundamental difference between Copernicus' cosmology and Ptolemy's consists in the fact, that they assume different cosmological hypotheses. Ptolemy's cosmology stems fromcosmology of Eudoxus, Callippus and Aristotle. Neglecting for the present some details let us only say, that, according to Ptolemy, the earth is immobile and placed in the centre of the Universe, and the stellar sphere, being mobile, has two motions:'the daily rotation around polar axis, and the long-period of 36000^{ey} (ey = Egyptian years of 365 days) around the axis of ecliptic.

On the other hand, Copernicus' cosmology stems from three sources: from the cosmological speculations of Pythagoreans, secondly, from the paradigmatic Aristotle's cosmology and finally from its critique made by the Buridan's school (John Buridan, Nicole Oresme, Albert of Saxony, Marsilius of Inghen).[42]

De caelo et mundoAristotelis.

As far as we know, it is Sosigenes (ca. 130) who was first philosopher, who, contrary to Aristotle's physics, thought that the celestial spheres do not revolve around the same center but about their proper centers. Moreover, he was sure that this new axiom is more true than the old axiom of geocentric astronomy. Moreover, contrary to Ptolemy, but without quoting him, Sosigenes emphasized that all celestial spheres must revolve uniformly around their proper centers. It is important, that the fundamental ideas of this book, being quoted extensively by Simplikios *In Aristotelis de caelo libros commentaria* and more or less extensively by Thomas of Aquino and Averroes commentaries of Aristotle's *De caelo or Metaphysica,* were known very well in the Middle Ages and Renaissance.

Copernicus knew this question from, for instance, Albertus de Brudzewo's *Commentary on Puerbach's Theoricae novae,* Averroes' *Commentaries on Aristotle's Metaphysics and on Aristotle's De coelo* et mundo, and certainly some Averroists' and Buridanists' commentaries to Aristotle (see Albertus de Brudzewo, *Commentariolum super Theoricas Novas planeta- rum Georgii Puerbachii,* Editit L. A. Birkenmajer, Cracovie, 1900.

For further information on the whole this subject see: Kraftz [1973], On the Averroes, Averroism and its reception by Byridanists see: Birkenmajer [1900] p. 89-98, [1924] p. 90-93; Rosen [1971] p. 194-195, f. 251; Markowski [1971], Markowski [1973] in Kurd- zialek, ... (ed.) [1973] p. 13-31.

This issue was very important not only for Copernicus (1473-1543) above mentioned philosophers but also for Ibn al-Haytham (965-ca. 1040) (Cairo) and the astronomers of the Maragha School (XIII-XIV C. north-western Iran).

As far as we know, it is Ibn al-Haytham who made the first explicit critique of physics of Ptolemy's theory. Especially, he condemned Ptolemy's use of equants as the violation of the principle of uniform circular motions. But this work was not translated into latin.

On Ibn al-Haytham see: Kraftz [1973], M. Shramm, *Ibn al-Haythams Weg zur Physik*, Wiesbaden, 1963; Ibn al-Haytham, *sx-Shukuk 'ala Battlamyus (Dubitationes in Ptole- maeurri)* edited by A. Sabra and N. Shehaby, Cairo 1971; S. Pines, Ibn al-Haytham's Critique of Ptolemy, *Proceedings of the Tenth International Congress of the History of Science,* Ithaca, 1962, Paris 1964, p. 547-550; B. R. Goldstein in *Journal for the History of Astronomy* 4, 1973, p. 138.

On Ibn al-Haytham and the astronomers of the Maragha School (13-14th C. northwestern Iran) see Swerdlow, Neugebauer [1984] vol. I. p. 44-48.

Therefore also, Copernicus' cosmology inherits the language and some issues of physics and cosmology of the Philosopher, that is Aristotle. The following examples illustrate this thesis: the notions and the issues of the natural motions, of the natural place in the universe and of the gravitation of bodies. But going on Buridanists footnotes, Copernicus uses these conceptions otherwise. While, for instance, for Aristotle, gravitation is the tendency of heavier bodies to go to the centre of the universe, for Buridanists and Copernicus himself, it is simply the tendency of heavier bodies to go the centre of spherical masses of matter.

> I believe that gravity is nothing but a certain natural desire, which the divine providence of the Creator of all things has implanted in parts, to gather as a unity and a whole by combining in the form of a globe. This impulse is present, we may suppose, also in the sun, the moon, and the other brilliant planets, so that through its operation they remain in that spherical shape which they display." {*On the Revolutions,* B. I, ch. 9, p. 18)

Moreover, to explain motion of the celestial spheres, Aristotle introduces the conception of the intelligences, spiritual movers, but Buridanists, the more general and more concrete conception of impetus. Buridan in his *Qestiones de Caelo et Mundo* states:

> One does not find in the Bible that there are Intelligences charged to communicate to the celestial spheres their proper motions; it is permissible then to show that is not necessary to suppose the existence of such Intelligences. One could say, in fact, that God, when he created the universe, set each of the celestial spheres in motion at it pleased him, impressing on each of them an *impetus* which has moved it ever since. God has therefore no longer to move these spheres, except in exerting a general influence similar to that by which he gives his concurrence to all phenomena. Thus he could rest on the seventh day from the work he had achieved, confiding to created things their mutual causes and effects. These *impetus* which God impressed on the celestial bodies have not been reduced or destroyed by the passage of time, because there was not, in celestial bodies, any inclination towards other movements, and there was no resistance which could corrupt and retrain these *impetus.* All his I do not give as certain; I would merely ask theologians to teach me that all these things could come about [...].[43]

[42] This critique was known and discussed by the astronomers and philosophers of the Arts Faculty at the University of Cracow in the fifteenth century. For example, Albert of Saxony's *Questiones in De Caelo of Aristotle's* was the one of the most important contemporary university books (in handwritten copies) used in Cracow (Markowski [1971]). Moreover, this critique was known also in the philosophical centres of Italy, since, for instance, at 1497 in Venice, soon after Copernicus arrival to Italy, it was printed the above mentioned work of Albert of Saxony.

[43] Translated by Crombie [1952] ,2, p. 69/70 from the Latin text published by Anneliese Maier, Zwei Grundprobleme der Scholastischen Naturphilosophie, Rome, 1951, p. 211-212.

In general, the *impetus* is an internal power of a body that enables to move it in the direction endowed by a mover during his action to body. The measure of the *impetus* is proportional to velocity and the quantity of matter. In the absence of resistance from the medium the *impetus* is constant. This is just the case of the motions of celestial spheres. Copernicus accepts such an understanding of the idea of *impetus* and, in the *De revolutionibus* (B.I, ch. 8, p. 17), uses the equivalent term *causa indeficiens* that is the constant physical cause, the unfailing cause. Moreover, with Buridanists, he knows also that the motion of falling heavier bodies is accelerated. But contrary to Buridanists Copernicus says nothing about the Aristotelian intelligences.

Next, Buridan and Oresme, using the idea of the relativity of local motion, considered only one macroscopic motion of the earth[44], that was the diurnal one. Contrary to them, Albert of Saxony considered the other possibility. Firstly, he let the daily motion of the stellar sphere be. Secondly, using the idea of the relativity of the local motion he ascribed to the earth, being in the middle of the universe, two long-period motions; namely, the uniform rotation of 36000^{ey} around the ecliptic's axis but with the opposite direction like it was primary in the Ptolemy's theory, and the non-uniform trepidation.[45]

But, Copernicus considered three (basic) motions: diurnal, annual and the so-called *motus declinationis*. It is clear that framing these motions by applying the principle of relativity of motion, Copernicus went on Buridanists' footnotes. But, as far as we know, Buridanists they did not consider the annual motion of the earth.

Moreover, the mobility of the earth has different status in Buridanists' and Copernicus' considerations. Let us emphasize, that the philosophers of the Buridan's school, similarly to Ptolemy (Almagest, B.I.Ch.7), did not treat this motion as real. Discussing with Aristotle's physics Buridanists pointed, that on the ground of observations, it is impossible to prove conclusively whether the earth or the stellar sphere moves. Oresme, for instance, in his *Le Livre du ciel et du monde* (1377) gave, among other things, such arguments[46]:

[44] We say 'the macroscopic motion' because Buridan considered also the fluctuation motions of the centre of gravity and the geometrical centre of the earth due to the erosion of the earth, the tides and the winds. In consequence, the centre of the Aristotelian world that is the centre of gravity of the earth, must fluctuate. Even from the point of view of the theoretical physics which we learn in University nowadays, such conception is very ingenious.

[45] See P. Duhem, *Le System Du Monde*, vol. 9, p. 359-362.

[46] Oresme, *Le Livre du del et du monde,* B. II, ch. 25 fol 138b-144b p. 521-537) ed. by A. D. Menut, A. J. Denomy, C. S. B, transl. A. D. Menut, Madison 1968.

(1) Contrary to Aristotle: The apparent rotation of the sky around the polar axis is not conclusive argument at all, since according to Witelo's *Perspective* the local motion is relative, and depends, using modem language, of the reference frame; it is this fact that was illustrated clearly by Witelo on the case of two boats.

(2) Contrary to Aristotle: If the earth were have diurnal motion, it would not cause a continuous wind blowing from the east at all, since not only the earth moves but also with the water and air.

(3) Contrary to Ptolemy: If the earth were have diurnal motion, it would have not to cause that an arrow or stone thrown vertically upwards from a certain place, would fall to other place being far to the west (what, according to Oresme, is not observed), since an arrow or stone, has two motions; one circular with the earth and second vertical.

Thus it results from considerations of Oresme what follows (p. 531.):

> The astronomical tables of the heavenly motions and all the books would remain as true as they are at present, save that, with respect to diurnal motion, one would say that it is apparently in the heavens, but actually in the earth; no other effect would follow or result from one theory more than from other.

Because two conceptions of the diurnal motion of the earth and diurnal motion of the sky do not change nothing in the apparent regularities of phenomena, one may only argue for the possibilities of the motion of the earth. However, in Oresme's opinion, it is possible to give several persuasive arguments or reasons favorable to the theory that the earth moves. Namely, it is much more reasonable to assume that the earth, and not the sky, makes rotation in a natural day, since:

(i) in such case all principal motions of celestial bodies should go in one direction from west to east (p. 533):

> [...] according to the philosophers and astronomers, it cannot be that all bodies move from west to east; but, if the earth moves as we have indicated, then all proceed alike from west to east — that is, the earth by rotating once around the zodiacal poles: the moon in one month, the sun (sic!) in one year, Mars in approximately two years, and so on with the other bodies. It is unnecessary to posit in the heavens other primary poles or two kinds of motion, one from the east to the west and the other on different poles in the opposite direction, but such an assumptions definitely necessary if the heavens move with diurnal motion.

(ii) according to Aristotle (De caelo, ch.V), the rest is nobler than the motion and the sky is much more perfect and nobler than the earth.

(iii) then the period of rotation of celestial bodies will increase with, the distance taken from the center of the earth (p. 535):

> Proceeding in this manner, the higher heavens make their revolution more slowly yet, [...] and this process continues up to the heaven of the fixed stars, which is motionless or makes its revolution very slowly, according to some in thirty-six thousand years or one degree in one hundred years.

(iv) it is more economical assumption (p. 535):

> All philosophers say that an action accomplished by several or large-scale operations which can be accomplished by fewer or smaller operations is done for naught. And Aristotle says in Chapter Eight of Book I [of De caelo] that God and nature do nothing without some purpose. Now, if it is true that the heavens move with diurnal motion, it becomes necessary to posit in the major bodies of the universe and in the heavens two contrary kinds of movements: one from east to west and the other from the opposite direction, [...] And with regard to diurnal motion, we must assume an excessively great speed; for, if we consider thoughtfully the height or distance of the heavens, their magnitude, and the immensity of their circuit, mindful that this circuit is traveled in but one day's time, no man could imagine or conceive how marvelously swift and excessively great, how far beyond belief and estimation their speed must be. Since, then, all the effects we see could be movement of the heavens aa smaller operation, namely, the diurnal motion of the earth, a very small body as compared with the heavens, and by so doing avoid the multiplication of operations so diverse and so outrageously great, the God and nature must have created and arranged them for naught; [...] Assuming the entire havens to move with daily motion and, in addition, assuming the eighth sphere to have a different motion, as the astronomers believe, then we must necessarily posit a ninth sphere moving only with diurnal motion. However, if we assume that the earth moves as stated above, then the eighth heaven moves with a single slow motion and it is consequently unnecessary to imagine a ninth natural sphere invisible and starless; for god and nature would have made this ninth sphere for naught since by another method, i.e., assuming that the earth to move, everything can exactly as it is.

Regarding the issue of simplicity, economy of nature, etc, the diurnal motion of the earth is more probable than the diurnal motion of the sky. However, because these two different conceptions explain the apparent phenomena equally well, the argumentation based on the simplicity, economy of nature, etc, is, according to Oresme, not conclusive but persuasive.

But, in spite of this argumentation, Oresme and all others philosophers of the Buridan's school, feel sure about the immobility of the earth and mobility of the stellar sphere, because these facts are certified by the authority of the Holy Scripture. Thereby their considerations about the mobility of the earth had in reality the status of the exercises of logic. Moreover, they were used for the apologetic aims.[47]

[47] After evaluating the evidence for the possibility of the daily motion of the earth

On the other hand, Copernicus uses the above mentioned Buridanistic critique of Aristotle's physics in a'positive way. First of all, contrary to Buridanists, he assumes three fundamental motions of the earth and accepts them as the hypothetical basis of his astronomical theory. However, this hypothetical celestial physics is only the qualitative beginning of his theory, since the sense of the hypothetical earth's motions is determined within the context of the whole Copernicus' theory. It is defined by the constructing of the astronomical models which, being based on Copernicus' hypothetical cosmology, use the language of Arithmetics, the Euclidian and Spherical Geometry. Finally, Copernicus is sure that the motions of the earth are proven, because, the predictions of his astronomical models are, in principle, consistent with observations and measurements. This is one of the pivotal Copernicus' thoughts. Therefore it is proper to analyse this issue more precisely.

In the first place, let us recall, that Copernicus' searches for a more general theory than Ptolemy's, starts from the methodological critique of the earlier astronomy based on the hypotheses of the immobile earth and the mobile stellar sphere. For instance, Copernicus criticizes Eudoxus, Callipus and Aristotle's homocentric astronomy for the fundamental, very big quantitative defects, because their theories can not predict changing brightness of the planets. On the other hand, he thinks that, the basis of the Ptolemaic astronomy (of deferents, eccentrics and epicycles) is defected by the existence, within its context, the fundamental qualitative fault. In Copernicus' opinion, namely, the using of the conception of *punctum aequans* (the point of equality, equant) and *circum aequans* (the circle of equality) is contradicted with the first principle of astronomy, that is the principle of absolute motion. This principle states that the stellar motions must be uniform relative to the proper centers. But, though, Ptolemy accepted this principle explicitly, nevertheless, many non-uniform motions in his theory were considered as uniform on the only fictitious circle of equality with respect to the abstract point called the point of equality. Therefore Copernicus states:

N. Oresme concludes: "However, everyone maintains, and I think myself, that the heavens do move and not the earth: For God established the world which shall not be moved, in spite of contrary reasons because they are clearly not conclusive persuasiones. However, after considering all that has been said, one conclude then believe that the earth moves and not the heavens, for the opposite is not clearly evident. Nevertheless, at first sight, this seems as much against natural reason as, or more against natural reason than, all or many of the articles of our faith. **What I have said by way of diversion or intellectual exercise can in this manner serve as a valuable means of refuting and checking those who like to impugn our faith by argument** [my bold]." (ibidem)

> Hence this sort of notions seemed neither sufficiently absolute nor sufficiently pleasing to the mind. *(Commentariolus,* p. 81)

In other words, in Copernicus' opinion, it is necessary to reject equants for the physical consistency of the bases of astronomy.[48]

Moreover, though, the Ptolemaic astronomy gives the considerably more precise predictions than the astronomy of homocentric spheres (since it predicts the changing brightness of the planets), nevertheless, in Copernicus' estimation, it is not quantitatively perfect. Listing these difficulties (among other things: the impossibility of determining a constant length of year, "the structure of the universe and the true symmetry of its parts"), Copernicus concludes:

> Hence in the process of demonstration or "method", as it is called, those who employed eccentrics are found either to have omitted something essential or to have admitted something extraneous and wholly irrelevant. This would not have happened to them, had they followed **sound principles (certa principia). For if the hypotheses assumed by them were not false, everything which follows from their hypotheses would be confirmed beyond any doubt** [my bold]. (On the *Revolution,* Preface, p. 4)

Taking that into consideration, it is obvious, that Copernicus must search for certain new hypotheses. Reading philosophical texts of the earlier philosophers he finds, that Pythagoreans ascribed to the earth daily and annual motions.

> And even though the idea seemed absurd, nevertheless I knew that others before me had been granted the freedom to imagine any circles whatever for the purpose of explaining the heavenly phenomena (ad demonstrandum phenomena astrorum). Hence I thought that I too would be readily permitted to ascertain whether **explanations sounder (firmiores demonstrationes) than those of my predecessors could be found for the revolution of the celestial spheres on the assumption of some motion of the earth** [my bold]. *(On the Revolutions,* Preface, p. 5)

But note, Copernicus says about the "sounder explanations" *(firmiores demonstrationes)* but he does not maintain that these explanations are the best ones or the ones and only. Moreover, noticing, that the hypotheses of the daily motion of the stellar sphere and immobility of the earth are, in some degree, incoherent with the core of Aristotle's physics, Copernicus concludes:

[48] However, it must be emphasized that Copernicus' rejection of equants is very refined problem. See Neugebauer [1968], Swerdlow [1973] p. 467—171, Swerdlow, Neuge- bauer [1984] p. 292-297.

> You see, then, that all these arguments make it **more likely (probabilior)** [my emphasis] that the earth moves than it is at rest. This is especial true of the daily rotation, as particularly appropriate to the earth. *(On the Revolutions,* I, ch. 8, p. 17)

Noticing that the hypothesis of the annual motion of the earth links many phenomena harmoniously, he concludes:

> This should be admitted, I believed, in preference to perplexing the mind with an almost infinite multitude of spheres, as must be done by those who kept the earth in the middle of the universe. On the contrary, **we should rather heed the wisdom of nature. Just as it especially avoids producing anything superfluous or useless, so it frequently prefers to endow a single thing with many effects** [my emphasis]. *(On the Revolutions,* I, 10, p. 20)

Hence, according to Copernicus, astronomical phenomena are better explained by the hypotheses of the daily and annual motions of the earth, and by the immobility of the stellar sphere than by the hypotheses of the daily motion of the stellar sphere, annual motion of the sun and by the immobility of the earth. Just therefore Copernicus' hypotheses are better. They are better because are more economical. Since Copernicus' astronomical models are based on the more economical cosmological hypotheses, Copernicus finds the new and the deeper order of astronomical phenomena. He sees the greater harmony and symmetry in the Universe.

> In this arrangement, therefore, **we discover a marvelous symmetry of the universe, and an established harmonious linkage between the motion of the spheres and their size, such as can be found in no other way. [...] All these phenomena proceed from the same cause, which is in the earth's motion** [my bold], *(On the Revolutions,* I, 10, p. 22)

However, though, this harmony and symmetry are greater than in the case of hypotheses of the immobility of the earth, and of the mobility of the stellar sphere, nevertheless, let us repeat it once again, in Copernicus opinion, his demonstrationes have not the status of the absolute, ontological or logical certainty. On the other hand, it does not mean at all, that they are false, or, that they are a mere fiction. To comprehend this issue better, let us see what Rheticus, the only one Copernicus' disciple, thought about it:

> Aristotle says: "That which causes derivative truths to be true is most true" *(Metaphysics* 993b 26-27). Accordingly, my teacher decided that **he must assume such hypotheses as would contain causes capable of confirming the truth of the observations of previous centuries, and such as would themselves cause, we may hope, all future astronomical predictions of the phenomena to be found true** [my bold]." (Rheticus, *Narratio prima*, p. 142—143)

From the above considerations it results that, according to Copernicus, the coherent cosmological hypotheses which lay at the basis of an astronomical theory, called the quasi-entities in the HDMCT, are not convenient fictions only. Similarly, theories based on such hypotheses are not convenient mathematical schemes only. The above statements result from the fact, that the sound, coherent hypotheses, defined within the context of a whole theory, carry the important, non-trivial information about the net of relations and a mathematical order which exists in the Universe.[49]

Continuing our subject, it is proper to add, that the principle of the inner perfection (or the coherence of cosmology), and the principle of the consistence of the theoretical predictions with observations, they are not enough to prejudge conclusively about the ultimate shape of astronomical models assuming the definite cosmology. Discussing three kinematically equivalent models of the long-period motion of the earth' abside line Copernicus notices:

> **Since many arrangements lead to the same result, I would not readily say which one is real, except that the perpetual agreement of the**

[49] Of course, such Copernicus' understanding of the status of the cosmological and astronomical hypotheses and mathematical (geometrical) models (1) was very far from the pure instrumental character endowed to mathematical astronomy by Proclus, Simplicius, Averroes, Aquinas and their scholastic followers and (2) was not based on the simple Aristotelian syllogism and especially on the convertibility of the reasons and conclusions or causes and effects. Contrary, the conception of syllogistic reasoning was the base of the critical estimate of the status of Ptolemy's and Ptolemaic and later Copernicus' astronomical theories. On the issue of syllogism and relating questions regarding medieval and Renaissance discussions on the methodology of astronomy see professor Crombie's splendid work Crombie [1994], vol. I p. 529-543.

It is proper to notice an another aspect of the issue under discussion. Copernicus' considerations regarding the status of the hypotheses and mathematical (geometrical) models were not completely anything unique. It is not an indifferent fact at all, that they were appreciated, for instance, by the authors of the subsequent scientific revolutions: Kepler (three Kepler's laws), Galileo (kinematics), Newton (dynamics and theory of gravitation), Poincare (Special theory of relativity) and Einstein (Special and the General theory of relativity). (For appropriate quotations regarding Kepler, Galileo, see for instance the studies of Grant [1962a, b]; **H.** Poincare, *Value of Science* (English translation, reprint, Dover, 1958), 140-141.) For instance, making sure that Osjander forged the Preface to the *De revolutionibus*, Kepler wrote: **"It is a most absurd fiction, I admit, that the phenomena of nature can be demonstrated by false causes. But this fiction is not in Copernicus. He though that his hypotheses were true, no less than did those ancient astronomers of whom you speak. And he did not merely think so but he proves that they were true. As evidence, I offer this work** [that is, *Astronomia nova* — my addition]." (in Rosen [1971] p. 24 n. 68.)

Moreover, Copernicus was not the first scientist (or, to be historically exact, philosopher-mathematician) at all, who understood the above mentioned methodological issues in such way — similarly earlier Ptolemy did in his *Almagest* and *Hypotheses Planetares.*

> **computations and phenomena (numerorum ac apparentium perpetua consonantia) compels the belief that is one of them.** *(On the Revolutions,* III, 20, p. 164)

Thus, it appears that Copernicus' knew the so-called principle of undetermination of theory by empirical facts.[50]

However, in spite of this difficulty, from the methodological point of view, it is justifiable to agree with Copernicus who writes in the *Commentariolus:*

> **Accordingly, lest anybody suppose that, with the Pythagoreans,** I have asserted the earth's motion gratuitously, he will find strong evidence here too in my exposition of the circles. For, the principal arguments which the natural philosophers attempt to establish the immobility of the earth rest for the most part on the appearances. All these arguments are the first to collapse here, since I undermine the earth's immobility as likewise due to an appearance. *(Commentariolus* p. 82).[51]

From the HDMCT's point of view, Copernicus proved the motion of the earth. He did it, because (1) he framed more coherent cosmology than Aristotle's and Ptolemy's; (2) on this ground, he created the qualitatively (in the mathematical sense) more economic models than Ptolemy's; (3a) the predictions of Copernicus' models were more consistent with phenomena than, Ptolemy's (c.140); (3b) the predictions of Copernicus' models with the redetermined parameters by Reinhold's *Prutenic Tables* (1551) were, according to many astronomers until to circa 1610, superior to the

[50] However such knowledge was not a fruit of his genius at all, but a part of standard philosophical and especially astronomical paradigm of those days. Since, as it was referred in Ptolemy's *Almagest,* Hipparchus discovered the fact that an eccentric and chosen properly the pair of a deferent and an epicycle are geometrically equivalent. And just this theorem is the basis of the above quoted Copernicus' statement. Moreover, the Medieval and Renaissance philosophers often discussed the logical consequences of this mathematical theorem. On this subject see Crombie [1994], vol. I, p. 529-543; Duhem [1969],

[51] Reading this quotation from Copernicus' *Commentariolus* it is worthy remembering about Ptolemy's conclusion from the first book of the *Almagest.* Just there, Ptolemy, after the considerations regarding the status of astronomy treated as the mathematical branch, the order of the theorems and cosmology (the spherical shape of the universe and the earth, the earth's immobility and central position in the universe, and mobility of the fixed stars) says:

"It will be sufficient for these hypotheses, which have to be assumed for the detailed expositions following them, to have been outlined here in such a summary way since they will finally be established and confirmed by the agreement of the consequent proofs with the appearances." *(Almagest,* I, 8, p. 12 in *The Great Books of the Western World,* Encyclopaedia Britannica, Inc., Chicago, London, Toronto, Geneva 1952, translated by R. Catesby Taliaferro.)

Alphonsine Tables (c. 1327) in the accuracy of predictions.[52] Furthermore, Copernicus did not err at all, when he comprehended the cosmological and mathematical hypotheses, their justification and confirmation in the described above sense.

The last-quoted Copernicus' opinion proves in addition, that Copernicus perfectly apprehends the problem which is called nowadays

The issue of the "theory-ladenness" of facts

Exactly the same as Einstein (see fn. 16 p. 15), Copernicus knows that a theory does prejudge what is observed finally. Moreover, just this subject is in the centre of his considerations. According to Copernicus, the apparent immobility of the earth and its observed central position in the universe, which were treated as real and ontological in the context of the primary sensory apperception and the geostatic cosmology, become only the appearance, the illusion of our senses.

Conclusion

Since Copernicus, firstly, uses the hypotheses; secondly, knows that the astronomical theory has complicated structure which the parts are not only the mathematical schemes but also physical, cosmological conceptions; thirdly, understands the so-called undetermination principle of theories by empirical facts and the so-called issue of theory-ladenness of facts perfectly, etc, we may conclude justifiably without follows: Copernicus understands the "modem" Hypothetico-Deductive Method very well.

Furthermore, on the ground of the above considerations it is evident, that, according to such a mathematical astronomer as Copernicus, it is not possible to reduce astronomy only to mathematical considerations, because this discipline has its own methodology, epistemology and ontology. Therefore, it is reasonable to say that, according to Copernicus, mathematical astronomy is to the core a philosophical branch. In such a

[52] Therefore, the *Prutenic Tables* were more widely adopted by astronomers to construct ephemerides.

On the other hand, Tycho Brahe who primary (c. 1573) thought that the *Prutenic Tables* are superior to the *Alphonsine Tables*, next (c. 1588) changed his opinion, since his systematic observations showed that the accuracy of the *Prutenic Tables* and the *Alphonsine Tables* are comparable. On this subiect see Moesgaard [1972] p. 33—35; Gingerich [1973] 43-62, esp. 53.

way the mathematical scheme of an astronomical theory, prima facie so non-philosophical, is conditioned by the philosophical conceptions in fact.[53]

4.2.2. Copernicus and the Method of Correspondence Thinking

It remains now to prove only that Copernicus, while searching for the new and more general theory than Ptolemy's, was consequently and constantly using the idea of a correspondence postulate, a correspondence relation and hence he was using the Method of Correspondence Thinking in a systematic way. Let us start our considerations with a brief discussion of the following issue:

Fundamental quasi-entities according to Copernicus' and geo-centro-static cosmologies

Rejecting Aristotle's and Ptolemy's arguments for an immobility and a central position of the earth in the Universe, and for the mobility of the stellar sphere, Copernicus assumed as a cosmological basis of his theory the (dialectically) opposite thesis, that the earth is movable and the stellar sphere is immovable. He considered these hypotheses as real, giving them, within the context of his theory, the quasi-absolute, quasi- ontological sense. Moreover, Copernicus, both in the *Commentariolus* and *De revolutionibus,* endowed the earth with three fundamental motions (they are listed below in the same order as in the *Commentariolus):*

(i) the annual revolution around the sun in the order of the zodiacal signs, that is, from west to east, with the period of one sidereal year[54]; this

[53] Such Copernicus' comprehensions of mathematical astronomy surpasses the neo-positivistic understanding of "exact sciences" considerably. Furthermore you may check, that so important mathematical astronomers as Ptolemy (see *Almagest* B. 1) and Nasir al-Din al-Tusi's (see *Tadhkira,* "Memorandum", B. I) thought exactly the same about mathematical astronomy. Let us illustrate this thesis by the quotation from the first Book of the *Tadhkira:* "Every science must possess the following: a) a subject which this science investigates, b) principles which are either self-evident or need to be proved in another science but are taken for granted in this science, and c) problems (mas'il) which are proved in this science. Now the subjects of astronomy (hay"a) are the simple bodies, both superior and inferior, in respect of their quantities, qualities, positions and inseparable motions. The principles of astronomy that need proof are demonstrated in three sciences: metaphysics, geometry and physics. The problems of astronomy are aimed at gaining knowledge of these bodies themselves, and of their shapes, arrangements (nadd), motions, and their quantities, distances and the reasons (illal) for the difference in their positions." (Sabra [1978], p. 124)

[54] The sidereal year is defined to be the interval between two successive returns of the sun (moving in the apparent orbit) to the same point among the stars, as viewed from the earth.

motion replaces the apparent motion of the sun towards east assumed by the "geostatic theories";

(ii) the daily rotation of the earth (with its circumjacent water and encircling atmosphere) about the polar axis in the order of the zodiacal signs, that is, from west to east; this motion replaces the first motion of the stellar sphere (the eighth sphere), that is, the daily motion of the stellar sphere in the opposite direction, assumed by the "geostatic theories";

(iii) the motion of the declination *(motus declinationis or inclinationis)-,* that motion has two components: (a) the variation of the obliquity of the earth' polar axis with respect to the plain of the ecliptic, and (b) the circular motion of the earth' polar axis towards west (= the westward motion of the earth' polar axis) with the period which is equal to one changing tropical year[55].

According to Copernicus, the motion of the earth can explain as well short - as long-period phenomena.

> Whatever motion appears in the firmament is due, not to it, but to the earth.
> (The *Commentariolus,* p. 81)

For instance, first and third motions together cause long-period alterations of some important astronomical magnitudes, which according to Hipparchus (161-126 B.C.) and Claudius Ptolemy (90-168), were uniform alterations (or were unchanged), and according to the later astronomers as Thabit Ibn-Qurra (836-901), John Linieres and John of Saxony — the authors of the Parisian version of the *Alphonsine Tables* (c. 1327), Georg Puerbach (1423-1461), Marcus Beneventanus (c. 1465- c. 1525) and Jan Werner (1468-1528) were non-uniform ones. These two motions of the earth's explain and describe, among other things, the phenomenon of the non-uniform change in stellar longitudes of fixed stars.[56] But, according to Hipparchus and Ptolemy, this change is uniform

[55] The tropical year is defined to be the interval between two successive passages of the sun through the vernal equinox.

[56] To describe the position of a star on the stellar sphere it is necessary to give two co-ordinates that is two arcs. But to do it one firstly must select a concrete reference frame. On the stellar sphere, one may differ two important circles: the celestial ecliptic ([in the heliocentric (geocentric) cosmology] the plain of the annual motion of the earth (the sun) extended to the sphere of firmament) and the celestial equator (the plain of earth's equator extended to the sphere of firmament). These circles intersects at the characteristic points called equinoctial points (vernal and autumn equinoxes). In the ecliptical reference frame, the above mentioned co-ordinates are called "stellar latitude" and "stellar longitude". The stellar latitude is the arc of circle drawn perpendicularly from a given point (= the position of a star on the celestial sphere) to the point of ecliptic. The stellar longitude is the arc reckoned from the vernal point to the point being the intersection of the circle of latitude with ecliptic.

and equal to (at least) 1 per 100^{ey} (Egyptian years of 365 days), and is explained by the second motion of the firmament (= motion of the eighth sphere) about the axis of ecliptic towards east with the period of (at most) 36000^{ey}. Next, according to Thabit Ibn-Qurra, the authors of the Parisian version of the *Alphonsine Tables* (c. 1327), Georg Puerbach, Marcus Beneventanus and Jan Werner, the change in stellar longitudes is non-uniform and long-period and is explained by the non-uniform resultant motion of the eighth sphere caused by the uniform motions of the subsequent celestial spheres (ninth, tenth or even eleventh[57]).

From the above mentioned three motions of the earth which are defined within the (whole) context of Copernicus' theory, the first and the third are problematical. Contrary to them, the second motion, that is, the daily rotation of the earth, was, in principle, already analysed by philosophers and astronomers before Copernicus. And this motion does not create any significant difficulties for Copernicus. Since, it was Ptolemy *(Almagest,* I, 7) who understood that any astronomical observations did not deny such possibility, and from this point of view, this Pythagorean hypothesis is simpler that of the firmament rotating about the immobile earth. However, the motion of the earth, which was possible from an astronomical point of view, was impossible on the ground of contemporary Aristotelian physics. Just therefore Ptolemy thought that this motion could not exist. Next, this physical objection was in great part smoothed away within the context of Buridanists' new physics.[58] But, before Copernicus, the motions of the earth (the daily, the annual, and the motion of the declination) were not considered together and/or were not interpreted as real.[59]

To show that Copernicus used the Method of Correspondence Thinking, for brevity, in this article, we will only consider the issue of the long-period phenomena and their theories which was the leading problem of the Renaissance astronomy. Taking up this subject we have in mind, for instance, the following fact. In Rheticus' *Narratio prima,* the chapter "The

[57] The eleventh sphere was used by Werner in 1522 in his *De motu octave sphaere tractatus primus.*

[58] See: N. Oresme, *Le Livre du ciel et du monde,* ed. by A. D. Menut, A. J. Denomy, C. S. B, transl. A. D. Menut, The University of Wiscontin Press, Madison 1968; and Crombie [1952] vol. II ch. 3, p. 47-84'

[59] Let us recall that long before Copernicus, Pythagoreans and Aristarchus of Samos, endowed to the earth the daily and annual motions, but they did not know the motion of the declination. On the other hand, the philosophers of Buridan's school did not consider the annual motion of the earth at all, and except Albertus of Saxony, they did not endow to the earth any long-period motions. (See p. 34).

Principal Reason Why We Must Abandon the Hypotheses of the Ancient Astronomers” starts with the below quoted words:

> In the first place, the indisputable precession of the equinoxes, [...] and the change in the obliquity of the ecliptic persuaded my teacher to assume that the motion of the earth could produce most of the appearances in the heavens, or at any rate save them satisfactorily. *(Narratio prima*, p. 136)

Discussing the issue of the long-period phenomena and their theories, we will analyze, firstly, the *Commentariolus* and next, the *De revolutioni- bus*. But, as to Copernicus, Ptolemy’s theory is the best of the all existing hitherto, analyzing Copernicus’ considerations we will compare them with the analogical of Ptolemy’s[60]

A. *Commentariolus*
The long-period phenomena according to Copernicus’ and the phenomena corresponding with them according to Ptolemy

Firstly, according to Copernicus and Ptolemy, respectively, the tropical year not being (being) constant is not (is) always equal to 365 $1/4^d$-$1/300^d$ that is 365^d $5^h55^m12^s$. Secondly, Copernicus and Ptolemy agree that the stellar longitudes of fixed stars taken with respect to the selected fixed stars[61] (the cardinal points, that is to the equinoxes and solstices) change during the long period of time since, according to Copernicus (Ptolemy respectively), the equinoxes (the fixed stars) move towards west (east). Thirdly, in accordance with Copernicus (Ptolemy respectively), the speed of this motion being non-uniform (uniform) magnitude, is not (is) always equal to 1 per 100^{ey} (Egyptian years of 365 days). Fourthly, according to

[60] Following the examples of: Thabit ibn Qurra, the authors of the Alphonsine Tables, Peurbach, Albertum de Brudzewo and Beneventanus, Copernicus was convinced about the corroborated by astronomical observations and measurements existence of the non-uniform variations of some astronomical phenomena which are impossible to express by terms of Ptolemy’s theory. But, firstly, he did not agree with these astronomers, about what phenomena are long-period and secondly, he simultaneously does not accept any known to him, theoretical generalizations of Ptolemy’s primary theory. Such decision is the next example which shows that Copernicus knows the issue of theory-ladness of facts.

Let us emphasize, that Copernicus was not the first Renaissance astronomer who assumed that the equinoctial points move. It already did Beneventanus in his own theory of the motion of the eighth sphere (see L. A. Birkenmajer [1901a] p. 146). But before Beneventanus and Copernicus analogously did Thabit Ibn-Qurra (836-901).

[61] In the *Commentariolus,* Copernicus didn’t select any concrete stars as the origin point. He did it in the *De revolutionibus* where y Arietis, first star in Ptolemy’s catalog of stars, played such a part.

Copernicus (Ptolemy respectively), the value of the angle between the ecliptic and the terrestrial equator not being (being) constant, is not (is) always equal to 23;51,20°.

The above regularities are the key for the understanding of Copernicus' methodological thinking properly. To make ourselves certain of the correctness of our considerations, let us quote Copernicus' opinion about the length of the tropical and sidereal years:

> Accordingly, since the equinoxes and the other cardinal points of the universe shift considerably, whoever attempts to derive from them the equal length of the annual revolution necessarily falls into error. Besides, different determinations of this length were made in different ages on the basis of many observations. Hipparchus computed it as 365 1/4 days, and al-Battani the Chaldean as $365^{d}\ 5^{h}46^{m}$, that is 13 $3/5^{m}$ or (sic!) (my addition) [13] $1/3^{m}$ less than Ptolemy. Hispalenius, on the other hand, increased al-Battani's length by the 20th part of an hour, since he determined the tropical year as $365^{d}5^{h}49^{m}$.
>
> Lest these differences should seem to have arisen from errors of observation, [let me say that] if anyone will study the details carefully, he will find that **the discrepancy has always corresponded to the shift in the equinoxes. For when the cardinal points moved 1 in 100 years, as was found in Ptolemy's time, the length of the year was then what Ptolemy himself reported. When however in the following centuries they moved with greater rapidity in opposition to lesser motions, the year became shorter by as much as the cardinal points' displacement increased. For by their swifter recurrence they encountered the annual motion in a shorter time** [my emphasis] .Therefore the derivation of the equal length of the year from the fixed stars is more accurate. Thus I used the Spike of the Virgin and discovered that the year has always been 365 days, 6 hour, and about 1/6 hour, the value also found in ancient Egypt. The same reasoning must be employed also with the other motions of the heavenly bodies because their apsides, which are likewise fixed in the firmament, with their true testimony make manifest the laws of the motions as well as heaven itself. *(Commentariolus* p.83/84)

To make a step forward in our analysis of the *Commentariolus,* it must be noticed [after Birkenmajer ([1900] p. 14, fn. 1, 2) and Swerdlow ([1973] p. 451-452)], that the above quotation, at the place marked by (sic!), lost some important words or even the whole sentence, since the original Copernicus' text is self-contradictory and, moreover, inconsistent with the fundamental source of data used by the *Commentariolus,* that is with the Regiomontanus' *Epitome In Almagestum Ptolemaei* (Venice 1496). Firstly, we see clearly, that 13 $3/5^{m} = 13^{m}36^{s}$ is not equal to 13 $1/3^{m} = 13^{m}20^{s}$. Secondly, we read in the *Epitome* (L. III, Prop, ij), that the length of the tropical year measured by Hipparchus was (a little shorter than) 365 $1/4^{d}$

= $365^d 6^h$, by Ptolemy, 365 (1/4 - l/300)^d = $365^d\ 5^h 55^m 12^s$ and by al-Battani, 365^d $5^h 46^m$, that is shorter 13 3/5^m than Ptolemy's value.[62]

Thus, we assume what follows.

(1) The length of the tropical year decreases during the time between Hipparchus and al-Battani and increases between al-Battani and His- palenius. The astronomers measured the following values: Hipparchus $365^d 6^h$, Ptolemy 365^d $5^h 55^m 12^s$, al-Battani the Chaldean 365d$5^h 46^m 24^s$ and Hispalenius $365^d 5^h 49^m$.[63]

(2) The speed of the motion of the equinoxes, called by Copernicus rate of precession, which is always (approximately) proportional (with the opposite sign) to the change of the length of the tropical year, at first increases and later decreases, in the above periods of time.

(3) In the Ptolemy's age, the rate of precession is equal to Ptolemy's value about l°/100^{ey}.

Next, we note, that, in the *Commentariolus,* Copernicus considers also the phenomenon of the change in obliquity of the ecliptic (or to be exact, the earth's equator). However, his considerations on that subject are much less detailed than on the change of the tropical year. Regarding the values of the changing obliquity of the ecliptic, Copernicus states only, that for his days this magnitude is about 23(^{1}/2)°. However such shortened character of the *Commentariolus'* remarks is dictated by the similar, of the *Epitome* (L. Ill, Prop, xviij). We read there, that this value is equal to 23;28° for our days and 23;51,20° for Ptolemy's. Hence, Copernicus knows that the obliquity of ecliptic decreased Ptolemy's times.

[62] The other interpretation is given by Rosen ([1971] p. 65 fh. 19, [1978] p. 97-98 fn. 96 in Nicolaus Copernicus, Complete Works, vol. III, Minor Works) who thinks that *Commentariolus'* text is correct, and Copernicus thought that Hipparchus and Ptolemy accepted the length of the tropical year equal to 365 174 days. We think, that this interpretation is not correct, since the *Epitome* (L. III, Prop, ii) states that the length of the tropical year measured by Hipparchus was (a little shorter than) 365 l/4^d = $365^d 6^h$ and by Ptolemy, 365(1/4 - 17300)^d = $365^d 5^h 55^m 12^s$.

[63] The later value, as Swerdlow ([1973] p. 452 fh. 2) suggests, may be the approximation of the length of the tropical year given by the *Alphonsine Tables* $365^d 5^h 49^m 16^s$.

Note here, that the precision of seconds is not very important for Copernicus in the *Commentariolus.* Hence, from the methodological point of view, it is clear, that Copernicus, in the *Commentariolus,* is interested in a qualitative (in a mathematical sense) analysis of his theory.

The long-period phenomena and the functions corresponding with them

According to Copernicus, astronomical phenomena are much morecompli- cated than it seemed to Ptolemy. Nevertheless,

Copernicus accepts Ptolemy’s results of measurements of the length of the tropical year, and of the speed of the motion of the equinoxes, that is, speaking more precisely, of the speed of the change in stellar longitudes caused by the second motion of the stellar sphere. Hence, for Ptolemy’s epoch, Ptolemy did not err at all. But, beyond it the predictions of Ptolemy’s theory do not save the phenomena. In other words, for Ptolemy’s epoch, Copernicus’ and Ptolemy’s theories of the change in longitudes of fixed stars should be observationally equivalent but beyond this period, non-equivalent. Moreover, in the analogous relation should be the obliquity of the earth’s equator defined within the context of Copernicus’ theory, and the obliquity of ecliptic defined within the context of Ptolemy’s theory.

On the grounds of the above Copernicus’ considerations, it is clear, that Copernicus’ functions modelling the length of tropical year, the speed of the change in stellar longitudes of fixed stars (measured from the selected fixed stars) and the changing obliquity of the ecliptic should be continuous:

(10)
$$F_C(t) \to F_C(t_{OP}),$$
$$t \to t_{OP}$$

F_C(f) - long-period function of the considered “observable” defined within the context of Copernicus’ theory,

t - time,

fOP - optionally chosen moment of time.

Therefore, in the limit case *t* —> £pt, Copernicus’ functions of the long-period phenomena and Ptolemy’s analogous ones, should be observa- tionallv or. in principle. numericallv equivalent. that is:

(ID
$$F_C(t) \approx F_{Pt}(t_{Pt}).$$
$$t \to t_{Pt}$$

But beyond this era (and maybe beyond their periodic replicas) these functions should be non-equivalent.

Moreover, from the *Epitome* (B. VII, Prop, v), Copernicus knew, that during 391^{ey} lasted between two observations of the star Spica by Timocharis and Ptolemy, its longitude increased 3;55°. Thus, during this interval of time, the mean speed of this (and others) star(s) was:

$$\omega_{T_i-Pt} = \frac{3;55}{391} \approx 0.097 \approx \frac{1}{100}\ [^\circ/_{ey}].$$

Thus, the speed of the change in longitudes of the fixed stars is nearly constant and equal 1/100 [$^\circ/_{ey}$] not only for Ptolemy's moment of measurement (or Ptolemy's age) but during the relatively long passage of time of about 400^{ey} lasted between Timocharis and Ptolemy. For such a reason, Ptolemy assumed that the value of the speed of the change in longitudes of the fixed stars was a constant of Nature being equal 1/100 [$^\circ/_{ey}$] And, in consequence, within the context of Ptolemy's theory, the function of the change in longitudes of fixed stars was always linear. Similarly, Ptolemy assumed that the value of the obliquity of ecliptic is always constant being equal to 23;51,20°. But, according to Copernicus, the values of the speed of the change in longitudes of the fixed stars and the obliquity of ecliptic change. Thus, from the logical point of view, these magnitudes are not constants of Nature. In consequence, within the context of Copernicus' theory, the functions of: the change in longitudes of fixed stars, and the change of the obliquity of ecliptic could not be linear. But, since, in the interval of time of about 400^{ey} lasted between Timocharis and Ptolemy, the speed of the change in longitudes of the fixed stars and the obliquity of ecliptic change very little, these magnitudes may be assumed as effectively constant. In consequence, during this period of time, the functions of the change in longitudes of fixed stars, and of the change of the obliquity of ecliptic should be effectively linear. In other words, Copernicus notices that his longperiod functions and analogous ... lasted between Timocharis (tT) and Ptolemy (tpt). It means that for any moment of time *t* taken from this interval of time of about 400^{ey} between

Timocharis and Ptolemy (ΔT_{T_i-Pt}):

(13a) $$t_{Pt} - \Delta T_{T_i-Pt} \leq t \leq t_{Pt}$$

(13b) $$F_C(t) \approx F_{Pt}(t).$$

But beyond these eras (and, maybe, beyond its periodic replicas) Copernicus' and Ptolemy's analogous functions are non-equivalent.

Noticing the above facts, we already know a lot, but it does not use up our subject. We have only listed the data of the *Commentariolus* regarding the issue of the long-period phenomena and have only said a few words about the fundamental consequences which follows from these data for the shape of Copernicus' functions modelling the above mentioned phenomena. But such a sort of mathematical regularities determines only a half of a theory (model), since every theory must both save phenomena and explain them.

It is the right time for analyzing the later question precisely.

Copernicus' sketchy theory of the longperiod phenomena

Though Copernicus accepts Ptolemy's data for Ptolemy's days, he rejects Ptolemy's model of explanations, that is, rejects Ptolemy's quasi-entities by means of which the phenomena are explained in Ptolemy's theory. Copernicus abandons, namely, the eastward motion (revolution) of the sphere of fixed stars of 36000^{ey} around the axis of ecliptic, and the conception that the obliquity of the ecliptic is constantly equal to 23;51,20°. In Copernicus' opinion, the long-term phenomena can be effectively explained by the motion(s) of the earth. Therefore he says:

> [...] it is the common opinion that the firmament itself has several motions.
>
> But even though the principle involved is not yet sufficiently understood, **it is less surprising that all these phenomena can occur on account of the earth's motion** [my bold], *(Commentariolus,* p. 83)

To explain and describe the long-period phenomena in the *Commentariolus,* Copernicus uses the so-called first and third motions of the earth. The first is clearly explained. It is the annual revolution around the sun in the order of the zodiacal signs, that is, from west to east, with the period of one sidereal year[64]; this motion replaces the apparent motion of the sun towards east assumed by the "geostatic theories". Much more complicated, there is the question of the third motion. Copernicus states on that subject the following.

> The third is the motion of the declination [at Rosen's translation — motion in declination]. For, the axis of he daily rotation is not parallel to the Grand Orb's axis, but is inclined [to it at an angle that intercepts] a portions of a circumference, **in our time about** $\mathbf{23^{31}/_{2}}^{o}$ [my bold]. Therefore, while the earth's center always remains in the plain of the ecliptic, that is, in the circumference of the circle of the Grand Orb, the earth's poles rotate, both of them describing small circles about centers [lying on a line that moves] parallel to the Grand Orb's axis. The period of this motion also is a year, but not quite, being nearly equal to the Grand Orb's [revolution]. The Grand Orb's axis, however, being invariant with regard to the firmament, is directed

[64] The sidereal year is defined to be the interval between two successive returns of the sun (moving in the apparent orbit) to the same point among the stars, as viewed from the earth.

toward what are called the poles of ecliptic. **The poles of the daily rotation would always be fixed in like manner at the same points of the heavens by the motion of the declination** [at Rosen's translation — motion in declination] **combined with the Orb's motion, if their periods were exactly equal. Now with the long passage of time it has become clear that his alignment of the earth changes with regard to he configuration of the firmament. Hence it is the common opinion that the firmament itself has several motions. But even though the principle involved is not yet sufficiently understood, it is less suprising that all these phenomena can occur on account of the earth's motion** [my bold]. I am not prepared to state to what its poles are attached. I am of course aware that, in more mundane matters a magnetized iron needle always points towards a single spot in the universe. It has nevertheless seemed a better view to ascribe the phenomena to a sphere, whose turning governs the movements of the poles. This sphere must doubtless be sublunar. *(Commentariolus,* p. 83)[65]

Moreover, with the question of the third motion, there are connected strictly the mentioned above nonuniform long-term changes of the tropical year and the long-term non-uniform changes in stellar longitudes. These phenomena are explained by the interaction of the first motion and third motions.

Reading the above quoted *Commentariolus* text once again, we may distinguish the general and special cases of Copernicus' theory.

In the general case, the effective period of the motion of the declination towards west, being a regularly changing magnitude, is always a little smaller than the constant period of the revolution of the earth.

However, if both these periods were exactly equal, the earth's axis would be always kept invariably directed towards the same points of the firmament. Therefore, in this first special case of Copernicus' theory, the earth's equator and hence the celestial one would not shift in relation to the fixed celestial ecliptic, that is, there would not be any motion (called by Copernicus, the precession) of equinoxes. Note, at once here, that such a case (reversed into geocentric cosmology) would be possible to model within the context of Ptolemy's theory by assuming that the period of the second motion (revolution) of the sky (the eighth sphere) is equal to infinity or is so long, that its measurement consequences cannot be observed.

[65] This magnetic analogy is realy very interesting. It leads straightly to magnetic explanations of the motion(s) of the earth given by: (1) Gilbert's *De magnete* (1600) - diurnal rotation; (2) Galileo's *Dialogo sopro i* due massimi sistemi del mondo Tolemaico e Copemicano (1632) - rejection of the third motion of the earth; (3) Kepler's *Astronomia Nova* (1609) - *Epitome astronomiae Copemicanae* (1619) - magnetic explanations of motions of planet See Russell [1972] p. 203-212.

Next, if the considered periods of earth's motions had different values but either was still constant, similarly to the case of Ptolemy's theory, the speed of the alteration in stellar longitudes would be constant. But, note, in addition that this special second case of Copernicus' theory would be observatory equivalent with Ptolemy's theory of the uniform long-period motion of the eighth sphere if Ptolemy's values of the above mentioned parameters were additionally adopted to Copernicus' theory and the initial astronomical epoch was adequately taken.

Moreover, it is important that the factual difference between the above mentioned periods of earth's motions, is very small. It must be so small, so that the expected, observable difference between the predictions of the general and the second special case of Copernicus' theory could appear after hundreds of years.

Thus, on the ground of the information taken from Copernicus' *Commentariolus,* we assume the following. To explain and describe the long-term phenomena, Copernicus assumes firstly, that the sidereal year is an adequate constant unit of time and the tropical year changes in long passage of time in a systematic way. Secondly, he substitutes the long- period motions of the eighth sphere with two short-period motions of the earth, the first and the third.

The first is the revolution of the earth on the circumference of the ecliptic of the sphere, called the Great Sphere;[66] this motion is from west to east and its period is equal to one constant sidereal year.

The third is the motion of the declination or the motion of the inclination *(motus declinationis or motus inclinationis^,* it is a resultant motion caused by two motions: (a) the variation of the inclination of the earth's polar axis with respect to the plain of the ecliptic (in the *Commentariolus* Copernicus doesn't describe precisely the time-dependence of this motion and he doesn't prejudge that this variation is uniform or non-uniform or/and periodic)[67]; (b) and the non-uniform circular motion of the polar axis towards west (= the westward motion of the polar axis); the effective period of this motion is equal to one changing tropical year.[68]

[66] Note that the (real) sun is not the centre of this sphere. This part is played by the mathematical point, called, using terminology of the geocentric cosmology, the mean sun. The radius of the Grand Sphere is equal to the mean distance of the earth to the sun.

[67] But if the change of obliquity is periodic, it is natural to assume, that (1) it is a long-period phenomena, since for short passage of time it seems to be constant and (2) the obliquity of ecliptic oscillates around a mean value.

[68] Such an understanding of the motion of the declination is based on the interpretations of Copernicus' statements which regard the connected each other issues of the variation of the obliquity of the polar axis, the changing length of the tropical, and the

It must be emphasized, that for a short passage of time, the periods of the above components of the motion of the declination may be regarded as (approximately) constant magnitudes. Just therefore, Copernicus' model of the longterm nonuniform changes of precession and obliquity of ecliptic, should be able also to explain and describe the uniform precession and constant obliquity of ecliptic measured during a relatively short passage of time.

We would like now to render more precisely and more clearly the substance of Copernicus' search for the new and more general theory than Ptolemy's. To fulfil that task, let us use the language of algebra and the elements of the differential and integral calculus.[69]

But, since, in the *Commentariolus,* Copernicus didn't describe precisely the change in the inclination of the earth's polar axis with respect to the plain of the ecliptic, we will fix our attention only on the model of the changing tropical year and the changing precession of the equinoxes.

Algebrization of Copernicus' sketchy theory of the precessional phenomena

According to Ptolemy, the speed of the change in stellar longitudes, noted here by copt, is always constant value equal (at least) to:

(14) copt - ^{lo}/L00^{ey}.

length of the stellar year, and the phenomenon of the changing precession of the equinoxes. It seems to me that such an understanding of the motion of the declination (1) expresses properly the sense of (in a some degree unclear) the statements from Copernicus' *Commentariolus* regarding this subject; (2) is coherent with the analogous (but clearer) statements from Rheticus' *Narratioprima* and Copernicus' *De reiolutionibus*-, (3) completes the interpretation given by Ravetz ([1964] p. 767) successfully.

In general, the model of the long-period phenomena described in the *Commentariolus* is in a some degree confusing. For instance, regarding Copernicus' model of the obliquity of the ecliptic, L. A. Birkenmajer [1900] states on the pages 200. 330 and 331 that it is the model of the constant obliquity. Concerning Copernicus' model of the precession, the same author states on the page 302 that it is the model of the nonuniform precession which is qualitatively but not quantitatively the same as the model given in the *De revolutionibus.* On the page 331, however, we read, that it is the model of the constant precession.

Next, these contradictory interpretations are discussed extensively by Swerdlow [1973] p. 449-450. Finally, he assumes that *Commentariolus'* model of the long-period phenomena is the model with the constant obliquity and the constant period of one mean tropical year. Such an interpretation is given also in Swerdlow, Neugebauer [1984] p 51.

[69] Of course, Copernicus did not know the differential and integral calculus. But, on the other hand, he used some primitive elements of it. This fact justifies our decision.

Therefore the change of the stellar longitude of every fixed star, noted Ppt, is the linear function of time:

(15) $$P_{Pt} = A_{Pt} + B_{Pt} \cdot t.$$

A_{Pt} – magnitude that characterizes the (initial) position of the star on the stellar sphere for $t = 0$,

$B_{Pt} = \omega_{Pt} = 1^{\circ}/100^{ey}$.

The considered phenomena are modeled within the context of Ptolemy's theory by the so-called second motion of stellar sphere with period, noted here by T_{Pt}, equal (at most) to 36000^{ey}:

(16) $$T_{Pt} = 36000^{ey}.$$

Now, let us consider the analogous subject in accordance with the *Commentariolus.*

In the first place, note that the change in stellar longitudes of the fixed stars is explained by the interaction of two motions of the earth: (1) the annual revolution of the earth towards east; its period, denoted here T_R, is equal to one constant sidereal year; (2) the circular motion of the earth' polar axis towards west (= the westward motion of the polar axis); the effective period of this motion, denoted here T_D, is equal to one changing tropical year. Because, according to the mathematical and philosophical paradigm of those days, every changing astronomical magnitude should be a certain periodic (and simultaneously simple) function, and the length of the tropical year changes in a small range, the changing period of the westward motion of the declination may be expressed in the form:

(17a) $$T_D(t) = T_{DM} + \Delta T_D \cdot f_{NU}(t, T_{NU}),$$

(17b) $$\Delta T_D \ll T_{DM} < T_R$$

(17c) $$|f_{NU}(t, T_{NU})| \leq 1$$

$T_D(t)$ - the changing period of the westward motion of declination which is equal to the changing value of the tropical year,

T_{DM} – the mean value of the period of the westward motion of the declination which is equal to the mean value of the tropical year, the amplitude of the

ΔT_D – periodic nonuniform changes of the length of the period of the westward motion of declination, which is equal to the amplitude of the periodic nonuniform changes of the length the tropical year,

$f_{NU}(t, T_{NU})$ - the nonuniform periodic function of time with the period T_{NU},

T_R - the constant sidereal year.

Moreover, note, that as the simplest but still general shape of the function fNu(t, TNU), the following form may be selected:

(17d)
$$f_{NU}(t, T_{NU}) = f_{NU}(\varphi_{0NU} + \frac{360°t}{T_{NU}})$$

φ_{0NU} – the initial phase.

Furthermore, note that, as the simplest nonuniform function fNu may be chosen a cosine or sine function. And, what is important for us, Copernicus, knowing the Cracow *Commentariolum Super Theoricas Novas Planetarum Georgii Purbachii* by Albertus de Brudzewo and the *Commentary on Euclid's Elements* by Proclus (mentioned in the *De revolutionibus N,* 25), can construct such function by usage of the set of certain circles which move uniformly. It is just that issue that is called by Copernicus the motion of the libration *(motus librationis).* He knows the solution of this issue already in the *Commentariolus* (where it is used explicitly in the issue of a planetary theory) and explains it precisely in the *De revolutionibus* (B. Ill, ch. 4).

Constructing such a function, Copernicus does it in agreement with the postulate of the absolute uniform motions. Moreover, according to the *De revolutionibus* (B.I. ch. 8, p. 17); these uniform circular motions are governed by the constant physical causes, the unfailing causes *(.cause indeficiens).* It is an important point, since Copernicus' *cause indeficiens* is the synonym of Buridan's *impetus.*

Next, on the ground of the earlier quoted Copernicus' statements about the length of the tropical year and some additional remarks (see p. 46-49), it results that the function fisiu decreases for moments of time between Hipparchus and al-Battani and increases between al-Battani and Hispalenius. Therefore, to estimate the order of the value of TNU, it is reasonable to assume the simplest possibility, that is:

(18)
$$T_{NU} \cong 2(t_{Bat} - t_{Hip}),$$

t_{Bat}, t_{Hip} - the moments at which al-Battani and Hipparchus made the measurements of the length of the tropical year.

Next, not embarking upon the detailed (chronological, theoretical) investigations, Copernicus knows from the *Epitome* (B. Ill, Prop, ij) that between Hipparchus' and Ptolemy's observations of equinoxes it lasted 285^{ey}, and between Ptolemy's and al-Battani's, $743^{ey}178^{d}$. Thus, between Hipparchus and Ptolemy, it lasted about 1028^{ey}. Thus, already writing the *Commentariolus,* Copernicus might know that:

$$T_{NU} \approx 2000^{ey}.$$

Now, using the data of the length of the tropical year, Copernicus should compute the mean value of the period of the westward motion of the declination (T_{DM}), and the amplitude of the nonuniform changes of the period of the westward motion of the declination (ΔT_D). In order to do it, he should find the mean value of the tropical year and the amplitude of its changes. However, he has one important problem. To determine these magnitudes correctly, it is necessary to have many results of measurements . But, he has only few. Therefore, in order to find the mean value of the tropical year, and thanks to it, the mean value of the period of the westward motion of the declination (T_{DM}), he cannot calculate the arithmetical mean. Instead of it, he should assume *ad hoc* that this value is approximately equal, for instance, to the length of the tropical year given by Hispalenius, that is:

(20) $$T_{DM} \approx 365^d 5^h 49^m,$$

Knowing this mean value and assuming in addition that the maximum value of the tropical year is approximately equal to $365^d 6^h$, we may find the approximated maximum amplitude of the nonuniform changes in the period of the westward motion of the declination (ΔT_D):

(21) $$\Delta T_D \approx 11^m.$$

Let us now recall that Copernicus' value of the period of the annual motion of the earth which is equal to the length of the sidereal year:

(22) $$T_R \approx 365^d 6^h 10^m,$$

Next, we note, that the speed of the change in stellar longitudes of fixed stars, denoted here by $\omega_C(t)$, may be expressed in Copernicus' theory by the form:

(23) $$\omega_C(t) = 360°(1/T_D(t) - 1/T_R),$$

T_R - the period of the earth's revolution around the mean sun equal to one sidereal year,

$T_D(t)$ - the period of the westward motion of the declination equal to one changing tropical year.

Since the period of the westward motion of the declination is a variable magnitude, the change in stellar longitudes, denoted here by Pc(£), may be expressed by the following formula:

(24) $$P_C(t) = \int_0^t \omega_C(\tau)\, d\tau = 360° \int_0^t \frac{1}{T_D(\tau)} - \frac{1}{T_R}\, d\tau$$

In order to get more valuable information from the above formulas (23)—(24), let us now use the expression for the length of the period of the westward motion of the declination TD(£) given by the formulas (17a)- (17c). Neglecting higher components than the first we can get:

(25) $$\omega_C(t) \approx \frac{360°}{T_{Cmean}} - \Delta_\omega f_{NU}(t, T_{NU})$$

(26) $$P_C(t) \approx \frac{360°}{T_{Cmean}} t - \Delta_\omega \int_0^t f_{NU}(\tau, T_{NU})\, d\tau$$

(27) $$T_{Cmean} \equiv T_R * \frac{T_{DM}}{T_R - T_{DM}}.$$

(28) $$\Delta_\omega \equiv 360° \frac{\Delta T_D}{T_{DM}^2}$$

Thereby, it appears that the interaction between two circular short- period motions of the earth, that is, between the annual motion [towards east with the period of one constant sidereal year] and the motion of the declination [towards west (= the circular motion of the earth's polar axis towards west) with the period of one changing tropical year], creates effectively the uniform and the nonuniform long-period components of the change in stellar longitudes of fixed stars. Using terminology of Copernicus from the *De revolutionibus,* we may call first component — the uniform precession, and the second — the libration of precession. Next, using the above computed values of TR and TDM we get:

(29) $$T_{Cmean} \approx 25063^{ey},$$

(30) $$\Delta_\omega \approx 1/132{,}90\ [°/ey].$$

In consequence, the speed of the change in stellar longitude of every fixed star (co;) and corresponding to it the instantaneous effective period (Ti), have the following the maximum, mean and minimum values:

(31a) $\omega_{Cmax} \approx 1/45{,}686 \approx 1/46\ [°/ey],$ $T_{Cmin} \approx 52646\ [ey]$

(31b) $\omega_{Cmean} \approx 1/69{,}619 \approx 1/70\ [°/ey],$ $T_{Cmean} \approx 25063\ [ey]$

(31c) $\omega_{Cmin} \approx 1/146{,}24 \approx 1/146\ [°/ey],$ $T_{Cmax} \approx 16447\ [ey].$

Next, let us note now that from the above general formulas (23)–(24) we can deduce two special cases (SC) of Copernicus' theory of precession:

(32, SC1) if $T_R = T_D$, than

$$T_C(t) = const_1 = \infty\ \&\ \omega_C(t) = const_2 = 0\ \&\ P_C = const_3;$$

(33, SC2) if $T_R > T_D$ & $T_D = \text{const}_1$, than
$$T_C(t) = \text{const}_1 < \infty \;\&\; \omega_C(t) = \text{const}_2 > 0 \;\&\; P_C(t) \sim t.$$

We see that in this case, Copernicus' function of the speed of precession and function of precession have the same character as Ptolemy's analogous functions. But, only for the below condition:

(34a)
$$T_D = \text{const}_1' = T_{Pt} = 36000^{ey},$$

these pairs of functions are observationally equivalent, that is:

(34b)
$$\omega_C(t) = 1/100\ [°/ey] = \omega_{Pt}, \quad P_C(t) = P_{Pt}(t).$$

But, let us now turn to the general case (GC) of Copernicus' theory of precession given by formulas (25-31c) and schematically described below:

(35, GC) if $T_R > T_D(t)$, than
$$T_{C\text{mean}} \neq T_{Pt} = 36000^{ey} \;\&\; \omega_{C\text{mean}} \neq \omega_{Pt} = 1/100\ [°/ey] \;\&\; P_C(t) \not\sim t \neq P_{Pt}(t).$$

At first sight, this general case of Copernicus' theory of precession differs completely from the analogous of Ptolemy's theory. However, this is only a logical illusion, since in this case, for the limit *t* -> tpt, the following condition is realized:

(36)
$$\omega_C(t) \underset{t \to t_{Pt}}{\longrightarrow} \omega_C(t = t_{Pt}) \approx \omega_{Pt} \approx 1°/100^{ey}$$

Moreover, this magnitude was approximately constant during the relatively long passage of time of about 400^{ey} lasted between Timocharis and Ptolemy. In such the case, Copernicus' function of precession should effectively be a linear function of time with the rate of precession approximately equal to Ptolemy's. However, beyond this period, Copernicus' function of precession should effectively differ from Ptolemy's linear function. To be exact, let us note, that, in fact, Copernicus' function of precession is always non-linear. Nevertheless, it may be treated as linear, but only for the relatively small interval of time in comparison to the value of the period of the nonuniform changes of the declination (and of the tropical year) fixed (initially) for about 2000^{ey}. But, to minimize the non-linear effects for every optional moment of time between Timocharis and Ptolemy, the following condition should be assumed. The function f_{Nu} reaches its maximum value (= 1) when, firstly, its argument is equal to 0, and secondly, it happens in the moment of time (approximately) between the moments of measurements of Timocharis and Ptolemy:

(37)
$$f_{NU}(\varphi_{0NU} + \frac{360° t_0}{T_{NU}}) = \text{maximum} = 1$$

when $\varphi_{0NU} + \frac{360° t_0}{T_{NU}} = 0$ and $t_0 \approx \frac{|t_{Pt}| + |t_T|}{2}$.

In this place, we may ask the following questions. What do mean the above considerations from the methodological point of view? What does mean the fact, that the two different theories, namely, Copernicus' nonlinear theory of precession of equinoxes and Ptolemy's linear theory of the second motion of the firmament, are observationally equivalent in a certain interval of time near Ptolemy's age, and they are observationally non-equivalent beyond it? We think that these facts prove that Copernicus' theory of precession was framed in conformity with the correspondence postulate so that this theory was linked with the analogous Ptolemy's theory by the proper correspondence principle.

Hence, already on the grounds of the *Commentariolus,* we are justified in stating that Copernicus long before Newton, Fresnel and Young, Planck, Bohr, Einstein or Heisenberg, used the Method of Correspondence Thinking (MCT).

Of course, this thesis may seem strange within the context of existing historico-methodological knowledge. Nevertheless, it is right. To make sure of that, we will look for its further corroboration in the *De revo- lutionibus*.

Finishing our analysis of the *Commentariolus,* note, that Copernicus has not created the exact theory of the long-period phenomena yet. To do it, he will have to make what follows. (1) To find the concrete shape of the function (= the set of circles moving uniformly around the proper centers) modelling the non-uniform changes of the tropical year or the precession. (2) To explain the change of obliquity of the earth's equator using the language of motions of the earth and to find the concrete shape of this function (= the set of circles), since, in principle, this problem was not touched in the *Commentariolus*. (3) To find the values of parameters of his models.[70]

Moreover, because Copernicus searches for the theory of the long-period phenomena, he has to know the exact observational data both for

[70] Moreover, he will have to frame theories of the long-period changes of the eccentricity of the earth, and of the position of the line drawn by points of the maximum and minimum distance of the earth from the sun. He will have to do it, because, in the *Commentariolus,* he accepted the values of the above magnitudes, similarly like of eccentricities of planets, given in the *Alphonsine Tables* and not in *the Almagest.* But, these values were different.

historical ages and his own days. But in order to achieve this purpose, he must make what follows. First of all, to find the chronological canon of different calendars used by the earlier Greek, Arabian and Europeans astronomers. Especially, to fix the true names of Egyptian months used by Ptolemy and his successors and the true names of Greek astronomers mentioned by Ptolemy's Almagest. And next, to harmonize precisely the chronology of these data. There were really very difficult questions because fundamental sources of historical astronomical data, as, for instance, Puerbach and Regiomontanus' *Epitome in Almagestum Ptolo- maei* (Venice 1496) and *Almagest* in the translation from Arabian text by Gerhard of Cremony (1154) (printed edition Venice 1515) were full of many many translation errors and modifications.[71] Secondly, he must make new observations. We read in the *De revolutionibus:*

[71] One of them is connected with Hipparchus who, in the *Epitome in Almagestum Ptolemai* by Puerbach and Regiomontanus, is called rightly by Puerbach, but is named Abrachis by Regiomontanus. Let us say more about it fixing our attention on the issue of the length of tropical year and the change in longitudes of fixed stars.

As we have seen, Copernicus, after the *Epitome* (III, Prop, ij), assumed in the *Commentariolus* that Hipparchus and Ptolemy measured the following values of the tropical year:

$$T_H = 365^d6^h, \qquad T_{Pt} = 365^d5^h55^m.$$

Let us now use these values and find, from the formula (25), the corresponding to them values of the SDeed of the chanse in the lonsitude. We set what follows.

$$\omega_H = 0{,}007 \approx 1/143\ [°/_{ey}],\ \omega_{Pt} = 0{,}010 = 1/100\ [°/_{ey}].$$

Moreover, from the *Epitome* (B. VII, Prop, iiij), Copernicus knew, that during 265^{ey} lasted between two observations of the star Spica by Abrachis and Ptolemy, its longitude increased 2 2/3°. Thus the mean speed of this (and others) star(s) during this interval of time was equal:

$$\omega_{A\text{-}Pt} = (2\ 2/3)/265 \approx 0.0010 = 1/100\ [°/_{ey}].$$

Moreover, from the *Epitome* (B. VII, Prop, v), Copernicus knew, that during 391^{ey} lasted between two observations of the star Spica by Timocharis and Ptolemy, its longitude increased 3;55°. Thus the mean speed of this (and others) star(s) during this interval of time was equal:

$$\omega_{T\text{-}Pt} = 3;55/391 \approx 0.097 \approx 1/100\ [°/_{ey}]$$

Next, after reading (about 1503-1515 A.D.) the Greek *Commentary to Aratos* by Theon (1503 Venice), Copernicus finds out that Abrachis is Hipparchus, in fact (see Birkenmajer [1900], p. 135/6 fn. 3). Hence, on the ground of above considerations, it is evident that Hipparchus' length of the tropical year must be (nearly) equal to Ptolemy's $365^d5^h55^m$ and not to 365^d6^h. Therefore Puerbach's and Regiomontanus' *Epitome*, and Copernicus' *Commentariolis* erred in this point.

It is an interesting that Copernicus will not notice this fact even in the *De revolutionibus* (III, 13). He states there, namely that Hipparchus thought that the tropical year is a little shorter than $365\ 1/4^d$, and that Ptolemy firstly measured that this difference is equal to 1/300 days. But Copernicus did knew the *Almagest* (Venice 1515), where (in b. III, ch. 1) it was stated clearly that Abrachis (= Hipparchus) measured $365\ (1/4 - 1/300)^d$ as first.

> For not so long ago under the Leo X the Lateran Council [1514—1517] considered the question of reforming the ecclesiastical calendar. **The issue remained undecided then only because the lengths of the year and month and the motions of the sun and moon were regarded as not yet adequately measured. From that time on,** at the suggestion of that most distinguished man, Paul, bishop of Fossombrone, who was then in charge of this matter, **I have directed my attention to a more precise study of these topics** [my bold]. (Ore *the Revolution*, Preface, p. 5—6)

Moreover, to frame theory of considered phenomena, Copernicus must harmonize data with assumed their geometric-arithmetical model. It was also very difficult stage since he had to make many calculations. Because the tasks mentioned above were not easy, Copernicus will searches them for many years separating the creation of the *Commentariolus* (c. 1510) and.De *revolutionibus* (c. 1516-1543).[72]

B. *De revolutionibus*

Cutting long story short, in the De *revolutionibus,* Copernicus realizes finally the task formulated already in the *Commentariolus* and after many years of his own theoretical and observational searches, he finds a more general theory of the long period-phenomena than Ptolemy's. Using cosmology of the mobile earth and the immobile stellar sphere, Copernicus selects ultimately the concrete shape of the searched functions, or to be exact, the sets of circles modelling the course of the long-period phenomena.

Fixing, for brevity, our attention only on the quantitative aspect, let us discuss Copernicus' function of the change in stellar longitudes of fixed stars, and function of the obliquity of the earth's equator.[73] Next, since, according to Copernicus, the change in stellar longitudes of fixed stars is due to the precession of equinoxes, that is, to the motion of equinoxes, Copernicus' function describing the changing stellar longitudes of the fixed stars will be called below the precession, briefly.

[72] The fundamental work about mentioned issues is L. A. Birkenmajer [1900], About the Copernicus' searches for the chronological canon see L. A. Birkenmajer [1900] p. 38-39, 139-147, 244-246, 279-282; on Copernicus' emendation to *Almagest* (Venice 1515) p. 242- 292; about Copernicus' own observations: p. 317-319.

[73] Copernicus' model of the precession and the obliquity of the earth's equator is described in the *De revolutionibus,* I, 11; III, 1-12. On this issue see also: the studies of Dobrzycki [1964], Moesgaard [1968], Swerdlow [1980], and Swerdlow and Neugebauer [1984], are referred.

Functions of: precession and speed of precession

According to the *De revolutionibus,* and, what is necessary to recall, in agreement with the *Commentariolus* (see p. 57-58 and the formulas (25-28), especially), the precession is composed of two parts: namely, the first, called the mean precession, is uniform, and the second, called the trepidation or libration of precession, is non-uniform. In the case of the *De I'evolutionibus,* the periods of the above mentioned components of precession, denoted by Tcmean and TNUP, respectively, are equal to 25816^{ey} and 1717^{ey}, respectively. The resultant Copernicus' precession is equivalent to the following function:

(38a) $$P_C(t) = A + Bt - C\sin(2\varphi)$$

(38b) $$\varphi = D + Et,$$

$P_C(t)$ – Copernicus' function of precession,
φ – function called the inequality; 2φ – function called the inequality of equinoxes or the double inequality,
t – time elapsed since the birth of Christ and the time unit is 1^{ey} (1 Egyptian year of 365 days),

(38c) $$A = 5;32 = 5{,}53(3)\ [^\circ],$$

(38d) $$B = \frac{360^\circ}{T_{Cmean}} = \frac{360}{25816}\ [^\circ/\varepsilon\psi]$$

(38e) $$C = 1;10 = 1{,}16(6)\ [^\circ],$$

(38f) $$D = 6;45 = 6{,}75\ [^\circ],$$

(38g) $$E = \frac{360^\circ}{2T_{Cmean}} = \frac{360}{3434}\ [^\circ/\varepsilon\psi]\ .^{74}$$

It is obvious that the above values of the parameters have not been assumed by Copernicus accidentally, but in order to square the predictions of his theory with the phenomena. Let us try to ascertain about this fact looking at the below two tables. The first shows the comparison

[74] See Birkenmajer [1900] p. 200; Dobrzycki [1965] p. 38; Moesgaard [1968] p. 134; Mercier [1975] p. 60; Swerdlow [1980] p. 217; Goldstein [1994] p. 193.

Moreover, let us note that from the above function of precession we can get Copernicus instantaneous speed of precession, denoted by co$_r$(f):

$$\omega_C(t) = B - 2CE\,(\pi/180^\circ)\cos[2(D + Et)].$$

Remembering about the analogous issue considered in the *Commentariolus* (see the formula (27) at the page 57 especially) we see, that in the *De revolutionibus*, Copernicus selects the simplest acceptable periodic and non-uniform function of time fNU between the values of precession derived from observations and computed from Copernicus' theory:

Observer	Time[^{ey}]	Report [°]	Computed [^{tf}
Timocharis	-294	2;20	2;18
Hipparchus	-128	4;00	4;01
Menelaus	+98	6; 15	6;15
Ptolemy	138	6;40	6;40
al-Battani	880	18;10	18;10
Copernicus	1515	27;14	27;14
Copernicus	1525	27;21	27;20 [75]

The second shows the comparison between the values of the mean speed of precession and its components, derived from observations (the indicator O) and computed from Copernicus' theory (the indicator C) on the ground of the data given in the first table. To clarity let us add that the mean speed of precession is defined in the following way:

(39)
$$\omega_{\Delta C} = \frac{\Delta P_C}{\Delta t} = \frac{P_C(t_1) - P_C(t_0)}{t_1 - t_0}$$

for $t \in \langle t_1, t_0 \rangle$ and $t_1 \neq t_0$

Observer	Δt [^{ey}]	ΔP_0 [°]	$\omega_{\Delta 0}$ [°/$_{ey}$]	ΔP_C [°]	$\omega_{\Delta C}$ [°/$_{ey}$]
Pt-Tim	432	4;20	1/100	4;22	1/99
Pt-Hip	266	2;40	1/100	2;39	1/100
Alb-Men	782	11;55	1/66	11;55	1/66
Alb-Pt	742	11;30	1/65	11;30	1/65
Cop-Alb	645	9;11	1/71	9;10	1/71 [76]

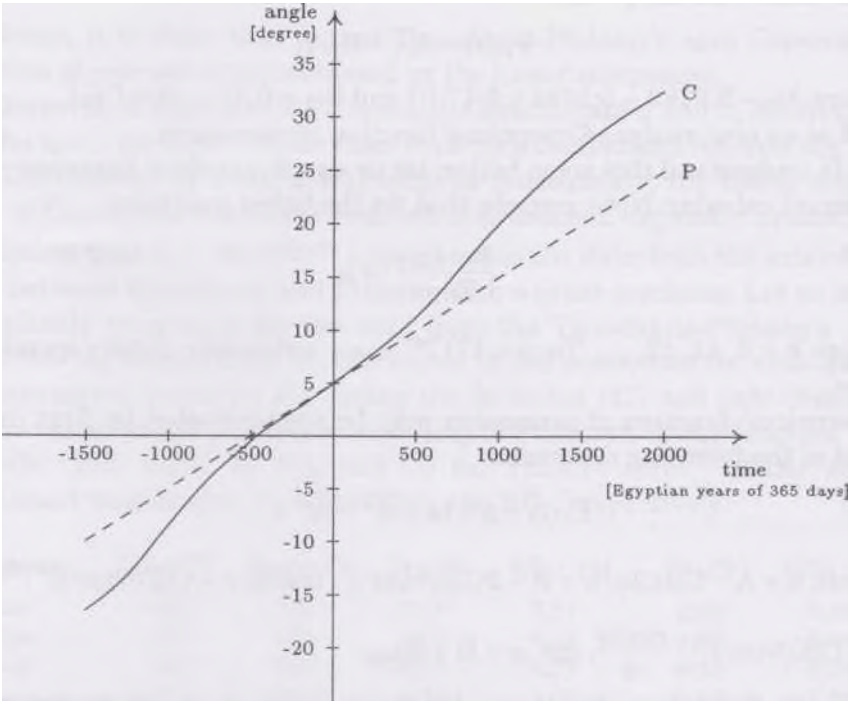
75 The shape of this table, in consideration of clarity, is taken from Swerdlow [1980] and Swerdlow, Neugebauer [1984]. The content of this table differs from the analogous of Swerdlow [1980] p. 219 and Swerdlow, Neugebauer [1984] p. 147 only slightly. The essence of this difference lies in two facts. Firstly, according to the *De revolutionibus* (B. Ill, ch. 2-12), we chose approximated data slightly different from ones chosen by them. Secondly, in our column "Time" are given the interval of time elapsed from the era of Christ in Egyptian years (of 365 days), while in Professors N. M. Swerdlow and O.Nuegebauer's column "Data" are given the data measured in Julian years (of 365 1/4 days) (compare Swerdlow, Neugebauer [1984] table I on p. 551). But such differences do not influence the accuracy of computations. Regarding the content of this table, moreover compare: Rheticus *Narratio Prima*, 116 in Rosen [1971], p. 116.

76 Regarding the shape of this table see first point of the former footnote. Regarding the content, this table is a development of the table of Swerdlow, Neugebauer [1984] on page 132 and is composed not only of the part called "Data in text" but also of "Computed". Contrary to Professors Swerdlow and Neugebauer, but in accordance with the approximated intervals assumed in the preceding table, we assume the value 742^{ey} and not 741^{ey}.

Hence, we see that Copernicus saves phenomena remarkably.

Let us note here only the one consequence resulting from these tables. For the Timocharis-Ptolemy's ages, except a small difference for Timo- charis's data, the graph of Copernicus' function of precession and the corresponding to it the graph of Ptolemy's do not differ essentially. But beyond this period of time, they do. Let us illustrate this fact graphically:

Copernicus' and Ptolemy's functions of precession: C(T) and P(T)



Thus one sees clearly "that Copernicus's precession **corresponds very closely** [my emphasis] to Ptolemy's from -300 to +200, covering the period of the observations used by Ptolemy [...]"[77]. In this place, we may and should ask the following question. What do the above facts mean from the methodological point of view, and especially, what do they mean in the light of the HDMCT? It means quite a lot, since they show that, in the *De revolutionibus,* Copernicus realizes a certain sort of correspon-

We do it since the first value was mentioned by Copernicus in tree chapters (*De revolutionibus*, B. III, Ch. 4,7,9) and the second one only in one {*De revolutionibus,* B. III, Ch. 2).

[77] Swerdlow[1980], p. 217-218.

dence principle which links Copernicus' and Ptolemy's functions of the change in stellar longitudes of fixed stars.

In order to make sure of this fact, let us now analyse the course of Ptolemy's and Copernicus' functions of the change in longitudes of fixed stars for a special interval of time from Timocharis to Ptolemy.

Firstly, we remember, that, according to Ptolemy, the speed of the change in longitudes of fixed stars is constant. Therefore, using the data given in first table on page 63 and the method of linear regression, we get such form of Ptolemy's function:

(40) $$P_{Pt}(t) = A_{Pt} + B_{Pt} \cdot t,$$

where $A_{Pt} \approx 5{,}2780 \approx 5;1641 \approx 5;17$ [°] and $B_{Pt} \approx 0{,}01 \approx 1/100$ [°/ey].

Let us now analyse Copernicus' function of precession.

To understand this issue better, let us use the modem language of the different calculus. Note, namely, that for the below condition:

(41) $$\frac{|t - t_{OP}|}{T_{NUP}} \rightarrow n$$

where $n = 0, \pm 1, \pm 2, \ldots$, $T_{NUP} = 1717^{ey}$ t_{OP} – optionally chosen moment of time.

Copernicus function of precession may be approximated by first component of the following expansion:

(42) $$P_C(t) = a + bt + ct^2 + dt^3 + \ldots$$

where $a = A - C\sin 2\varphi$, $b = B - 2C[E(\pi/180°)] \cdot \cos 2\varphi$, $c = C[E(\pi/180°)]^2 \cdot \frac{\sin 2\varphi}{2}$, $d = C[E(\pi/180°)]^3 \cdot \frac{\cos 2\varphi}{6}$, for $\varphi = D + E \cdot t_{OP}$.

From these formulas it results many interesting consequences. For instance, the all coefficients of the above expansion oscillate with period of 1717^{ey}. Next, the changing coefficient 'b', being the instantaneous speed of precession, has the following maximum, mean and minimum values:

(43a) $b_{max} = B + 2CE \approx 1/55$ [°/ey] for $t = 922{,}8875 + n1717$ [ey],

(43b) $b_{mean} = B \approx 1/72$ [°/ey] for $t = 364{,}8625 + n1717/2$ [ey],

(43c) $b_{min} = B - 2CE \approx 1/103$ [°/ey] for $t = -64{,}3875 + n1717$ [ey],

where $n = 0, \pm 1, \pm 2, \pm 3, \ldots$.

Let us compute now the values of the above coefficients a, b, c, d only for the special moment of time for Timocharis-Ptolemy's ages when the inequality of equinoxes, is equal to zero, that is, when:

(44) $$2\varphi = 0 \Leftrightarrow t_0 = -64{,}3875^{ey}\ .^{78}$$

For this special moment of time we get:

(44a) $$a \approx 5{,}2584 \approx 5;1531 \approx 5;16\ [°]\ ,$$

(44b) $$b \approx 0{,}0097 \approx 1/103\ [°/ey]\ ,$$

(44c) $$c = 0\ [°/(ey)^2]\ ,$$

(44d) $$d \approx 1{,}2 * 10^{-9}\ [°/(ey)^3]\ .$$

Hence, it is clear, that during Timocharis-Ptolemy's ages Copernicus' function of precession is dominated by the linear component.

Moreover, it appears that Copernicus' coefficients a and b, determined for the to = - 64,3875^{ey}, differ than Ptolemy's coefficients A_{pt} and B_{pt} [see formula (40a-b)] in a slight way only. In consequence, the linear component of Copernicus' nonlinear function of precession expanded around the moment of time t_o = -64,3875^{ey} approximates the data from the interval of time between Timocharis and Ptolemy with a great precision. Let us show it explicitly computing for the data from the Timocharis-Ptolemy's eras the following magnitudes: (1) the values of the precession for such linear approximation, noted by P_{CL} [using the formulas (42) and only (44a-b)], (2) the values of the precession according to Ptolemy's linear function [the formula (40)], noted by P_{pt}, and (3) the relative error[79] of the above mentioned magnitudes, noted by RE_{CL} and RE_{pt} respectively:

Observer	Time [ey]	Report [°]	P_{CL} [°]	RE_{CL} [%]	P_{Pt} [°]	RE_{Pt} [%]
Tim	−294	2;20	2;25	3,57	2;20	0,00
Hipp	−128	4;00	4;01	0,42	4;00	0,00
Men	+98	6;15	6;13	0,53	6;16	0,27
Pt	+138	6;40	6;36	1,00	6;40	0,00

From this table we may read, among other things, that the precision of the linear component of Copernicus' function of precession expanded around the moment of time *to* = -64,3875^{ey} is the best between Hipparchus (-128^{ey}) and Menelaus (+98^{ey}), a bit weaker for Ptolemy (+138^{ey}) and [* 7]

[78] Let us note here, that this moment of time is approximately in the middle between Timocharis (= -294^{ey}) and Ptolemy (= 138^{ey}).

[79] By the relative error of a magnitude X which measurement value, determined by a certain measurable procedure, is equal to X_M, and a theoretical value, computed within the context of a certain model, is equal to X_T., we assumed the magnitude RE(X): RE(X) = 100% $|(X_T-X_M)/X_M|$.

the worst for Timocharis (-294^{ey}). Thus we see that the relative error increases when the interval of time counted from $t_0 = -64{,}3875^{ey}$ increases.[80]

It becomes thus evident, that for the limiting condition:

(45)
$$\frac{|t - t_0|}{T_{NUP}} \to 0 \quad \text{and} \quad t_0 = -64{,}3875^{ey}$$

Copernicus' non-linear function of precession approaches to Ptolemy's linear function:

(46)
$$P_C(t) \xrightarrow[\frac{|t - t_0|}{T_{NUP}} \to 0]{} P_{Pt}(t)$$

Hence, for the condition (46), Copernicus' function of the instantaneous speed of precession, denoted by $\omega_C(t)$, approach to Ptolemy's analogous function, denoted by $\omega_{Pt}(t)$:

(47)
$$\omega_C(t) \xrightarrow[\frac{|t - t_0|}{T_{NUP}} \to 0.]{} \omega_{Pt}(t).$$

In other words, the linear approximation of Copernicus' non-linear function of precession modells the course of precession for Timocharis - Hipparchus - Ptolemy's ages.

Moreover, it is worthy noticing that for the case when

(48)
$$\frac{|t-t_0|}{T_{NUP}} \to n, \quad n = 1, 2, \ldots \quad \text{and} \quad t_0 = -64{,}3875^{ey}$$

Copernicus' function of precession [Pc(t)] is also very well approximated by a straight line. However this line doesn't fuse with Ptolemy's line [PPt(f)], but is parallel to it:

(49)
$$P_C(t) \xrightarrow[\frac{|t - t_0|}{T_{NUP}} \to n]{} \parallel P_{Pt}(t)$$

where the sign '$\to\parallel$' means – "approaches for being a parallel to".

[80] In addition, let us note that if Copernicus' function of precession be expanded around, for instance, Ptolemy's data of measurement (+138), we will get that the relative error of the linear approximation of Copernicus' function is smallest and near or equal to zero just for the surroundings of Ptolemy's data.

It means, that though for the condition (48) Copernicus' function of precession doesn't approach to Ptolemy's, nevertheless, for the more general condition:

(50)
$$\frac{|t-t_0|}{T_{NUP}} \to n, \qquad n = 1, 2, \ldots \qquad \text{and} \qquad t_0 = -64{,}3875^{ey}$$

Copernicus' function of the instantaneous speed of the precession, [coc(Q], approaches to Ptolemy's analogous function [copt(t)]:

(51)
$$\omega_C(t) \to \omega_{Pt}(t).$$
$$\frac{|t-t_0|}{T_{NUP}} \to n$$

Furthermore, from the above described properties of Copernicus' non-linear function of precession it results the other interesting conclusion. It appears, namely that Ptolemy's theory of "precession" with its constant values of parameters might be applied only for a relatively small period of time = 400^{ey}:
a) dining Timocharis-Hipparchus-Ptolemy's ages;
b) for every other epoch = 400^{ey} redetermining the values of the initial phasis or/and the speed of precession of the Ptolemy's theory. In such both cases, the non-uniform and non-linear character of the astronomical phenomena (verified by the many ages' astronomical observations and accepted by many Medieval and Renaissance astronomers) had an imperceptible influence on the values of the parameters of Ptolemy's function of "precession".[81]

Now we should ask the following question: What do mean the above mentioned properties of Copernicus' function of precession from the methodological point of view? However, not being acquainted with the HDMCT, we might say practically nothing more about this subject. But knowing it, we see at once what follows. The restriction of the applicability of Ptolemy's theory that is dictated by Copernicus' theory, is characteristic for the correspondence relation linking two theories. Similarly, the same question occurs in the case of the correspondence relation which links, for instance, the STR (Special Theory of Relativity) and the CM (Classical Mechanics), the QM (Quantum Mechanics) and the CM. In consequence:
(i) the period $T_{NUP} = 1717^{ey}$, being the fundamental constant of the non-uniform alterations of Copernicus precession, must be and is the

[81] It explains also why the different values of the rate of precession were determined by astronomers during long passage of time, for instance, Timocharis 1772^{ey}, Hipparchus and Ptolemy 17100^{ey}, al-Battani $1^{o}/66^{ey}$, Copernicus 177 l^{ey}.

analogue of the fundamental constant of the STR or the QM: c (speed of light) or h (Planck's constant), respectively;
(ii) the condition (45), being the analogue of, for instance, the limiting condition $v/c \to 0$ defined within the context of the STR, must be and is the limiting condition of Copernicus' theory of precession.[82]

Of course, we should remember also, that Copernicus and Ptolemy explain the phenomenon of precession in a completely different way, because they assume different cosmology, that is, they accept different quasi-entities. But, on the other hand, we know, that exactly the same happens, for instance, in the case of the correspondence relation which links the above mentioned theories: the STR and the CM or the QM and the CM.

Having the above mentioned facts in view, we conclude finally that Copernicus' theory of precession of equinoxes is linked with Ptolemy's theory of the long-period motion of fixed stars by the correspondence relation. Let us write it using the schematic formulation introduced earlier (see the formula (4) on page 20):

(52)
$$\langle P_C, P_{Pt}; \frac{|t - t_0|}{T_{NUP}} \to 0 \rangle \,,$$

(53)
$$\langle \omega_C, \omega_{Pt}; \frac{|t - t_0|}{T_{NUP}} \to n \rangle \,.$$

where $t_0 = -64{,}3875^{ey}$, $T_{NUP} = 1717^{ey}$, $n = 0, 1, 2, \ldots$.

Copernicus' function of the obliquity of the earth's equator and Ptolemy's function of obliquity of the ecliptic

Now let us say a few words about the one of the most fundamental magnitudes of spherical astronomy, that is, the obliquity of the earth's equator, according to Copernicus, or the obliquity of the ecliptic, according to, for instance, Hipparchus and Ptolemy.[83]

[82] Let us note two things. Firstly, regarding the speed of the precession, the limiting condition is more general, because is defined by the condition (61). Secondly, it is possible to make the analogous considerations for the period of the mean of precession T_{up} that is, in the *De revolutionibus,* equal to 25816^{ey}. If it was assumed that the amplitude of the non-uniform precession is equal to zero, and that $T_{UP} \to \infty$ or $t / T_{UP} \to 0$, we would get that the value of the precession vanishes to zero. It is exactly as astronomers thought before Hipparchus.

[83] Let us recall, that the magnitude of the obliquity of the earth's equator or the obliquity of the ecliptic, is so important, because for a given geographical latitude it determines many phenomena seen from the earth. Let us give three examples, which may

According to Ptolemy, the obliquity of the ecliptic, denoted by ept, is a constant magnitude:

(54) $$\varepsilon_{Pt} = 23;51,20°.$$

Contrary, according to Copernicus, this magnitude, or to be exact, the obliquity of the earth's equator, denoted by ε_C, is a periodic variable. Its oscillations, called in those days the librations or trepidations, are described by the mentioned below formula:

(55) $$\varepsilon_C(t) = \varepsilon_M + \Delta\varepsilon\cos(D + Et),$$

$\varepsilon_C(t)$ – Copernicus' function of the obliquity of the earth's equator,
t – time elapsed since the birth of Christ and the time unit is 1^{ey} (1 Egyptian year of 365 days),
$\varepsilon_M = 23;40 = 23,6(6)$ [°],
$\Delta\varepsilon = 0;12 = 0,2$ [°],
$D = 6;45 = 6,75$[°],
$E = 360/3434$ [°/ey] .[84]

From this formula it results, among other things, that the changing Copernicus' function of the obliquity of the earth's equator has the following maximum, mean and minimum value:

(56a) ECmax = 23;52 [°] for t = -64,3875 + n3434 [ey],

(56b) ECmean = 23;40 [°] for t = 794,1125 + n1717 [ey],

(56c) ECmin = 23;28 [°] for t = 1652,6125 + n3434 [ey],

and $n = 0, \pm1, \pm2, \pm3, \ldots$

Hence, at first sight, Ptolemy's magnitude and Copernicus' differ completely since the former is constant and the latter is variable. But, though, it is the logical truth, nevertheless, it is not the whole truth. To see this statement clearly, let us compare the value of the obliquity of the earth's equator computed from Copernicus' theory and derived from observation:

be interesting for every mortal: the maximum and minimum values of the length of shadow thrown by the unit gnomon (that is perpendicular to the locally flat surface of the earth), the maximum and minimum values of daylight, the position of the tropics and the polar circle on the terrestrial globe.

[84] See, for instance, Moesgaard [1968] p. 134; Swerdlow [1980] p. 220.

Observer	Time H	Report [°]	Computed [°]
Timocharis	-294	23;51,20	23;50,57
Aristarchus	-262	23;51,20	23;51,13
Hipparchus	-128	23;51,20	23;51,55
Ptolemy	+138	23;51,20	23;51,11
al-Batatni	+880	23;35	23;38,07
Arzachel	1070	23;34	23;34,11
Prophatius	1300	23;32	23;30,24
Copernicus	1515	23;28,30	23;28,22
Copernicus	1525	23;28,24	23;28,19 [85]

For brevity, let us illustrate only the results obtained for Timoeharis-Hipparchus-Ptolemy's ages by the quotations from Rheticus' *Narratio Prima* and next from Copernicus'He *revolutionibus'.*

> In the time of Timocharis, Aristarchus, and Ptolemy **the change in the obliquity was very slow, so that they believed the maximum arc of declination to be invariable** [my bold], always having the value of 11/83 of a great circle [11/83 *360° = 47°51'20", which makes the obliquity 23°51'20"]. (Rheticus, *Narratio Prima,* p. 118.)

> Ptolemy reported that the obliquity of 23°51'20" had not changed at all in the 400 years before him since Aristarchus of Samos. **Hence this shown that it then stayed nearly steady around the limit of maximum obliquity, when of course the precession of the equinoxes was also having its slowest motion** [my bold]. (*On the Revolutions*, III, 6, p. 129.)

Hence we see that, though, according to Copernicus, the obliquity of the earth's equator changes, nevertheless, its alteration is not accidental. The shape and parameters of Copernicus' function of variable obliquity of the earth's equator, are chosen in such a way that, to imitate, as it is possible the best, Ptolemy's constant value of the obliquity of the ecliptic in Timocharis-Aristarchus of Samos-Ptolemy's ages.

It is proper to ascertain about this fact in another way. Firstly, let us make the expansion of the above Copernicus' function around an optionally taken moment of time top.

[85] Regarding the shape of this table see first part of the footnote 75. Considering the basic content, unlike Swerdlow [1980] on page 220-221 or/and Swerdlow, Neugebauer [1984] on page 147 we assume: firstly, for Aristarchus approximated interval of time (_262[ey]) and not the date (-279"), secondly, for Hipparchus approximated interval of time (-128[ey]) and not the date (-145[iy]), and, in addition, we place in our table the measurement of Timocharis (—294[ey]). Our computations differs slightly from Professors N. M. Swerdlow and O. Neugebauer's. The biggest difference is for Aristarchus 27", much more less for Hipparchus 3" and for others 1" at most. Regarding the content of this table, moreover compare: Rheticus *Narratio Prima* in Rosen [1971], p. 118-119.

For the condition:

(57) $$\frac{|t - t_{OP}|}{2T_{NUP}} \to n, \quad n = 0, 1, 2, 3, \ldots$$

we get:

(58) $$\varepsilon_C(t) = \varepsilon_M + \Delta\varepsilon \cdot \cos\varphi[1 - \delta(t)],$$

where, $\varphi = D + Et_0$,

$$\delta(t) = [E(\pi/180°]\mathrm{tg}\varphi(t - t_{OP}) - [E(\pi/180°)]^2 \frac{\mathrm{tg}\varphi(t - t_{OP})^2}{2} +$$

$$+ [E(\pi/180°)]^3 \frac{\mathrm{tg}\varphi(t - t_{OP})^3}{6} + \ldots$$

Now let us restrict our interests only for $n = 0$, that is, for the period including the Timocharis' (-294^{ey}) and Ptolemy's ($+138^{ey}$) eras. From the above formulas it results, among other things, that the instantaneous speed of the change of the obliquity of the earth's equator is the smallest when the function of inequality (p is equal to zero. We already know that, for Copernicus' data (D, E and T_{NUP}), it occurs when $t_o = -64{,}3875^{ey}$. Moreover, we know also that this moment of time is not chosen accidentally by Copernicus at all, because it is approximately midway between Timocharis' (-294^{ey}) and Ptolemy's ($+138^{ey}$) eras and the more between Aristarchus of Samos' (—262^{ey}) and Ptolemy's ($+138^{ey}$). And, just for this moment of time, Copernicus assumes that the maximum of obliquity of the earth's equator of 23;52° happens. Next, Copernicus' function of the obliquity of earth's equator is even. Therefore, we see in the table on page 71, that the values of the obliquity of earth's equator computed for Aristarchus of Samos' (-262^{ey}) and Ptolemy's ($+138^{ey}$) eras, are approximately equal: 23;51,13° and 23;51,11°. These values differ slightly with 23;50,57° computed for Timocharis's era (-294^{ey}). Thus, during Timo- charis-Aristarchus of Samos-Ptolemy's epoch the obliquity of the earth's equator changes in very small range of about 0:01° around the maximum value of23;52°.

Next, let us take the following facts into account. The half of the period of time of 400^{ey} elapsed between Aristarchus of Samos (-262^{ey}) and Ptolemy ($+138^{ey}$) (and similarly of 432^{ey} elapsed between Timocharis and Ptolemy) is relatively small in comparison to the period of the non-uni- form alterations of the obliquity of the earth's equator of 3434^{ey}. Moreover, the amplitude of the non-uniform changes of the obliquity

$(\Delta\varepsilon = 0;12°)$ is small in relation to the mean value of the obliquity $(\varepsilon_M = 23;40°)$.

In consequence, for the moments of time which get close to the Timocharis', Aristarchus', Hipparchus' and Ptolemy's dates of the measurements of the obliquity of the ecliptic (and, to be exact, the all analogous dates differed on the integer multiple of 3434^{ey}) Copernicus' value of the obliquity of the earth's equator approaches (approximately) to Ptolemy's constant value of the obliquity of the ecliptic. Thereby it appears, that the period of time between Timocharis, Aristarchus of Samos and Ptolemy, defines the limit region of Copernicus' theory of the obliquity of the earth's equator.

Simultaneously, as we remember, Copernicus explains the above mentioned phenomena by two motions of the earth and not by two motions of the firmament, as Ptolemy did.

Taking the above mentioned facts into account, we conclude: the correspondence principle links Copernicus' law of the obliquity of the earth's equator and Ptolemy's law of the obliquity of the ecliptic. In agreement with the assumed convention (the formula (4) page 20), let us write it schematically:

(59)
$$\langle \varepsilon_C, \varepsilon_{Pt}; \frac{|t - t_0|}{2T_{NUP}} \rightarrow n \rangle.$$

where $t_0 = -64{,}3875^{ey}$, $T_{NUP} = 1717^{ey}$, $n = 0, 1, 2, \ldots$.

Next, because according to Copernicus, the value of the obliquity of the earth's equator oscillates, the all functions which depend on this magnitude, also oscillate. Copernicus understands it perfectly. In the *De revo- lutionibus* we read:

> Now the ancient mathematicians used to divide the earth into seven climes by means of the several parallels of latitude passing, for example, trough Merone, Seyne, [...] and so on. [These parallels were selected on a threefold basis:] , the difference and increase in the length of the longest day [in the specified localities during the course of a year]; the length of the shadows observed by means of sundials at noon on the equinoctial days and the two solstices of the sun; and the altitude of the pole or width of each clime. **These quantities, having partly changed with time, are not exactly the same as they once were. The reason is, as I mentioned, the variable obliquity of the ecliptic, which was overlooked by previous astronomers. Or, to speak more precisely, the reason is the variable inclination of the equator to the plane of the ecliptic. Those quantities depend on this inclination. But the altitudes of the pole, or the latitudes of the places, and the shadows on the equinoctial days agree**

with the recorded ancient observations. This had to happen, because the equator follows the pole of the terrestrial globe. Therefore those climes likewise are not draw and bounded with sufficient precision by means of any impermanent properties of shadows and days. On the other hand, they are delimited more correctly by their distances from the equator, which remain the same forever. But that variation in the tropics, although it is quite small, in southern localities allows a slight difference of days and shadows, which becomes more perceptible to those who travel north [my bold], (On *the Revolutions,* **II, 6, p. 61)**[86]

But, as we know, for the moments of time which get close to Timocharis- Ptolemy's ages the values of Copernicus' function of the obhquity of the earth's equator approach (approximately) to Ptolemy's constant value of the obliquity of the ecliptic. Hence these functions are hnked by the correspondence principle. Just therefore, the all functions which depend on the obliquity of the earth's equator have the analogous property. Thus Copernicus' functions that depend on the variable obliquity of the earth's equator and Ptolemy's functions that depend on the constant obhquity of the ecliptic, are hnked by the correspondence principle. In agreement with the assumed convention (the formula (4) page 20), let us write it schematically:

(60) $$\langle f_C(\varepsilon_C), f_{Pt}(\varepsilon_{Pt}); \frac{|t - t_0|}{2T_{NUP}} \to n \rangle$$

where $t_0 = -64{,}3875^{ey}$, $T_{NUP} = 1717^{ey}$, $n = 0, 1, 2, \ldots$.

On the grounds of the above considerations, we reason that Copernicus' theory of the long-period phenomena and Ptolemy's theory of the correspondence phenomena, are hnked by the correspondence principle.[87] Let us write it schematically:

[86] The strength of this beautiful quotation will increase more if, after Gamow's Mr Tompkins, we imagine that the period of the non-uniform alterations is very small and the amplitude of these variations is considerably grater than 0;24°. Let us emphasize: such intellectual experiments are indispensable for every theorist. For methodological reasons, Copernicus also had to use them.

[87] Thus it is clear that these theories, being geometrically and formally non-equivalent, are, in principle, observationally and numerically equivalent for Timocharis-Ptolemy's ages. However, beyond this region, they are geometrically, formally but also numerically and observationally non-equivalent. At least this question differentiates Copernicus and Albert of Saxony. Hence, at least for this reason it is necessary to reject the opinion, that Copernicus' theory is formally, geometrically, numerically and observationally equivalent with Ptolemy's.

(61)
$$\langle T_C, T_{Pt}; \frac{|t - t_0|}{T_{NUP}} \to n \rangle$$

where $t_0 = -64{,}3875^{ey}$, $T_{NUP} = 1717^{ey}$, $n = 0, 1, 2, \ldots$.

Hence, Copernicus systematically used the Method of Correspondence Thinking (MCT).

Fundamental conclusion

Copernicus, in the process of creating of his astronomical theory that would supersede Ptolemy's, consequently and constantly used not only the Hypothetico-Deductive Method but also the Method of Correspondence Thinking. Therefore he used the method called the Hypothetico-Deductive Method of Correspondence Thinking (HDMCT). Especially, he applied the correspondence postulate by help of which found the proper correspondence principles linking his theory and Ptolemy's.[88] Moreover, Copernicus used the HDMCT in the same style as, for instance, Niels Bohr, Einstein, Schrodinger and Heisenberg and many other modem theorists while creating their theories which supersede the previously valid ones.

Therefore, it appears, that, in substance, the same deeply mathematical spirit and ethos determined searches made by Copernicus, who was an astronomer, that is, according to the Greek tradition, a mathematician, and by modem creative theoretical physicists as Newton, Einstein, Heisenberg, Hawking, etc.

Many important different conclusions result from the above discovered historical fact. For brevity, only a few of them will be analysed or mentioned below.

4.3. But there was the genuine Copemican revolution!

From the methodological point of view "the Copemican revolution", understood now narrowly as the framing by Copernicus his theory, was

[88] There are the more arguments for our thesis that Copernicus used the hypothetico-deductive method of correspondence thinking systematically because he did it also considering the possibility of the annual and daily motions of the earth and the issue of the planet. Already in my paper given in the XIXth International Congress of History of Science 22-29.08.1993, Zaragoza (Spain) I maintained and gave some proofs that Copernicus when analysed the possibility of the annual and daily motions of the earth, used the correspondence relations, correspondence parameters, etc. For instance, the fourth Commentariolus' assumption is nothing but the definition of a correspondence parameter, called by me, the correspondence parameter of the annual motion of the earth, and the statement of its value. This subject will be explained widely in my next article.

"a genuine scientific revolution". It occurred for the following simple reason. Copernicus, searching for a more general theory than Ptolemy's, used all methodological elements of the HDMCT. Especially, he used the correspondence postulate and thanks to it, framed some untrivial correspondence principles that link his theory with Ptolemy's. And, though, for instance, Copernicus' and Ptolemy's theories of the long period phenomena are not geometrically equivalent, nevertheless, they are observationally equivalent in the special limit case for the assumed values of the correspondence parameters of the formulated correspondence principles. Moreover, Copernicus' theory was not only the product of imagination. In Copernicus' times, it had the reliable empirical basis. Copernicus, namely, in agreement with the Medieval and Renaissance astronomy, assumed that the different results of observations and measurements of the analogous or the same objects or phenomena made by the best astronomers from Antiquity to Renaissance were determined well. Therefore just, in order to save phenomena, Copernicus' theory was based on the results of observations and measurements from Antiquity to Renaissance times.

But, as it has been mentioned at the chapter devoted to the HDMCT, the correspondence principle of theories is a good determinant of "a scientific revolution" in the field of the so-called exact sciences. Note now that, according to theoretical physicists, "a scientific revolution", on the level of discovered regularities, is, in principle, very conservative and, on the levels of quasi-entities and assumed mathematical language, may be very revolutionary. In the case of Copernicus' own revolution, this revolution was conservative for the following reasons. Firstly, Copernicus used paradigmatic axioms of sphericity of celestial bodies and of uniform circular astronomical motions. Secondly, he accepted the historical and new data in order to save phenomena from Antiquity to Renaissance times. Lastly and especially, he formulated some correspondence principles that link Copernicus' and Ptolemy's theories (models) of the long period phenomena in such a way, that these theories are observationally equivalent for about four hundred years before Ptolemy.

On the other hand, Copernicus' theory, regarding the level of assumed quasi-beings, was extremely revolutionary. It didn't only consist in the change of the reference system since Copernicus' cosmology of mobile earth and immobile sky completely destroyed the geocentric vision of the Universe, the integrated system of the Greek cosmology, physics and philosophy that was assimilated and developed by the Medieval and Renaissance Christian Europe.

Many different consequences resulted from this fact for the whole European culture, for astronomy and physics, scientific and popular cosmology, humanistic philosophy and theology.

Regarding (humanistic) philosophy and popular cosmology, the hypothesis of the mobile earth carried people away from the centre of the Universe. In a short historical perspective, it caused great intellectual, theological anxiety and opposition to Copernicus' theory, in a long, it was the beginning of the conflict between belief and science and, at the same time, the beginning of the crisis of religious thinking. Therefore it was needful to find new philosophy of man and God but only after many years, it would clear for theology that the reading of the Bible in the light of the geostatic and geocentric vision of the Universe is wrong.[89]

Concerning more scientific questions, Copernicus' theory caused directly:

(i) the formulation of geocentric models equivalent to Copernicus' - c. 1568-C.1594: by Caspar Peucer (1525-1602), Joannes Praetorius (1537- 1616) and Joannes Magini (1557-1617);[90]

(ii) the formulation of geoheliocentric theories or the so-called Tychonic or semi-Tychonic systems - c. 1578-c. 1651: by Tycho Brahe (1546-1601), Paul Wittich (c. 1546-1586), Helisaeus Roślin (1548-1616), Nicholas Rey- mers Bar (Ursus) (1551-1600), Duncan Liddel (1561-1613), Simon Mayr (Marius) (1573-1624) and Giambattista Riccioli (1598-1671);[91]

[89] On the related questions see: Stimson [1917] p. 39-48; Kuhn [1957] p. 188-200; N. M. Wildiers "Weltbild und Theologie Vom Mittelalter bis Heute" Zurich, Einsiedeln, Koln: Beziger Verlag, Part II; Galileo Galilei, 350 ans d'histoire, sous le direction de Mgr Paul Poupard, Desclee, Tournai 1983; The Galileo Affair: A Meeting of Faith and Science. Proceedings of the Cracow Conference 24 to 27 May 1984, Specola Vaticana - Citta del Vaticano 1985; J. Życiński, *Sprawa Galileusza,* Wydawnictwo Znak Kraków 1991.

[90] The issue of the long-period model of precession and the change of the obliquity considered were by Caspar Peucer (1525-1602) in *Hypotyposes orbium coelestium* ... (Argentorati 1568: Theodosius Rihelius), p. 516-517; similar solutions were accepted by J. Magini *(Tabulae secundorum mobilium coelestium* (1585)), and Christopher Clavius (1537-1612).

The issue of planetary theory was considered, for instance, by Johannes Praetorius (1537—1616) in *Theoricae Planetarum per homocentrepicyclos respondentes placitis Copemi- ci Hypotheses Astronomicae* (1592), *Compendiosa enarratio Hypothesium Nic. Copernici, Earundem insuper alia dispositio super Ptolemaica principia* (1594).

On PeucePs conception see Westman (ed.) [1975], p. 24—27. On Magini's see Dobrzycki [1965], p. 39; on Praetorius and his conceptions Westman [1975], p. 289-305.

[91] It was so very important subject for astronomy in the turn 16th century that the mentioned above (in the main text) astronomers, except Riccioli, were connected with the great quarrel about the priority of the discovery of the Tychonic system.

See Ch. Schofield "The Tychonic and semi-Tychonic world systems" in R. Taton, C. Wilson (eds.) *The General History of Astronomy,* Vol. 2, Cambridge: Cambridge University Press 1989, p. 33-43; O. Gingerich and R. S. Westman [1988]; I. B. Cohen, *Album of*

(iii) the development of new physics - Giovani Battista Benedetti (c. 1530- c. 1590), Wiliam Gilbert (1544-1603), Galileo Galilei (1564-1642), Johannes Kepler (1571-1630), Rene Descartes (1596-1650), Giovani Alfonso Borreli (1608-1679), Christian Huyghens (1629-1695), Robert Hooke (1635-1703), Issac Newton (1642-1727).

Since these facts are direct consequences of Copernicus' own revolution, it is justifiable to say about "the Copemican Revolution" understood in a broad sense as the revolution in astronomy and physics, scientific and popular cosmology, humanistic philosophy, etc. But, in this place, you might oppose. After all, many Copernicus' measuring data and theoretical conceptions appeared wrong in the light of Brahe's new method of measurements, of Kepler's three laws (1609, 1619) and next Newton's calculus, dynamics and theory of gravitation (c.1665-1687). From this point of view, it seems right that Copernicus' theory is old fashioned, and even absurd and irrational.[92] But note, Copernicus' theory was neither Keplerian nor Newtonian. On the other hand, within its own context, Copernicus' theory was not perfect[93] (but which theory is?).

Science, From. Leonardo to Lavoisier 1450-1800, Charles Scribner's Sons New York 1980, p. 44-15.

[92] Such negative readings of an old theory (even the best), it is possible to make for many episodes of the History of the so-called exact sciences. It is necessary only to make following things. Firstly, to use new methods of measurements. Secondly, to make a methodological critic of old measurements. Thirdly, on the ground of first and second steps, to falsify an empirical basis of an old theory. And lastly, (about seventy years after the origins of the old theory) to take as the critical hermeneutics of the old theory, the subsequent revolutionary theories interpreted ontologically.

[93] Especially, the theory in latitudes there was the weakest point of Copernicus' system. It was Kepler who understood it very well and who stated on this subject the following. "For, being overjoyed that the apparent latitudes increase when the earth approaches the [other] planets, Copernicus still did not have the courage to discard the remaining additional Ptolemaic latitudes [which this approach by the earth would not produce]. On the contrary [for the purpose of accounting for these too] Copernicus imagined that the planes of the eccentrics oscillate. As a result of these oscillations the angle of the inclination [which is constant and fixed in Ptolemy] would vary. And this [oscillating angle would conform] not to the rules governing the motions of its own eccentric but [and this is like a monstrosity] [to those governing the motion] of the earth's orbit, which has absolutely nothing to do with [the planet's latitudes]." *(Gesammelte Werke,* III, 141 in E. Rosen, "The Achievement of Copernicus", *SC* XIII 1975 p. 128—129)

But, though Copernicus' theory in latitudes is week, nevertheless, there exists a certain possibility to defend Copernicus. Such possibility was noticed by Professor K P. Moesgaard who conjectured that "Copernicus' book was incomplete, less then half done. I also think, that Copernicus' was quite aware of that, and that furthermore this was the reason why Copernicus would not publish the book before Rheticus came to him." (SC XIII, p. 156)

On the other hand, though Kepler very criticized Copernicus' theory in latitude, he very appreciated Copernicus' cosmological thinking.

Nevertheless, from the historical and methodological points of view, it was not so bad that we might say that "a genuine astronomical revolution" was not Copemican but Keplerian and Newtonian, and that in astronomy or mathematical astronomy "there was not the Copernican revolution!" It is impossible to accept such theses, because, in the light of the HDMCT, the above mentioned critical reading of the Copernicus' achievements is too literal. It simply neglects methodological and historical contexts. For instance, Galileo and Kepler, who very criticized many Copernicus' conceptions, were Copemicanists. Kepler, who, using the HDMCT, developed the magnetic theory of gravitation and replaced perfect circles by ellipses, states in the *Epitome of Copernican Astronomy* (1618-1621):

> By what right do you make this also a part of Copernican astronomy, since that author abided by the opinion of the ancients concerning perfect circles?
>
> I admit that this formulation of the hypotheses is not Copernican. But because the part concerning the eccentric circle is subordinate to the general hypothesis which employs the annual movement of the Earth and the stillness of the sun: therefore the name comes from the more important part of the hypothesis. Moreover, this small part "of the hypothesis is bound up with necessary arguments arising from the repose of the sun and the movement of the Earth, the doctrines of Copernicus; and so this part has a good title for being referred to Copernicus.[94]

Moreover, Kepler, who very criticized Copernicus' theory in latitude, says about this question in *Astronomia Nova* (1609) the following:

> Copernicus, unaware of his own riches, made it his entire undertaking to expound Ptolemy, not the nature of things, to which, however, **he had come the closest of all (the investigators)** [my bold; ...].[95]

For the above historical and methodological reasons, it should be clear that the Copemican revolution was not only a genuine revolution in physics, cosmology and philosophy, but also, at first, in astronomy: It was the genuine astronomical revolution that was the beginning of the Scientific Revolution of 17th century. The scientific essence of the Copernican Revolution lays, in the rejecting the instrumentalistic interpretation of astronomy (so common during the Middle Ages and Renaissance), and in the unification of the celestial and terrestrial physics. The former found its expression, for instance, in the title of Kepler's *Astronomia Nova*

[94] Kepler, *Epitome of Copemican Astronomy, V, translated by Charles Glenn Wallis in The Great Books of the Western World*, vol. 16, Encyclopdia Britannica, Chicago 1952, p. 967.

[95] Kepler, *Gesammelte Werke,* III, 141 in E. Rosen, "The Achievement of Copernicus", *SC* XIII 1975, p. 128.

seu Physica Coelestis (The New Astronomy or Physics of the Heavens. The later found its mature form in Newton's *Philosophiae Naturalis Principia Mathematica* (1687).

4.4. HDMCT and the falsification of earlier hermeneutical readings of Copernicus' works

On the ground of the above considerations, it seems that, discussing the issue of the genesis, status and reception of every theory belonging to the so-called exact sciences, and, especially, Copernicus' theory, it is necessary to abandon the following threefold approach:

(la) Though, according to Copernicus, "Mathemata mathematicis scri- buntur" ("Mathematics is written for mathematicians"), nevertheless, the detailed analyses of mathematical considerations are not necessary to understand properly the methodological, philosophical and cosmological aspects of Copernicus' astronomical (= mathematical) works and the genesis of these works.[96]

[96] In general, this thesis was the fundamental but latent assumption of many and different kinds of works (biographies, case studies, historical syntheses, philosophical analyses) regarding the history and philosophy of the so-called exact sciences. In the particular case of the Copernican question, this assumption underlines, for instance, the works of Duhem [1908, Engl, transl. 1969], Wasiutyński [1938], Butterfield [1950], Kuhn [1957], Koestler [1959], Blumenberg [1975, Engl, transl. 1987], Hallyn [1987, Engl.transl. 1990],

For instance, Kuhn in his very important book *The Copernican Revolution. Planetary Astronomy in the Development of Western Thought* [1957] states on p. 134: "All but the introductory First Book is too mathematical to be read with understanding by anyone except a technically proficient astronomer. We must deal with its essential technical contribution in relatively nonmathematical paraphrase [...]"

Next without a detailed mathematical analysis of Copernicus' work he may say, for instance, that: **"Neoplatonism** [with its mystique conception of mathematical simplicity, the symbolic identification of the sun and God, etc] **completes the conceptual stage setting for the Copernican Revolution** [my bold], at least as we shall examine it here. For an astronomical revolution it is a puzzling stage, because it is set with so few astronomical properties. This absence, however, is just what makes the setting important. **Innovations in a science need not be responses to novelties within that science at all. No fundamental astronomical discovery, no new sort of astronomical observation, persuaded Copernicus of ancient astronomy's inadequacy or of the necessity for change. Until half a century after Copernicus' death no potentially revolutionary changes occurred in the data available to astronomers. Any possible understanding of the Revolutions's timing and of the factors that called it forth must, therefore, be sought principally outside of astronomy, within the larger intellectual milieu inhabited by astronomy's practitioners** [my bold]." Kuhn [1957] p. 132.

Knowing the HDMCT, it is impossible to accept the dichotomous division assumed by Kuhn: internal factor: astronomical observations — external factor: philosophy, since the

(Ib) To comprehend properly the progress of the so-called exact sciences, the genesis of the new theories (framed by the concrete scientists at the concrete moments of history) and their justification, it is sufficient to analyse only the mathematical schemes and results of measurements and to ignore less exact and more philosophical, more methodological questions.[97]

core of astronomy there are the specific methodology and quasi-ontology. Next, since, from Ptolemy's times, there were many new observations (the novelties — using Kuhn's language) which were inconsistent with Ptolemy's models, there existed two important problems in the Renaissance astronomy: (1) the problem of the Calendar that was connected with the searches for a long-period theory of astronomical phenomena; (2) the problem of a planetary theory — connected with the searches for a medium-period theory. Moreover, contrary to the central thesis of Kuhn's *The Copemican Revolution,* the Renaissance astronomers thought that former was more important than later. Moreover, former was complicated than later, but both, in my opinion, were analysed using the HDMCT.

[97] I think, that is the case of Dreyer [1906,53], Neugebauer [1957], Swerdlow [1973], Swerdlow and Neugebauer [1984].

For instance. Professor N. M. Swerdlow [1973] in his brilliant article states on p. 435 that: "The principle of uniform circular motion as Copernicus uses it is a mechanical principle about the rotation of a sphere and nothing more. It should not be understood as a philosophical, much less metaphysical, principle about the motion proper to the substance of the heavens. **Speculations about such things do not belong to the domain of mathematical astronomy** [my bold]."

The more instructive is the following quotation from Professors N. M. Swerdlow and O. Neugebauer's discussion on the purpose and limitations of their brilliant book *The Mathematical Astronomy in the De revolutionibus* [1984]: "What follows is intended as an examination of Copernicus's mathematical astronomy from a technical, rather than a philosophical or sociological, point of view. **While everything in *De revolutionibus* may not be mathematical astronomy, most of it certainly is** [my bold], and thus this limited exposition may provide a foundation of elementary knowledge to assist scholars who wish to study more profound subjects for which we are not qualified. We have nothing to say about whether Copernicus was a good German or a good Pole, of whether he was a Priest, a Pythagorean, a Platonist, a Neoplatonist, a Hermetist, of the spelling of his name, the title of his book, of whether he liked or disliked common people, and how such things may affected his astronomy. Historians and philosophers have already written more than we have been able to read about the meaning of Osiander's preface, the aesthetics of the heliocentric theory, whether the theory is heliocentric or heliostatic or geokinetic, about the influence of ancient philosophers whose writings were entirely lost, of medieval philosophers whose writings were entirely neglected, of Renaissance philosophers who thought the sun beautiful (and who does not?), and we have neither the desire nor the learning to add to this extensive literature. All of what we have to say about Copernicus's life is contained in the preceding biography, and all of what we have to say about his physical astronomy is contained in the preceding general essay. **It is our, perhaps naive, belief that when Copernicus wrote that mathematics is written for mathematicians, he meant it, and that his work may be understood within the history of mathematical astronomy** [my bold]." vol. I. p. 94

Though I share many of the above mentioned critical opinions, I do not think that astronomy should be comprehended only as a combination of purely calculational questions

(Ic) Regardless of the detailed historical and methodological questions it is possible to judge completely rightly the value of Copernicus' works and his scientific method.[98]

Next, it seems that the below interpretations relating to the genesis, status and reception of Copernicus' theory are not right.

(1) Framing the astronomical theories, described in the *Commentario- lus* and the *De revolutionibus,* Copernicus was seize with a veritable obsession of the uniform circular motion and of the sphere as the perfect shape![99]

and results of measurements (what Professors assume) because also the methodological issues of the hypothesis and of the justification of theory and, in general, methodology of the so-called exact sciences, belong to the core of a creative practice in the field of the mathematical astronomy. Therefore, regarding the genesis of Copernicus' conceptions of the motions of the earth, for instance, I do not think that (the passages or) the (whole) writings of the ancient and medieval philosophers had nothing influence on Copernicus. Regarding the question of the justification of Copernicus' theory, it does not reduce only to the results of measurements but also to the correspondence relations which link Copernicus' and Ptolemy's theories. Therefore, in my opinion, Professors Swerdlow and Neugebauer are not right when, for instance, discussing the issue of justification of Copernicus' theory they fall into psychologism (see, for instance, Swerdlow and Neugebauer [1984] vol. I. p. 20—21). Compare also the next points.

[98] In general, this thesis was the fundamental but latent assumption of many opinions on Copernicus made by modern scientists. It is the case, for instance, of Infeld [19551. Werle [19741 esDeciallv D. 322-325. Heisenberv [1935. 1936. 1946. 1949. 19591. in

[1936] on p. 7–9, in [1959] p. 45–47, [1975] in Gingerich (ed.) [1975] p. 225–226; Кудрявцев [1973] p. 79–81.

[99] See Butterfield [1950] p. 29, 30, Koestler [1959] p. 201, 211, Koyre [1961] p. 10.

For instance, Butterfield on p. 29-30 writes: "Not only is Copernicus prodded and pressed into overturning the old system by **a veritable obsession for uniform circular motion** [my bold; ...], but in facing the two biggest problems of his systems, that dynamics of it and the question of gravitation, he gives an unexpected turn to the discussion by **a very similar obsession in regard to the sphere as the perfect shape** [my bold]."

According to Koestler [1959] p. 201 **"Copernicus' obsession with circles** [my bold] reaches its climax" in the *De revolutionibus,* B. Ill, Ch. 4, where he considers the issue of the *motus librationis.*

The echo of such thinking we may find also in Koyre [1961] when on p. 10 he says that thanks to Kepler's ellipse "the **obsession with circularity** [my bold] was partially overcame **(though it could never be completely so in the case of a closed universe** [my bold]) [...]"

Such an interpretation does not hold water because completely ignores the mathematical and physical conception of symmetry. Following such footnotes, we might say, surely, that Einstein, De Sitter, Lemaitre, efc also were seize with the obsession of sphericity, because they, within context of the General Theory of Relativity, also used the idea of sphericity in their cosmological considerations on the Universe. But the true reason is different. The "non-spherical, non-circular theories" are mathematically and physically more complicated than the "spherical, circular ones". Therefore, the formers were not framed before the later when it became clear that the "spherical, circular theories" couldn't save phenomena.

(2a) The fictionalistic or instrumentalistic interpretations of the scientific hypothesis made by Averroes, Averroists and Scholastics (St. T. Aquinas, J. Buridan, N. Oresme, Albert of Saxony, P. d'Ailly, Osjander and Bellarmine) is sound.[100]

(2b) The dichotomous distinction: the Ptolemaic mathematical astronomy - cosmology (= celestial physics), treated as the distinction between the mathematical schemes, which have only the instrumental character, and the cosmological considerations, which are only speculations about the fundamental (ontological) structure of the Universe — is sound.[101]

[100] Duhem [1908, Engl, transl. 69] (especially ch. Conclusion), [1954].

Since the HDMCT is not the method by help of which the ultimate ontology of the Universe is built up, it is not possible to share the following opinion: "Despite Kepler and Galileo, we believe today, with Osiander and Belarmine, that the hypotheses of physics are mere mathematical contrivances devised for the purpose of saving the phenomena. But thanks to Kepler and Galileo, we now require that they save *all the phenomena* of the inanimate universe *together."* (Duhem [1969], p. 117)

But, on the other hand, E. Rosen is right when he writes: "We are not limited to the choice offered Duhem between realism and fictionalism: any proposition or hypotheses is either the Ultimate Truth or a mere fiction. We may properly accept a hypothesis as the best statement at the moment and be ready to revise or to reject it when fresh empirical data require a modification of it, or a rival and superior hypothesis emerges to replaces it." (Rosen [1971], p. 33; see also Grant [1962a], Goddu [1990])

The other thing it is the importance of Osjander's preface in the dissemination of Copernicus' theory. This process is closely connected with the fictionalistic interpretation of Copernicus' geometrical models, the usage of Copernicus' calculations and with the issue of the reduction of Copernicus' models to the geocentric cosmology. On these subjects see, for instance, very valuable articles and discussions in Gingerich (ed.) [1975], especially p. 355-457; Westman (ed.) [1975], especially p. 213-353'

[101] See Dreyer [1906, 53] p. 149; Duhem [1908, Engl, transl. 1969] (especially ch. Conclusion, p. 114), [1954] p. 40; Kuhn [1957] p. 104-105, 184; Koestler [1959] p. 73-75; Price [1957] p. 199; Koyre [1961] p. 24-25 f. 44; Me Mullin [1984] p. 35-37.

From the HDMCTs point of view it is impossible to accept Duhem's remark: "The Greeks clearly distinguished, in the discussion of a theory about the motion of the stars, what belongs to the physicist — we should say today the metaphysicians — and to the astronomer. It belonged to the physicist to decide, by reasons drawn from cosmology, what the real motions of stars are. The astronomer, on the other hand, must not be concerned whether the motions he represented were real or fictitious; their sole object was to represent exactly the relative displacements of the heavenly bodies." (Duhem, [1954], p. 40)

Within the context of the HDMCT, it is clear that, for instance, Ptolemy comprehended the issue of the status of the cosmological hypotheses and the astronomical models created on this ground similarly like Copernicus, Kepler, Galileo, etc. (see footnote 52). The basic important difference consists in the usage of the different cosmology. And, after the studies of Hartner [1964] and Goldstein [1967], it appeared that Ptolemy's theory, described in the *Almagest* and the *Hypotheses Planetares*, is based, in principle, on the Aristotelian cosmology of finite and plenitude Universe. The issue of the planetary distances, the dimension of the universe (of about 20000^{tr} - terrestrial radii), and the rejection the Pythagoreans cosmology, are related with this subject strictly. It is therefore so important

(3a) Since the local motion is relative, Copernicus' theory and Ptolemy's and Tycho Brahe's should be formally, geometrically, mathematically equivalent.[102]

(3b) Since the local motion is relative Copernicus' cosmological hypotheses there are the simple (non-dialectical) negations of the false cosmological hypotheses assumed by Aristotle and Ptolemy. Therefore Copernicus' theories and Ptolemy's are not incommensurable.[103]

that Ptolemy's answers were paradigm during Middle Ages and Renaissance. (The last mentioned issue - see van Helden [1985], p. 15-39).

[102] See Schiapareli [1878] p. 85, Dreyer [1953] p. 363, Kuhn [1957] p. 204, A. R. Hall [1956] p.370, Price [1959] p. 203, A. R. Hall and M. B. Hall [1959] p. 254, Hoyle [1973] p. 79, Koyre [1973] p. 78 f. 7.

Let us illustrate this thesis by the quotation from Schiapareli [1878] given in Koyre [1973] p. 78 f. 7: "The great conflict between the Ptolemaic and Copernican systems revolved round the same physical and cosmological principles [as in Antiquity-Koyre's addition]. **Both systems are capable of being used to represent the phenomena, the one as well as the other. From the geometrical point of view they are equivalent; they agree with each other just as well as with the eclectic system of Tycho Brahe** [my bold]."

But, since, according to Copernicus, for instance, the precession is non-uniform, and, according to Ptolemy and Tycho Brahe, it is uniform (having the different rates of the precession in Ptolemy's and Brache's theories), it is obvious that Copernicus', and Ptolemy's, and Brahe's theories are not formally, geometrically or mathematically equivalent.

You may find the repercussions of the above mentioned wrong opinion (on the equivalence of Copernicus' and Ptolemy's theories) even in the beautiful book written by the eminent Professors N. M. Swerdlow and O. Neugebauer. And though they show many times that the various analogous Copernicus' and Ptolemy's models are formally or geometrically nonequivalent, nevertheless, they write, for instance: **"It is obviously indifferent to spherical astronomy whether the apparent rotation of the celestial sphere is due to its own motion or to the rotation of the earth. Copernicus can therefore take over his spherical astronomy unchanged from earlier sources** [my bold], and in this particular field there had been some progress, at least in the computation of useful tables, since Ptolemy." (Swerdlow and Neugebauer [1984], p. 104)

However, according to Copernicus (Ptolemy), the fundamental magnitude of spherical astronomy there is the obliquity of the earth's equator (the obliquity of the ecliptic). But Copernicus' and Ptolemy's models of this obliquity are not formally, geometrically or mathematically equivalent. According to Copernicus, the obliquity of the earth's equator changes continuously, and, according to Ptolemy, the obliquity of ecliptic is constant. On the other hand, they are observationally equivalent for about four hundred years before Ptolemy. Next, as we showed, these magnitudes are linked by the certain correspondence principle. In consequence, the correspondence principle links Copernicus' spherical astronomy and Ptolemy's.

[103] The above mentioned (formal, geometrical or observational) equivalence was surely the reason for which P. K. Feyerabend, rejecting the critique made by P. K. Machamer (see *Studies in the History and Philosophy of Science*, May 1973, p. 11), emphasized that: "... Machamer misunderstands even those ideas which I still hold. I never said, as he assumes I did, that any two rival theories, so-called 'universal', or 'non-instantial' theories,

(4a) Copernicus had not any conclusive proofs for the mobility of the earth, since Copernicus' and Ptolemy's theories were observationally equivalent or/and it is impossible to give them on the ground of the Aristotelian physics, or/and it is impossible to give them for the logical reasons; therefore:

(4b) Copernicus' acceptance of his cosmological hypothesis as physically true was illogical.[104]

if interpreted in a certain way, could not be compared easily. More specifically, **I never assumed that Ptolemy and Copernicus are incommensurable. There are not** [my bold](Feyerabend [1975], p. 115)

It is impossible to accept such thesis. It results from the following facts. According to Feyerabend, such theories as Quantum Mechanics and the Classical Mechanics, the Special Theory of Relativity and the Classical Mechanics, are incommensurable. On the other hand, theorists know that such theories are linked by some correspondence relations. Next, we showed that Copernicus' theories and Ptolemy's also are liked by some correspondence relations. Moreover, these theories use completely different quasi-entities, etc. Hence, Copernicus' theories and Ptolemy's are incommensurable in the same sense as, for example, Quantum Mechanics and the Classical Mechanics, the Special Theory of Relativity and the Classical Mechanics.

[104] (4a) E.g. Duhem [1908, Engl, transl. 1969]; Butterfield [1950] p. 27-31, especially p. 30-31; Kuhn [1957] p. 144-145, 150, 154-155; Koestler [1959] p. 148, 153; A. R. Hall [1964] p. 64; Swerdlow and Neugebauer [1984] p. 20-21, 60. (4b) Duhem [1908,69]; Grant [1962b] p. 220; Kuhn [1957] p. 184, 132; A. R. Hall [1964]; Swerdlow and Neugebauer [1984].

For instance, according to Butterfield [1950] p. 27: Copernicus "provided a neater geometry of the heavens, but it was one which made **nonsense of the reasons and explanations that had previously been given to account for the movements in the sky** [my bold]." Next on the p. 30-31: **"Within the framework of older system of ideas, Copernicus was unable to clinch his argument** [my bold]. To the old objection that if the earth rotated its parts would fly away and it would whirl itself into pieces, **he gave an unsatisfactory answer** [my bold] -— he said that since rotation was for the earth a natural movement, the evil effects could not follow, for the natural movement of a body could never be one that had the effect of destroying the nature of that body. **It was the argument of a man who still had one foot caught up in Aristotelianism himself, though perhaps precisely because it seems to be archaic to us it was more appropriate to those conservative people whose objections required to be met in the sixteenth century** [my bold]."

Kuhn [1957] p. 145: **"Those arguments** [that is Copernicus' arguments for the mobility of the earth] **are profoundly unconvincing** [my bold]. Except when they derive from mathematical analyses that Copernicus failed to make explicit in the First Book, they were not new, and they did not quite conform to the details of the astronomical system that Copernicus was to develop in the later books. **Only a man who, like Copernicus, had other reasons for supposing that the earth moved could have taken the First Book of the *De revolutionibus* entirely seriously** [my bold]."

Next, since Copernicus was mathematical astronomer, from the point (2a) and (2b) (given in the main text) should result that Copernicus should be an instrumentalist and not a scientific realist, whom was. Moreover, for instance, Duhem [1969] p.63: "according to Copernicus could not give any proofs for the motion of the earth for logical reasons, since, as

(5) The complicated reception of Copernicus' theory, and especially the framing of the geoheliocentric theory by Tycho Brahe (1546-1601) is the proof for the irrational character of the progress of the so-called exact
• **105**
sciences.

(6) At the light of the achievements of Tycho Brahe (the new measuring instruments) and of Newton (the theory of gravitation and dynamics), the issue of the long-period phenomena, which was searched seriously by Medieval and Renaissance astronomers and by Copernicus himself, was only the fiction, only the error of the measurements.[106] Therefore (except

scholastics learned, it is always possible to frame many different theories saving the **same** phenomena. But, Duhem thought that, it was the case of Copernicus' and Ptolemy's theories. Thus, according to Duhem [1969] p.63,113, when Copernicus and Copernicanists, (as Rheticus, Galileo and Kepler) thought that the motion of the earth is physically true hypothesis, they made the logical mistake. They 'stubbornly to stuck to an illogical realism... (Duhem [1969] p. 113)'.".

Hall [1964] p. 64: "It is an obvious principle of logic that the unknown cannot be demonstrated from the unknown. Measured by this standard, Copernicus' argument in favour of heliocentricity are illogical."

Grant [1962b] p. 220: "It was Copernicus who, by an illogical move first mapped the new path and inspired the Scientific Revolution by bequeathing to it his own ardent desire for knowledge of physical realities."

Swerdlow and Neugebauer [1984] write on p. 20-21: "[Copernicus] had not been able to proved the motion of the earth, but only argue with greater or lesser persuasiveness for it plausibility [...]"

I oppose to the above interpretations, because: (1) On the level of celestial physics or cosmological hypotheses (= quasi-entities), Copernicus' argumentation was not surely worse than Aristotle's and Aristotelian, since Copernicus' cosmology was not the original Aristotle's cosmology but a sort of Buridans' that it used the idea of impetus. (2) It is nonhistorical to criticize Copernicus for the fact that he is not the author of, for instance, Keplerian or Newtonian physics. (3) Copernicus' and Ptolemy's theories were linked by the correspondence relations. (4) Copernicus' "illogical realism" is a standard thing for today's theoretical physics when we discuss about electrons, quarks, etc.

105 According to Koyre [1948], p. 30 and Gingerich and Westman [1988] p. 43: "The history of scientific thought is not logical. The fact that Tycho Brahe came after Copernicus shows that history is not logical because, rationally, he should have appeared before him."

I think, that it is impossible to accept this thesis since to see the new deep connections and the new harmony of astronomical phenomena it was necessary to use, at first, as a basis of astronomical theory, a new physics. But, it is Copernicus' physics that has this property and not Brahe's — and such just was the opinion of Galileo and Kepler.

On Tycho Brahe see: Moesgaard [1973]: "Copernican Influence on Tycho Brahe", *SC* V 1975, p. 31-55; Westman: "Three responses to the Copernican Theory: Johannes Praeto- rius, Tycho Brahe, and Michael Maestlin", in Westman, R. S. [1975] p. 313-329.

Moreover, not only Tycho Brahe went by Copernicus' footnotes. The same did all authors who framed the Tychonic or semi-Tychonic systems. See point 2 page 83.

106 There exist the following three fundamental reasons for which this statement seemed to be entirely legitimated.

the works of L. A. Birkenmajer [1900, 1901a] and Duhem's Le Systeme du Monde (especially, vol. II, IV) up to the works of Dobrzycki [1964], Ravetz [1965], Moesgaard [1968], Swerdlow [1980], Swerdlow and Neugebauer

Copernicus' theory of the changing precession and the changing obliquity of earth's equator. "I have also noticed that in the longitudes [of the stars] the intricacy of the nonuniformity is not as great as Copernicus believed. For, what he imagined in this regard insinuated itself through defects in the observations, both ancient and recent. For at present the fixed stars traverse 1E, not in 100 years in accordance with his computation, but in only [years]. In the past they always uniformly completed very nearly this [motion], if the observations of [our] predecessors are properly delimited, with only a trivial irregularity, arising accidentally from another source." *Astronomiae instauratae mechanica* (Wandsbek, 1598; Opera omnia, Copenhagen, 1913-1929, V, 113, lines 9-17) in Rosen [1978], p. 384.

In consequence, Tycho Brahe, contrary to Copernicus, assumed that the change in precession and the change in obliquity are uniform, and thus, the precession and the obliquity are periodically linear functions of time. Moreover, having made the critical analyses of astronomical data and accuracy of measurement equipments used beforehim, Tycho Brahe assumed different values of constants of these phenomena than Ptolemy. Hence, according to Brahe, Ptolemy had to err in his measurements.

(2) The linear dependence' of the above mentioned functions, as seemed to historians, was corroborated in the theory of gravitation by Newton *(Principia).*

(3) According to the positivistic and neo-positivistic philosophy of science, which is still accepted by many historians of the so-called exact sciences, there exist the pure facts.

Hence, it seemed rational to think that the periodically non-linear effects were the ordinary errors of the measurements.

But such a thesis is not fully right and these errors of measurements were not ordinary.

For many the Medieval and Renaissance astronomers like Copernicus, Ptolemy, called the Astronomer that is the greatest astronomer, could not err in his measurements. But during the Middle Ages and Renaissance, the professional astronomers determined different values of the change in precession and in obliquity of ecliptic. The changes of these magnitudes were not accidental, but, non-uniform (see, for instance, Copernicus' table on p. 63, and on p. 71). In consequence, the Medieval and Renaissance astronomers thought that, during the long-period of time, they observed the non-uniform changes in the precession and in the obliquity. Next, according to the paradigm of astronomy since Plato's times, every astronomical phenomenon is periodic and must be explained in the language of circles. Thus, seeing the non-uniform change in the precession and in the obliquity, astronomers, and among them Copernicus, had to assume that these change is cyclic.

But, though, the Medieval and Renaissance astronomers erred regarding the non-uniform long-period changes, Tycho Brahe was not fully right in this subject too. Since, according to the Newtonian theory of gravitation (which explains the above phenomena by the gravitational interactions of the Sun, the Moon and planets for the earth's axis and equator), the function of the precession of equinoxes and the function of the obliquity of earth's equator are not constant functions of time but, non-linear and periodic. But **in the first order approximation** these functions are consisted with following two parts. The first is (periodically) linear and the second periodically non-linear. But note, in the case of the non-linear function of precession, its linear component is called by the convention just *precession,* and the periodically nonlinear the *nutation.* The period of the first is about 26000 years and, of the second about 9 years. The analogous features has the function of the obliquity, *(verte)*

[1984]) the issue of planet was the only one worthy noticing by historians and, especially, by philosophers of the so-called exact sciences.[107]

(7a) The "genuine astronomical or scientific revolution" was not Coper- nican but Keplerian (three Kepler's laws framing on the base of Tycho Brahe's new measurements) and Newtonian (theory of gravitation and dynamics).

(7b) There was not the Copernican revolution![108]

On the above questions see: J. F. Montucla, *Histoire des Mathematiques,* Tome Quatridme, Nouveau Tirage, Paris: Albert Blanchard, 1960, p. 215-233; W. M. Smart, *Textbook on Spherical Astronomy,* Cambridge University Press, Cambridge, Sixth ed. 1977, Chap. X "Precession and Nutation", p. 226-248; C. Wilson, "D'Alambert versus Euler on the Precession of the Equinoxes an the Mechanics of Rigid Bodies", *Archive for History of Exact Sciences,* vol 37, Number 3, 1987.

[107] It is the case of, for instance, Dreyer [1906, 53], Butterfield [1949], Hall [1954], Koyre [1957, Engl, transl. 1961], Price [1957], Kuhn [1957], Hanson [1964, 1973], Feyer- abend [1975], Lakatos and Zahar [1975], Me Mullin [1984], Cohen [1985], Chalmers [1981, 1983], Curd [1982,1983], Hutchison [1983], Petroni and Scolamiero [1986], Margolis [1993],

The identification of the problem of a planetary theory with the whole history of astronomy was the reason for which Dreyer reprinting in 1953 his book published in 1906 under the title *History of the Planetary systems from Thales to Kepler* changed its title for *A History of Astronomy from Thales to Kepler.*

From the methodological point of view, the problem of planetary system was important for the Medieval and Renaissance astronomers and Copernicus himself, but more important and more difficult was the problem of the long period-phenomena. Moreover, it is the HDMCT, that was used to analysed both of these problems.

[108] (7a) It is the case of Dreyer [1906, 53] p. 344, Butterfield [1950] p. 30, Price [1957] p. 198-199, Koestler [1959] p. 148, Neugebauer [1957] p. 205-206, [1968] p. 205—206, Cohen [1985] p. 105-125 esp. 123-125.

For instance: Butterfield [1950] p. 30 "Once we discovered the real character of Copernican thinking, we can hardly help recognizing the fact that the genuine scientific revolution was still to come."

Dreyer [1953] p. 344: "Copernicus did not produce what is now-a-days meant by 'the Copernican system' ".

Price [1957] p. 198-199: "Although Copernicus made a fortunate philosophical guess, his work as a mathematical astronomer was uninspired. From this point of view his book is conservative and a mere re-shuffled version of the *Almagest.* Above all, it introduced many false trails that must have hindered the acceptance of he one good point. In the domain of mathematical astronomy the first major advance after Ptolemy was made, not by Copernicus, but by Kepler."

Neugebauer [1957] p. 205-206: "There is no better way to convince oneself of the inner coherence of ancient and medieval astronomy than to place side by side the Almagest, al-Battani's *Opus astronomicum* and Copernicus's *De revolutionibus.* Chapter by chapter, theorem, table by table, these works run parallel. With Tycho Brahe and Kepler the spell of tradition was broken. The very style in which these men write is totally different from the classical prototype. Never has a more significant title been given to an astronomical work than Kepler's book on Mars: *Astronomia Nova.*" *(verte)*

Finishing here our analysis of the Copemican issue, note, however, what follows. Though, in my opinion, it has been shown in this paper, that Copernicus' scientific methodology, being based on the HDMCT, is quite well (since the same was used by many eminent modem theorists), nevertheless, **it hasn't been said above that:** (a) Copernicus was, simply, the greatest scientific genius, (b) who being, as it was once said commonly, the "father of the modem astronomy", (c) is also the "father" of the Hypothetico-Deductive Method of Correspondence Thinking (HDMCT) and (d) therefore, he is the "father" of the scientific methodology.

And such conclusions havn't been drawn for the fundamental reason. Before Copernicus, the HDMCT was systematically used by many astronomers of Renaissance, Medieval and Ancient times! For instance, it was used by Johannes Werner (1468-1522), Marcus Beneventanus (c. 1465 - c. 1525), John Linieres and John of Saxony — the authors of the Parisian version of the *Alphonsine Tables* (c. 1327), the authors of the Toledian version of the *Alphonsine Tables* (c. 1272), astronomers of the Maragian school (Margha, west-north Iran XIII-XIV c.), Thabit ibn Qurra (c. 836- 901) (Bagdad), Ptolemy (90-168), Hipparchus (161-126 B.C.), Aristarchus of Samos (c.280 B.C.), Callippus (c. 370-C.330 B.C.), Eudoxus (c. 408- c. 355 B.C.), etc.

Thus, also the below theses are not right:

(8a) Copernicus is the father of the scientific methodology.[*] [109]

(8b) Mature exact sciences develop by the correspondence principle. But the idea of the correspondence principle was not known before modem times. It originated after Copernicus.[110]

Koestler [1959] p. 148: "What we call the Copemican revolution was not made by Canon Koppernigk. His book was not intended to cause a revolution."

(7b) It is the thesis of I. Bernard Cohen, who, on the ground of the critical analyses of Copernicus' achievements made by Price [1957], Neugebauer [1968] and Swerdlow [1973], states in his very well book *Revolution in Science* [1985] the following. "If there was a revolution in astronomy, that revolution was Keplerian and Newtonian, and not in any simple or valid sense Copemican." (Cohen [1985], p. 125).

The critique of these theses has being given in the main text. See p. 76-80.

[109] This opinion was and is paradigmatic for con temporary physicists. It was shared also by some historians and philosophers of science. See Infeld [1955], Grand [1962a], Werle [1974] especially p. 322-325, Heisenberg [1975] in Gingerich (ed.) [1975] p. 219-236, especially p. 225.

For instance, Heisenberg [1975] p. 225 states: "In the scientific work of our present century we still follow essentially the method that had been discovered and developed by Copernicus, Galileo, and their successors in the 16th and 17th centuries."

[110] Heisenberg [1935, 1936, 1946, 1949, 1959], in [1936] on p. 7-9, in [1959] p.45-47; KyflpsmneB [1973] p. 79-81, Krajewski [1974] p. 135-136, [1977] p. 88-91, [1985] p. 294-298.

(8c) To find more general models of astronomical phenomena than Ptolemy's, the Medieval and Renaissance astronomers did not use an original scientific method at all.[**][111]

5. The General conclusions for practicing the history and the philosophy of the so-called exact sciences

From the whole this story, this lesson of History, let us draw the following conclusion. The history and philosophy of the so-called exact sciences are connected indissolubly, what, the famous Lakatos' statement, renders, well:

> Philosophy of science without history of science is empty; history of science without philosophy of science is blind.[112]

This thesis holds especially well in the case when we want to describe a historical process of the origin of certain theory which belongs to the domain of the so-called exact sciences. It happens therefore, that the historians gather systematically only these facts, which they able to

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following. (1) Mature exact sciences develop by the correspondence principle. (2) It is the case of theory of relativity and quantum mechanics which are linked with classical mechanics for u/*c* —> 0 and *h* -> 0 respectively. (3) In such limiting cases, these theories do not change laws of classical mechanics. (4) But these classical laws were discovered by scientists as Galileo, Kepler and Newton, who went on Copernicus' footnotes. (5) On the other hand, Copernicus theory is not linked with the old science of Aristotle-Ptolemy by any correspondence principle, therefore or since it was Copernicus who created the solid basis for modern physics.

[111] Professors Neugebauer and Swerdlow state: "It should be noted that while **the models of nonuniform precession or variation of the solar eccentricity have nothing corresponding to them in Ptolemy's work, the methods by which they were discovered were not original** [my bold]. Rather they were found by applying Ptolemy's procedures for derivation of parameters, or slight modifications of them, to new observations of the sun or a few zodiacal stars, and finding results not in agreement with the Almagest. The theories were then invented to account for the discrepancies." (Swerdlow and Neugebauer [1984] p. 43.)

However, it is impossible to share this opinion, since, in the light of the HDMCT, it appears that the Medieval and Renaissance astronomers used exactly the same method which was applied by contemporary physicists to formulate, for example: (1) the law of radiation of the black body - Planck (1900), (2a) first quantum theory of atom - Bohr (1913), (2b) its relativistic development - Sommerfeld (1915), (3) the quantum mechanics — Heisenberg and Schrodinger (1924-1927), (4) the Special and General theory of gravitation - Einstein (1905, 1905-1917)), etc. And it was the Hypothetico-Deductive Method of Correspondence Thinking which was used in all these cases.

[112] I. Lakatos, "History of Science and its rational reconstructions", in *Philosophical Papers*, vol. I, ed by J. Worral and G. Curie, Cambridge UP, London 1978, p. 102.

understand and interpret in a certain way. However, they always do it in the light of the assumed or accepted (often unconsciously) certain philosophy. On the other hand, they are the philosophical searches those permit to formulate new philosophical languages, new notional apparatuses. These new notional apparatuses will be better than the old ones, if thanks to them it is possible to deeper understand the known facts or/and to notice new ones. But, this philosophy cannot be chosen by historians accidentally. In the case of the so-called exact sciences, it should stem from the scientific practice of creative scientists who are busy with the so-called exact sciences, that is, with physics, astronomy, etc. Therefore, this philosophy must refer as well to the strictly mathematical, reckoning problems, as to the heuristic procedures of the framing of theories, the procedures of the making measurements, etc, since these all issues define the specificity of the so-called exact sciences. The better this philosophy will be known, the more interesting and true, the history of the so-called exact sciences will be, and vice versa.

Seeing the importance of the above issues, we have a chance to avoid the weaknesses of the "non-philosophical history" and "non-historical philosophy" of the so-called exact sciences, preserving their good points. Thanks to it we may write, in some degree, a new and, what is important, more true, more integrated history and philosophy of the so-called exact sciences.

6. Summary

In this article, we have introduced the Hypothetico-Deductive Method of Correspondence Thinking (HDMCT), which is consisted of the Hypothetico-Deductive Method (MCT) and the Method of Correspondence Thinking (MCT). This method is used consequently by creative scientists who practice the so-called exact sciences, that is, astronomy, physics, etc. Next, using the HDMCT, in its hermeneutic light, we have read Copernicus' texts in a new way. Thanks to it, as I think, they have been discovered or, better explained some important methodological facts, which were not (though might be) noticed or were not properly understood by historians or/and philosophers of science up to now. For instance, Copernicus used the correspondence postulate systematically and thanks to it he created theory which is linked with Ptolemy's by some correspondence relations. Speaking metaphorically, it is in this manner, that the history came full circle[113]:

[113] The formulation 'the history came full circle' is one of the example of the so-called notional diffusion, on the one hand, and the linguistic relict, on the other. These issues are

Copernicus is no less modern than Bohr, Einstein, Heisenberg and Hawking, and these last are no less ancient than Copernicus.

On these grounds, it is impossible to agree with the critique of Copernicus' scientific method and the results obtained thanks to applying this method. In short, after Copernicus' genuine revolution was the genuine Keplerian, the Galilean, the Newtonian revolutions, which are called together the Copemican revolution. But, the Copernican revolution was neither the last the global scientific revolutions nor first. Before it there was the Greek scientific revolution with the discovery of the idea of *cosmos* and with researches concentrated in the field of astronomy, optics and harmonics.

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Nicolaus Copernicus, *Complete Works,* vol. Ill, *Minor Works,* Edited by Paweł Czartoryski, Translation and Commentary Edward Rosen with the assistance of Erna Hilfstein, Warsaw-Cracow, Polish Scientific Publishers 1978;

Mikołaj Kopernik, *O obrotach sfer niebieskich. Księga pierwsza.* Ustalenie tekstu łacińskiego R. Gansiniec, przekład M. Brożek, komentarz A. Birkenmajer, redaktor A. Birkenmajer, Warszawa, PWN 1953.

connected strictly with an interaction of the so-called exact sciences with the human sciences. Namely, in this case, the notional diffusion due to the generalization of the idea of periodicity of the astronomical phenomena, which was expressed by many ages by the mathematical idea of circle, for the every aspect of the human life, both individual and social. The deterministic and circular recurring fate, it is the essence of astrology — and, without any doubt, astrology has very interesting history about which, however, we said nothing in this article. (On that subject see, for instance, the classical work by L. Thorndike, History of Magic and Experimental Science, vol. 1-8, 1921-1958; P. Zambelli, *Mit hermetyzmu i aktualna debata historiograficzna,* transl. for Polish P. Bravo, Warszawa

1994). With this subject, there are closely connected, among other things, the ideas of: 'το χυχλω', 'circulatio'; 'μεταβολη', 'revolutio'; 'ανακλωσιζ', 'vicissitudo'. (On this subject see, for instance, A. Nowicki, *The central categories of Bruno's philosophy* (in Polish), PWN, Warszawa, 1962; H. Arendt, *On revolution*, Viking Press, New York, 1963.)

The formulation 'the history came full circle' is the linguistic relict of astrology which was based on the ancient periodic astronomy. The Universe has seen in the light of present-day science: the General Theory of Relativity, Quantum Mechanics and the theory of chaos, is mathematically more rich than the Universe of Pythagoreans, Plato, Stoics, Averroists, and the simple idea of periodicity is one of many.

Mikołaj Kopernik, O *obrotach sfer niebieskich.* Przekład M.Brożek (ks.I). S. Oświęcimski (ks. II-VI); Komentarz A. Birkenmajer, J. Dobrzycki, Redaktor J. Dobrzycki, Warszawa-Kraków, PWN 1975.

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