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Indifference Curve Analysis: The Correct and the Incorrect

JEL Classification: *B40*

Keywords: *Indifference curve; convex; concave; budget line*

Abstract: *The thesis of this paper is that when the indifference curve is concave to the origin, the optimal point on the budget line is not the corner solution on the highest (most north eastern) indifference curve, the analysis all too often offered in the literature, but, rather, somewhat paradoxically, the lowest (most south western) indifference curve. The economics profession, as articulated through the megaphone of intermediate micro economics texts, offers a mixed result in this regard. Some few do offer a correct analysis, but many do not; others ignore the issue entirely. The contribution of the present paper and the aim of our research is to provide a correction of the widespread but erroneous indifference curve analysis that appears in many of our intermediate microeconomics texts. Our methodology*

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is one of contrast: we offer what we see as both the correct and the incorrect versions of indifference curve analysis.

Introduction

We maintain that there is a lacunae in the economics profession with regard to indifference curves. There is a very small minority of economists who properly utilize this analytic tool; the majority, however, err in this regard. However, our literature search uncovers no correctives of these errors that are widely committed; very much the opposite is true. That is the intended contribution of the present paper.

In the first section, we set out what we believe is the correct indifference curve analysis. The second section is devoted to a commentary on, and critique of, the mistaken indifference curve analysis offered by numerous leaders of our profession.

Methodology of the Research

Our methodology is to set out what we see as the correct interpretation of indifference curves and compare this with alternatives found in microeconomics textbooks. The aim of our research it to provide a corrective to a literature that would otherwise mislead students (and professors).

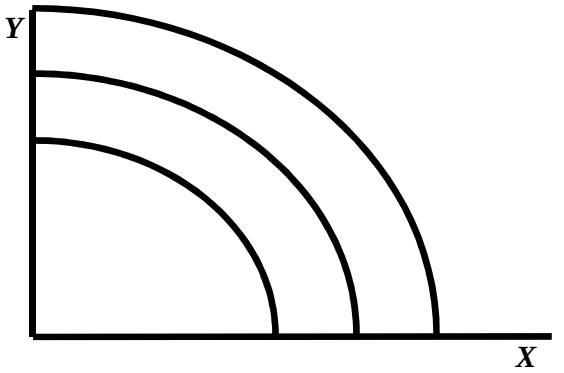
Indifference curve analysis

Our critique concerns the mistaken corner solution for concave-to-the-origin[1] indifference curves (Diagram 1). But, before we specify this error, let us review indifference curves. The ordinary downward sloping, convex-to-the-origin[2] indifference curve (Diagram 2) used to teach the concept to beginning students, is only one small part of the edifice. In this case we make the artificial assumption that more of a good is always preferred to less and that both goods in the two dimensional version of this diagram are subject to diminishing marginal utility. But, we all know full well that this is not the truth, and certainly not the whole truth and nothing but the truth. That is, in reality, when we relax these artificial[3] assumptions, too much of

[1] Or, for short, concave indifference curves.
[2] Or, for short, convex indifference curves
[3] Pedagogical?

a good thing can become a bad thing. Not only is marginal utility diminishing, it can also become negative; then, a good becomes a bad, or trash, or a garbage "good." When these truths are incorporated into the analysis, we arrive at diagram 3, which depicts a family of circular indifference curves, consisting in our diagram, of seven members. It is important to realize that they are labeled in order of *decreasing* desirability. That is, i7>i6>i5>i4>i3>i2>i1; location on i1 is the least preferred position. In contrast, the satiety point, i7 is the *most* preferred position.

Diagram 1. Concave-to-the-Origin Indifference Curves



Source: own work.

Diagram 2. Convex-to-the-Origin Indifference Curves

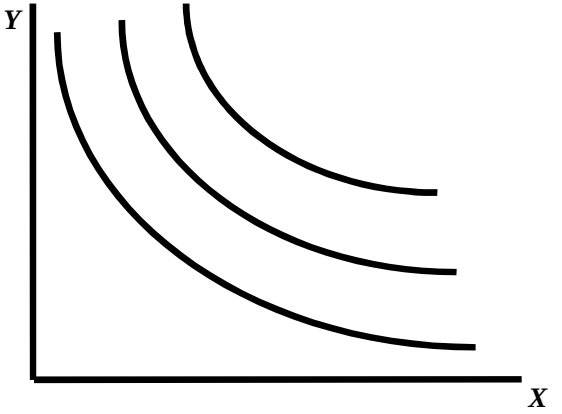


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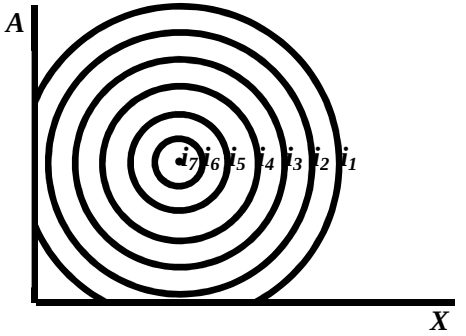
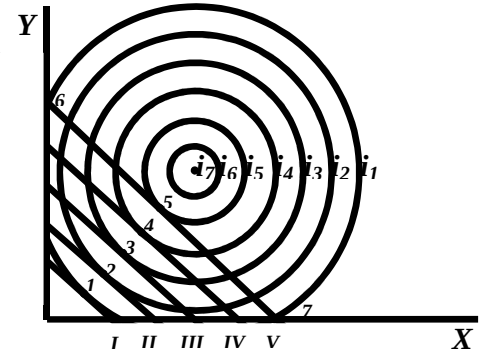
Diagram 3. Family of Circular Indifference Curves



Source: own work.

In our diagram 4 we illustrate the tangency points between the budget line and the indifference curves. We do so for the "ordinary" part of the indifference curve, which is done in all intermediate texts. The only departure from normal practice is that we show these tangency points for the *entire, circular* indifference curve set,[4] not just the downward sloping convex part.

Diagram 4. Family of Circular Indifference Curves with Budget Lines

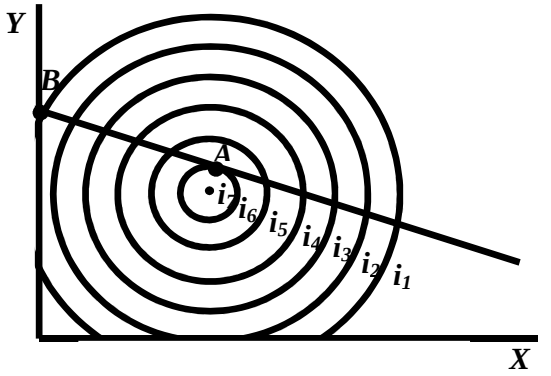


Source: own work.

[4] The only texts in our sample that utilize circular indifference curve families are Varian (2006, p. 43, figure 3.7) – our diagram 6; Boulding (1966, p. 605, figure 136) – our diagram 32; Mathis and Koscianski (2002, p. 60, figure 3.13) – our diagram 31; Vickrey (1964, p. 37, figure 6) – our diagram 33 and McCloskey (1982, p. 27, figure 2.2) – our diagram 34. This is greatly to their credit, as these are the only ones to offer this crucially important aspect of indifference curves.

This leads us to our diagram 5, the high point of our entire analysis. We again employ our circular family of indifference curves. This time, however, the budget line is tangent to indifference curve i6 at point A and intersects indifference curve i1 at point B. Which is preferable? Clearly, A>B, since A lies on indifference curve i6, B lies on i1, and i6>i1. To be sure, one can easily err, here. On can think that since part of i1 lies up and to the right of all of i6, i1 is preferable to i6. But this would be a grave error, as we have established that the closer to the satiation point, the more preferable the indifference curve and, surely, i6 is closer to this point, i7, than is i1. Let us look at this indifference curve set as if it were a three dimensional contour map, with i7 as the very tip top of the mountain. When viewed from this perspective, i6 is the next highest ridge on this mountain, and i1 is at the very bottom of the hill, at ground level.[5]

Diagram 5. Circular Indifference Curves with Tangency and Interception



Source: own work.

Commentary and critique of indifference curve analyses

For the purposes of illustrating our claim of widespread error, we utilize a random selection[6] of intermediate microeconomics textbooks. Why choose this literature? We do so because textbooks are the amalgamation,

[5] For a paper by the same authors on this general topic, see Block and Sotelo (2012).

[6] The microeconomics textbooks that happened to be on the office shelves of the second mentioned author, and on those of his friend, colleague and most important, next door neighbor at Loyola University, Prof. William Barnett II, to whom we owe a debt of gratitude for his discussions with us about these issues. All errors in the paper of course lie solely with the authors.

the summary, the encyclopedia as it were, of the entire profession. Into the text books go, at any given time, the knowledge of practicing economists. Why intermediate texts? For two reasons. One, all too many introductory textbooks either do not include indifference curves at all, or, offer a very perfunctory treatment of them, one that does not reach the error we propose to expose. Two, there is no such thing as an advanced textbook in economics.[7] There are only introductory and intermediate ones. Anything beyond that is consigned to the journal literature or books and treatises.

We now consider a sample of textbooks that discuss indifference analysis. They are divided into the "excellent", the "good," the "incomplete," and the "erroneous" categories. The first (unfortunately small) set of authors gives excellent analyses, as the name implies; they see the indifference curve in its fullness: as a full circle. Authors in the second category make no errors, but are problematic in that while they appreciate that too much of a good thing can become a bad; they fail to portray the indifference curve as the circle it is and/or should be. Entries in the third pretty much ignore the entire issue by assuming it away; we characterize them as "incomplete" not because they make any explicit error; rather, because they do not fulfill what we regard as the duty of an economist to tell the entire truth, or at least try to deal with all important aspects of a given subject. The fourth do indeed meet the challenge directly; not for them prevarication and avoidance of difficult subjects. But they get it wrong. Some compound their error by offering evidence that they interpret the indifference curve map as a circular one, but then violate these, their own insights. We shall consider these in reverse order. First the erroneous, then the incomplete, whereupon the good, ending with the excellent.

A. The erroneous

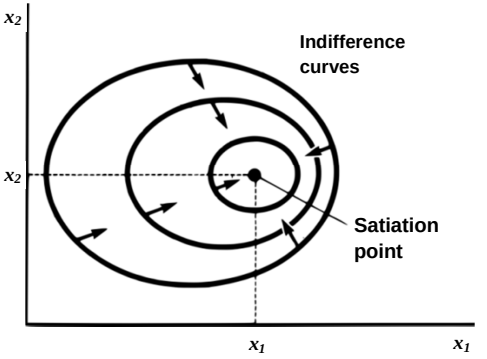
Varian

Varian (2006, p. 43), starts out on a high note: "We sometimes want to consider a situation involving satiation, when there is some overall best bundle for the consumer, and the 'closer' he is to that best bundle, the better off he is in terms of his own preferences." Precisely. So far, so good, for without the insights incorporated into this "circular" way of looking at indifference curves (Varian, 2004, p. 43, figure 3.7, repeated in our diagram

[7] This charming modesty is a credit to our discipline.

6)[8] it would be difficult to show the error we wish to focus on. However, unhappily for Varian (2006, p. 82, figure 5.8, repeated in our diagram 7) he undermines his earlier, correct point, by stating that "The optimal choice is the boundary point, Z, not the interior tangency point, X, because Z lies on a higher indifference curve." This is *very* curious in that Varian (2004, p. 43, figure 3.7, repeated in our diagram 6) quite correctly sees the satiation point as the optimum for the consumer, and the circular (or elliptical) indifference curves closer to this point as preferable to those that lie further away from it. Even his very arrows point in that direction. Yet, it is as if a different Varian (2006, p. 82, figure 5.8, repeated in our diagram 7) writes when he says that Z lies on a "higher indifference curve." No, Z lies on a *lower* curve than X. X, after all, is closer to the satiation point, than is Z. That is, indifference curve i1 (the one closest to the origin) is the highest or best indifference curve, and i4 (the indifference curve furthest away from the origin) is the lowest, or worst. (Varian did not label his indifference curves; we do this for him.) It is all well and good for those economists who do not see the circularity of indifference curves to make this error. But, for Varian to do so is really inexplicable. The others, see below, err, but without benefit of the insight that indifference curves are circular. The Varian of p. 43, figure 3.7 (repeated in our diagram 6) sees this very clearly, even brilliantly. But the Varian of p. 82, figure 5.8 (repeated in our diagram 7) falls into error, despite his earlier analysis which should have precluded him from doing any such thing.

Diagram 6. Satiated Preferences

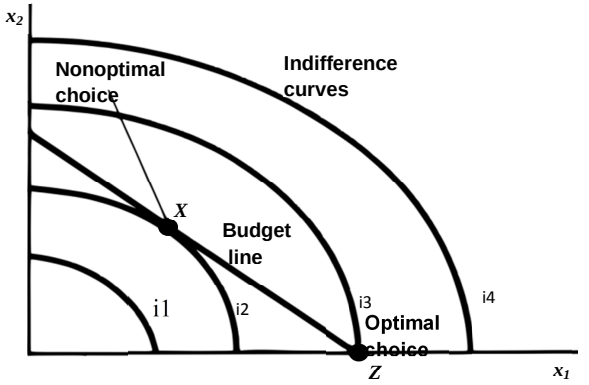


The bundle (x_1, x_2) is the satiation point or bliss point, and the indifference curves surround it.

Source: Varian (2004, p. 43, figure 3.7).

[8] This and all subsequent diagrams are shown with the original descriptions from their respective authors.

Diagram 7. Optimal Choice with Concave Preferences



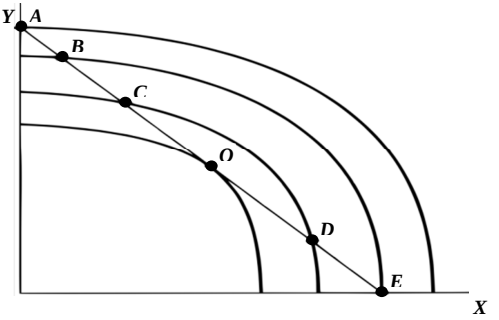
The optimal choice is the boundary point, Z, not the interior tangency point, X, because Z lies on a higher indifference curve.

Source: Varian (2006, p. 82, figure 5.8).

Landsburg

Landsburg (2011, p. 59, ex. 3.10, repeated in our diagram 8) commits outright error with his claim that "Nonconvex indifference curves always lead to a corner solution. The consumer pictured here will choose point A, which is on the highest indifference curve." Not a bit of it. Rather, A is located on the *lowest* indifference curve in that mapping. This can be shown by looking at this set of indifference curves as the two dimensional representation of a three dimensional map of a mountain, where its height decreases as we move from the indifference curve that contains O to C to B until we finally reach that on which A is perched, at ground level. If the consumer is confined to straight line budget line AOE, he will choose O rather than A, because O is placed on a *higher* indifference curve than is A. If the consumer is unconfined, and may range widely, choosing whatever combination of goods/bads available to him, he will stay within this budget line, and locate himself at the satiation point, which is not shown by Landsburg.

Diagram 8. Consumer Choice with Nonconvex Indifference Curves



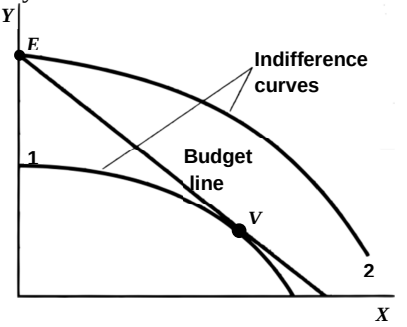
Nonconvex indifference curves always lead to a corner solution. The consumer pictured here will choose point A, which is on the highest indifference curve.

Source: Landsburg (2011, p. 59, ex. 3.10).

Mansfield and Yohe

Mansfield and Yohe (2004, p. 55, fig. 2.15, repeated in our diagram 9) are properly placed in the same category as Landsburg (2011); these authors, too, commit explicit error. In this figure of theirs, which is identical to Landsburg's, they mislead readers by stating: "Under these conditions … (of nonconvex indifference curves) … utility is maximized by specialized consumption of only one good – shown by point E. The tangency point V would be a utility-minimizing allocation because the budget line lies above the indifference curve."

Diagram 9. Utility Maximization with Nonconvex Indifference Curves



Excluding the assumption that the marginal rate of substitution of X for Y declines as the consumption of X increases allows for the possibility of nonconvex indifference curves. Under these conditions, utility is maximized by specialized consumption of only one good—shown by point E. The tangency point V would be a utility-minimizing allocation, because the budget line lies above the indifference curve.

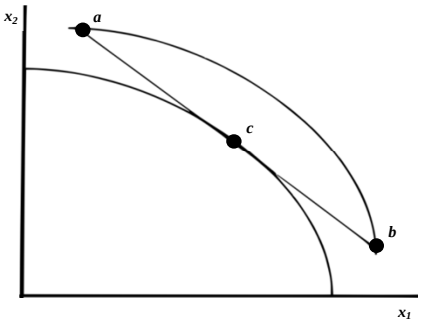
Source: Mansfield and Yohe (2004, p. 55, fig. 2.15).

But no. Indifference curve 1, upon which is perched V, is clearly closer to the satiety point than is indifference curve 2, which includes point E. True, V is a tangency point and E is not, but this is irrelevant to consumer choice. Equally, it cannot be denied that indifference curve 2 lies up and to the right of indifference curve 1. However, the only thing that counts is closeness to the satiety point, not the directions on the compass.

Schotter

Schotter (1994, p. 42, fig 2.13.a, repeated in our diagram 10) also deserves to be characterized as "erroneous." In his view, "Bundle *c* is a weighted average of bundles *a* and *b*, but it yields a lower utility level because it is on an indifference curve that is closer to the origin." This is almost Varian-esque in the extremity of the error. Schotter, like Varian, incorrectly claims that a or b are preferable to c, because a and b are both located on an indifference curve that lies in the northeast direction from c. Or, to put matters a bit more felicitously, c is actually preferable to a or b because c is closer to the point of *satiation.*

Diagram 10. Bowed-out Indifference Curves Violate the Convexity of Preferences



Bundle *c* is a weighted average of bundles *a* and *b*, but it yields a lower utility level because it is on an indifference curve that is closer to the origin.

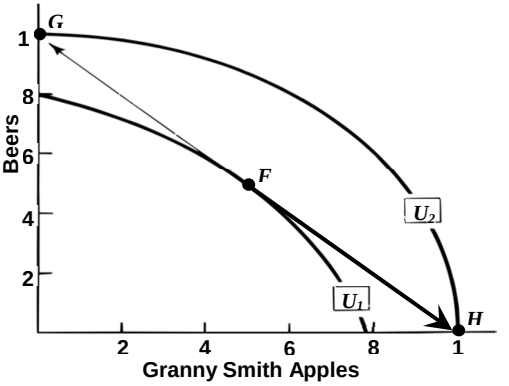
Source: Schotter (1994, p. 42, fig 2.13.a).

Friedman

Friedman (1990, p. 137, figure 6-2 b, repeated in our diagram 11) also makes an explicit error. He falsely claims that "The trade of 5 apples for 5

beers makes both parties better off, since both point G (10 beers) and point H (10 apples) are preferred to point F (5 of each)." Nothing could be further from the truth. There is only one possible reason for claiming that indifference curve U2, on which G and H lie, is preferable to indifference curve U1, on which F may be found. And that is the fallacious notion that U2 lies up and to the right of U1, or northeast of the latter. They key here is not the compass, but the vertical *height* of the three dimensional surface upon which the two dimensional indifference curves project (or lie, or rest). And, how is height of the mountain determined in two dimensions? Simple: the curve closest to the bliss or satiation point, which in this case is clearly Friedman's U1, not his U2.

Diagram 11. Indifference Curves, Endowments, and Trade



The figure shows a situation for two individuals with the same tastes (set of indifference curves: U_1 and U_2) and identical endowments (point F: 5 beers and 5 apples each). The trade of 5 apples for 5 beers makes both parties better off, since both point G (10 beers) and point H (10 apples) are preferred to point F (5 of each).

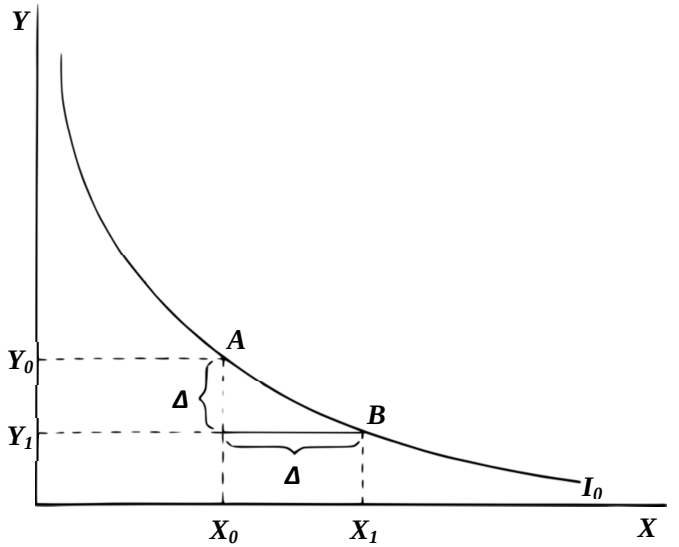
Source: Friedman (1990, p. 137, figure 6-2 b).

Quirk

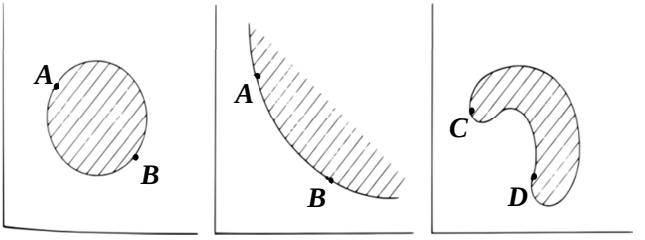
Quirk (1982, pp. 83-84, figure 4-5, a, b, c, repeated in our diagram 12) also commits an explicit error, albeit of a somewhat different type. This author states: "The sets are indicated by the shadings. The first two sets are examples of convex sets…" This is true enough of diagram b, if we take "convex" to mean convex with respect to the origin, the usual interpretation. And, it is also an accurate description of what is going on with *part* of diagram a, the section of this circular indifference curve that lies below A and to the left of B. But what of the other section of this circular indifference

curve, that part that lies above and to the right of A, and above and to the left of B. Surely, from the perspective of the origin, this latter segment is *concave*, not convex. Also curious is this author's mention of "sets," when he is clearly referring to indifference curves.

Diagram 12. Marginal Rate of Substitution, Convexity, and Indifference Curves



1. The marginal rate of substitution is positive. If X is increased, then Y must be decreased if the consumer is to remain indifferent.
2. The marginal rate of substitution decreases as X increases. It takes less Y to substitute for a given loss of X, the more of X that is available to the consumer.



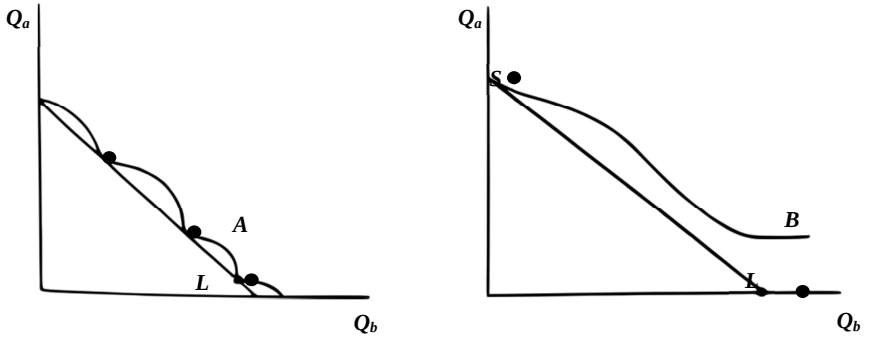
A simpler term may be substituted for "diminishing marginal rate of substitution". This simpler term involves the notion of a convex set. A set is convex if, given any two points in the set, a straight line connecting these points is also in the set (see figure above).

Source: Quirk (1982, pp. 83-84, figure 4-5, a, b, c).

Due and Clower and others

Due and Clower (1966, p. 67, figure 5-7A – our diagram 13) deserve inclusion in this category since they do mention concave (parts of) an indifference curve, but commit an error with regard to them. These authors state: "…suppose that an indifference curve is of the character of A in Figure 5-7A…, being convex in part, concave in part. With budget restraint line L, there are three distinct points of tangency and thus three equilibrium points, all yielding the same level of satisfaction, but the analysis does not indicate which of these will be selected by the consumer." True, true. With regard to the concave sections of A, these are disequilibria. At any point along these sections, only the same satisfaction as at the three equilibrium points mentioned by Due and Clower may be garnered, but, at greater expense, since more resources are used. However, with respect to these convex portions, if we stipulate that the individual must locate on budget line L, then a *higher* indifference curve can be reached, south west of these concave portions.

Diagram 13. Partially Convex Indifference Curves and Marginal Rate of Substitution



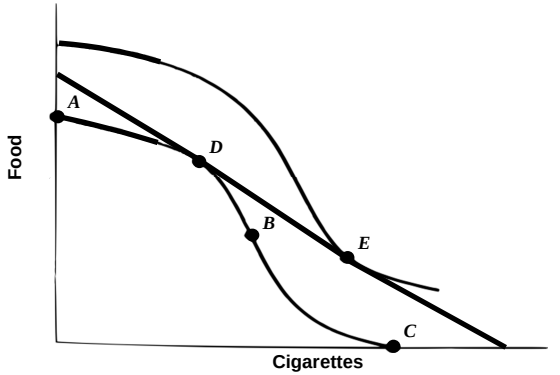
Suppose that an indifference curve is of the character of A, being convex in part, concave in part. With budget restraint line L, there are three distinct points of tangency and thus three equilibrium points, all yielding the same level of satisfaction, but the analysis does not indicate which of these will be selected by the consumer.
Or, suppose that the budget restraint and indifference curve appear as in B. There is one equilibrium point S, but this is not a point of tangency.

Source: Due and Clower (1966,p. 67, figure 5-7A).

The same analysis is offered by Bernheim and Whinston (2008, p. 135, figure 5.8, repeated in our diagram 14); Nicholson (2005, p. 98, figure 4.3 – our diagram 15), Stonier and Hague (1964, p. 48, figure 14 – our diagram 16), Varian (1990, p. 77, figure 5.4, see our diagram 17 in which we label this author's indifference curves for him, Wetzstein (2005, p. 60, figure 3.6

– our diagram 18), and Perloff (2012, p. 98, figure 4.11b – our diagram 19). Indeed, apart from using different labels for the points and curves, all of these figures are identical. And, thus, the same criticism employed against Due and Clower applies to them as well. The problem with all of the authors in this section is that there is an element of radical uncertainty involved in their presentations. It is exceedingly difficult, nay, impossible, to determine from their diagrams where the satiation point lies. And, without it, their guess is as good as ours, as to which of these points is actually more preferable than the other. In the text of this paper, our statements on this matter are but guesses. Of all of the books mentioned in this paragraph, Stonier and Hague (1964) is the only one that has a single indifference curve. These authors are correct when they state (1964, p. 48): "A consumer will never find it worthwhile to remain on a concave range of an indifference curve, however small that range may be. Since the marginal significance of the good he is buying is increasing, it will pay him to go on buying more and more of it until the indifference curve becomes convex again…" In contrast, the other authors mentioned in this paragraph, with indifference curve that have both convex and concave sections, show a family of indifference curves. And for these authors, but not Stonier and Hague (1964), the radical uncertainty mentioned above, applies.

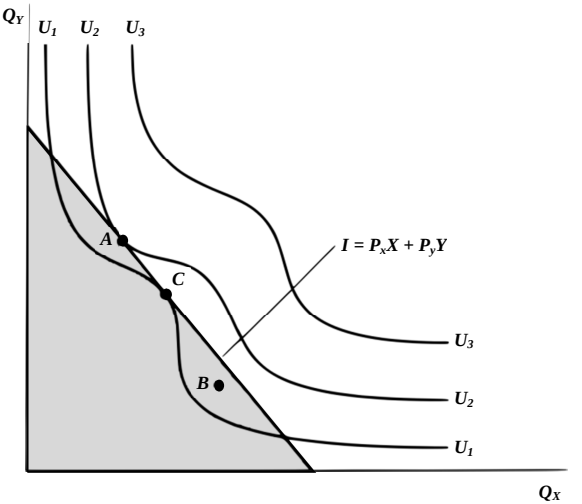
Diagram 14. Indifference Curves without Declining Marginal Rate of Substitution



Marlene allocates money between cigarettes and food. Because cigarettes are addictive, her indifference curves do not have declining MRSs. She chooses bundle E rather than bundle D, despite the fact that both bundles satisfy the tangency condition.

Source: Bernheim and Whinston (2008, p. 135, figure 5.8).

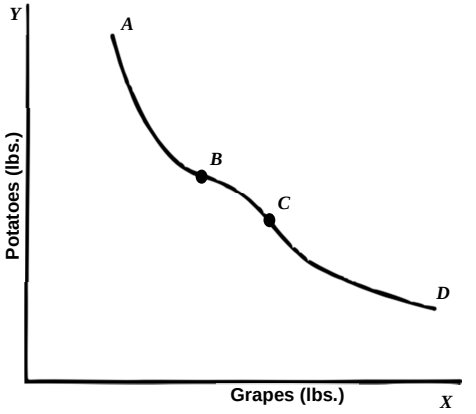
Diagram 15. Indifference Curves for which Tangency Conditions do not ensure a Maximum



If indifference curves do not obey the assumption of a diminishing MRS, not all points of tangency (points for which MRS = $P_x + P_y$) may truly be points of maximum utility. In this example, tangency point C is inferior to many other points that can also be purchased with available funds. In order that the necessary conditions for a maximum (that is, the tangency conditions) also be sufficient, one usually assumes that the MRS is d

Source: Nicholson (2005, p. 98, figure 4.3).

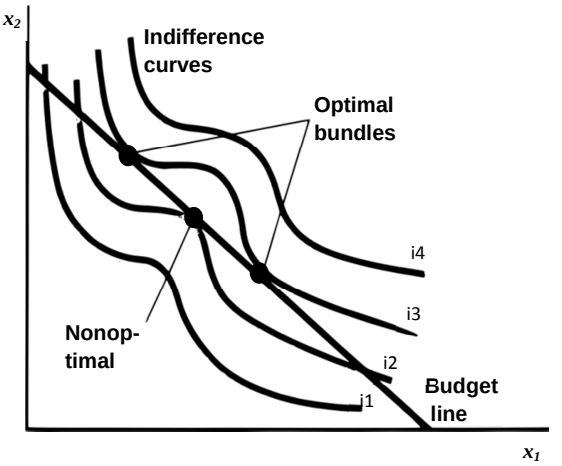
Diagram 16. Concavity vs Convexity of Indifference Curves



The ranges of the indifference curve above between A and B and between C and D display normal conditions with diminishing marginal significance of grapes in terms of potatoes. But, the range between B and C is abnormal. The marginal significance of grapes in terms of potatoes increases as the consumer has more grapes.

Source: Stonier and Hague (1964, p. 48, figure 14).

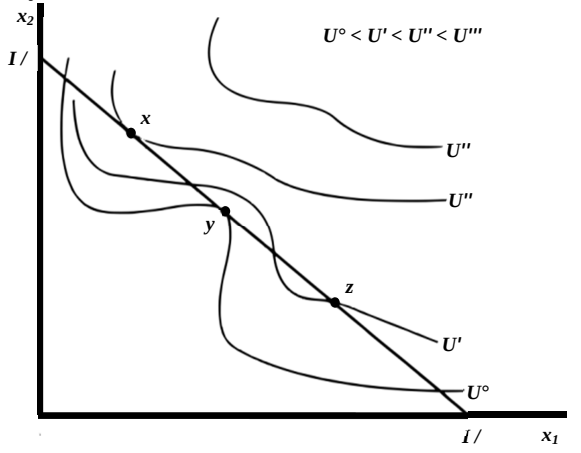
Diagram 17. Indifference Curves with More than One Tangency



Here there are three tangencies, but only two optimal points, so the tangency condition is necessary but not sufficient.

Source: Varian (1990, p. 77, figure 5.4).

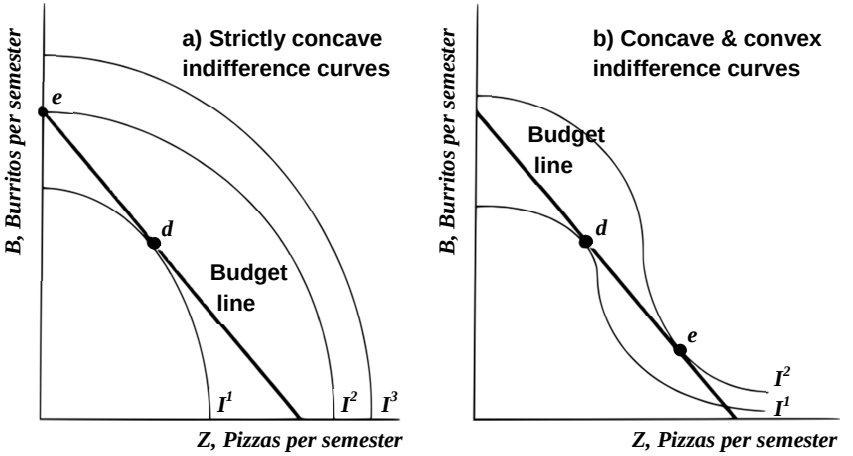
Diagram 18. Utility Maximization with Nonconvex Indifference Curves



The tangency at y is not a utility-maximizing point.

Soure: Wetzstein (2005, p. 60, figure 3.6).

Diagram 19. Optimal Bundles on Convex Sections of Indifference Curves



a) Indifference curve I^1 is tangent to the budget line at bundle d, but bundle e is superior because it lies on a higher indifference curve, I^2. If indifference curves are strictly concave to the origin, the optimal bundle, e, is at a corner.

b) If indifference curves have both concave and convex sections, a bundle such as d, which is tangent to the budget line in the concave portion of indifference curve I^1, cannot be an optimal bundle because there must be a preferable bundle in the convex portion of a higher indifference curve, e on I^2 (or at a corner).

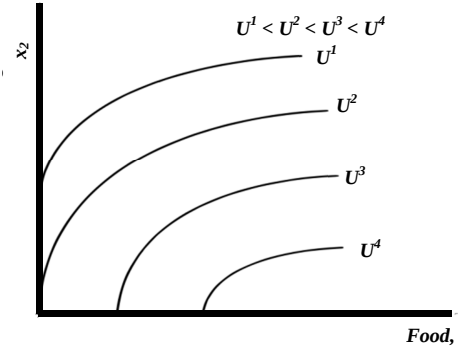
Source: Perloff (2012, p. 98, figure 4.11b).

Wetzstein

Wetzstein (2005) offers a rather mixed view of indifference curves. Wetzstein (2005, p. 40, figure 2.14 – our diagram 20) is absolutely correct in labeling his indifference curves in this order: U4>U3>U2>U1. Why? Because U4 is closest to the bliss point, and U1 is furthest away. All well and good, so far. But then what are we to make of his (Wetzstein, 2005, p. 36, figure 2.10 – our diagram 21) where he maintains that the "household prefers extremes to averages, given indifference curves with concave preferences"? In the text this author explains: "If a household's preferences were represented by strictly concave preferences, as illustrated in Figure 2.10 (our diagram 21) the household would prefer extremes to averages. In the figure, the household prefers the extreme commodity bundles (x"1, x"2) and (x'1, x'2) to the average [(x'1+x''1)/2, (x'2+x''2)/2]." However, this does not make sense. Here, Wetzstein tells us, U2>U1. But, it is U1, not

U2, that is closer to the bliss point. Whatever happened to the correct analysis of 2005, p. 40, figure 2.14 (our diagram 20) where U4>U3>U2>U1? Similarly, this author forgets his correct insight of 2005, 40, figure 2.14 (our diagram 20) when he moves to 2005, p. 61, figure 3.9 (our diagram 22). In this latter case he again inverts matters, claiming, falsely, that U”>U’>U0, and that, thus, the corner solution on the x2 axis, namely, I/p2, is preferable to point A. But no, it simply is not true that U”>U’>U0. Rather, U0 is the highest indifference curve, since it is closest to the satiety point.

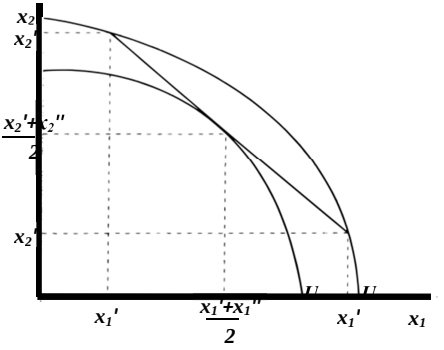
Diagram 20. Indifference Curves for Bad Commodities



For a household to be willing to purchase more of the bad commodity (cigarettes), it must be compensated with more of the good commodity (food).

Source: Wetzstein (2005, p. 40, figure 2.14).

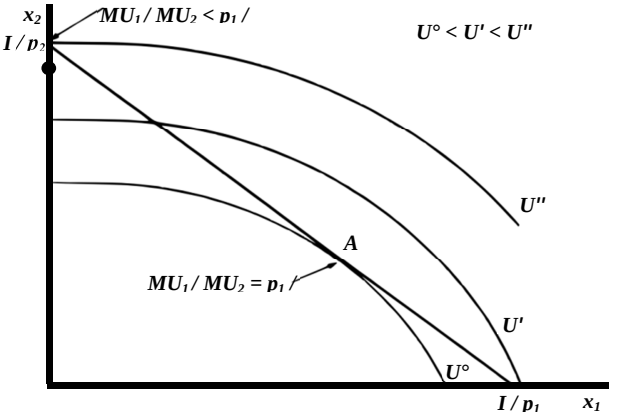
Diagram 21. Indifference Curves with Concave Preferences



The household prefers extremes to averages.

Source: Wetzstein (2005, p. 36, figure 2.10).

Diagram 22. Corner Solution with Strictly Concave Preferences



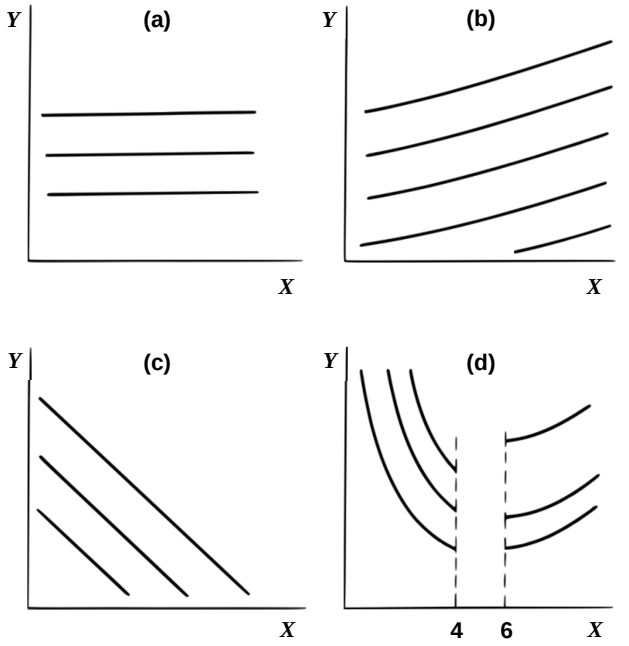
A household's utility is maximized by consuming the extreme bundle containing only x_2 and none of x_1.

Source: Wetzstein (2005, p. 61, figure 3.9).

Stigler

Stigler (1966, p. 50, figure 4-2 – our diagram 23), is good, particularly part d. He makes the usual simplifying assumptions about continuous divisibility and both commodities being desirable, but, then, happily, contradicts the latter of these two with this insightful figure. Figure 4-2a (our diagram 23) shows the case where the consumer places no value on X at all, 4-2b shows X as an outright garbage good, and Stigler's (1966, p. 49) description of 4-2d is too good not to quote: "A person who considers less than 4 units of X as desirable, would never be caught dead with 4 to 6 units (a difficult feat), and would find more than 6 a nuisance." How can these masterful insights be reconciled with the assumption that both commodities are desirable?

Diagram 23. Various Types of Indifference Curves

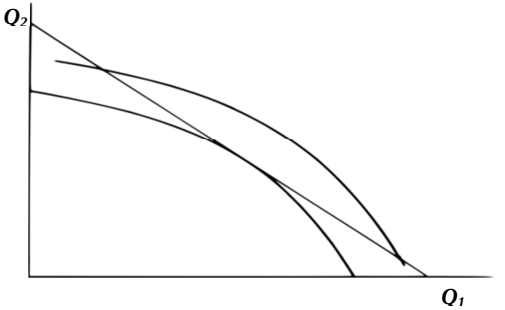


Source: Stigler (1966, p. 50, figure 4-2).

Henderson and Quandt

Henderson and Quandt (1958, p. 15, figure 2-4a – our diagram 24), also get it quite wrong. In their view: "… if the indifference curves are concave from below … the consumer's optimum position is given by a corner solution… the point of tangency represents a situation of minimum utility, and the consumer can increase his utility by moving from the point of tangency toward either axis. He consumes only one commodity at the optimum." These authors write as if the satiety point simply does not exist.

Diagram 24. First- and Second-Order Conditions with Concave Indifference Curves



In the case of indifference curves concave from below, the first-order condition for a maximum is satisfied at the point of tangency between the price line and an indifference curve, but the second-order condition is not. Therefore, this point represents a location of minimum utility, and the consumer can increase his utility by moving from the point of tangency toward either axis. He consumes only one commodity at the optimum.

Source: Henderson and Quandt (1958, p. 15, figure 2-4a).

Stonier and Hague

The treatment of Stonier and Hague (1964, p. 48, figure 14 – our diagram 16) of this diagram of theirs is worth citing at length:

"As one moves along the indifference curve from A to B, the marginal significance of grapes in terms of potatoes is declining. The consumer is prepared to give up progressively fewer potatoes in order to obtain further grapes. Once B is reached, however, a reversal takes place. The consumer suddenly becomes prepared to give up increasing amount of potatoes to obtain each further pound of grapes. This goes on until point C is reached, when conditions again change and the consumer becomes once more willing to give up fewer and fewer potatoes for each additional pound of grapes. The ranges of the indifference curve between A and B and between C and D display normal conditions with diminishing marginal significance of grapes in terms of potatoes. But the range between B and C is abnormal. The marginal significance of grapes in terms of potatoes increases as the consumer has more grapes.

"Our assumption, that all indifference curves are convex to the origin, rules out the possibility that there could be increasing marginal significance even over small ranges of indifference curves. This does not seem unreasonable. But in any case… a consumer can never be in an equilibrium position, buying some of both goods, at any point on an indifference curve which is concave to the origin. Similarly, a consumer will never find it

worthwhile to remain on a *concave range* of an indifference curve, however small that range may be. Since the marginal significance of the good he is buying is increasing, it will pay him to go on buying more and more of it until the indifference curve becomes convex again… One can therefore take consolation from the fact that even if there are isolated parts of indifference curves where marginal significance is increasing, these can never be possible positions of equilibrium."

True, very true, provided that the good we are speaking about is *not* a garbage good. And, also, when the budget line hits the satiation point, which is but a very small circle, it is just as tangent to the convex part of the indifference curve (well, point) as it is to the concave part. Indeed, there is now no difference.

Perloff

Perloff (2012, p. 81) states: "An indifference curve of this shape (concave to the origin) is unlikely to be observed. Lisa would be willing to give up more burritos to get one more pizza, the fewer the burritos she has." Yes, that is one meaning of this shaped indifference curve: increasing marginal utility. But, this is also compatible with garbage goods, and it is by no means "unlikely" that "Lisa" will experience that phenomenon. Perloff's (2012, p. 98, figure 4.11a – our diagram 19A) analysis of this diagram is as follows: "Indifference curve I1 is tangent to the budget line at Bundle d, but Bundle e is superior because it lies on a higher indifference curve, I2. If indifference curves are concave to the origin, the optimal bundle, e, is at a corner." But no, it is d, not e, that is at a higher indifference curve, I1, since I1 is closer to the saturation point (not shown here) than either I2 or I3, which lies further to the northeast.

The author continues (2012, p. 98, figure 4.11b – our diagram 19B): "If indifference curves have both concave and convex sections, a bundle such as d, which is tangent to the budget line in the concave portion of indifference curve I1, cannot be an optimal bundle because there must be a preferable bundle in the convex portion of a higher indifference curve, e, on I2…"

Again, we beg to differ. At least assuming concavity throughout, I1 is preferable to I2, because the former is closer to the satiety point,[9] than the

[9] Not shown here by Perloff, but lower and to the left of I1.

latter. If there is an exogenous requirement that the individual locate himself on the budget line, then d is preferable to e, since d lies on a *higher* indifference curve than e. If there is no such constraint, then the optimal location, compatible with the budget line is *inside* it, again at the satiety point. Nor is there any justification for mandating that the consumer locate on the budget line: fewer resources may be spent at the satiety point than on the budget line, yielding higher satisfaction.

B. The Incomplete

Then, there are several textbooks whose indifference curve analysis is guilty of failures of omission; of failure to tell the truth, the whole truth, and nothing but the truth. No explicit error is committed in these cases, but their presentations are incomplete; very much so. It is entirely possible that had these authors addressed themselves to the issue raised here, they would have been forthcoming with the correct analysis. However, in making the simplifying assumption that more is always preferred to less, it implies the false conclusion that there is no such thing as a satiation point, nor a garbage good; in so doing these economists are failing to tell the entire story. Under this rubric must be listed Frank (2008, pp. 65-75); Liebhafsky, (1968, pp. 161-222); Leftwich (1973, pp. 70-74, 104-108), Hope (1999, p. 6-22)[10]. These authors are all guilty of no more than making simplifying assumptions on the basis of which they conclude that indifference curves are convex. Let us consider some of them in more detail.

Ferguson

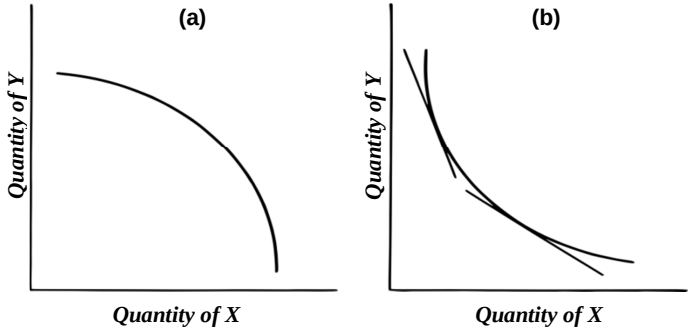
According to Ferguson (1972, p. 29): "… indifference curves are *concave from above* – that is, an indifference curve must lie above its tangent at each point, as illustrated by panel b, Figure 1.3.2 – our diagram 25. This implies that indifference curves cannot look like the curve constructed in panel a of that figure." Well, yes. If you are going to *assume* that more of every good is always and every preferred to less, then of course indifference curves cannot be concave.[11] But to make this simplifying assumption is to abstract from a basic element of reality, and thus render the model,

[10] However, this author Hope (1999, p. 144, figure 6.1) does at least mention risk adversity as a negative good).

[11] With respect to the origin, is the way most commentators phrase the matter, not convex from the perspective of above, as Ferguson would have it.

needlessly, less realistic. This would be all good and well in an introductory textbook. It is our contention, however, that this simply will not do for advanced students.

Diagram 25. Indifference Curves Concave from Above



Source: Ferguson (1972, p. 29, Figure 1.3.2).

Lancaster

Lancaster joins the crowd of economists who cop out with simplifying assumptions. He (1974, p. 222) states: "We now assume: *Goods are things the consumer prefers more of. That is, he always prefers a collection having more of both goods or more one good and no less of another*…. There are things, like garbage, that all consumers prefer to have less of…" His footnote 5 states at this point: "We have implicitly assumed what is technically referred to as *nonsatiation*, that there is no quantity of any good that will lead the consumer to have no interest in an additional amount." All well and good. None of this can be denied. However, in making such simplifying assumptions in what is presumably a cutting edge text, Lancaster avoids the more complex issues discussed in the present paper.

Thompson

According to Thompson (1989, p. 69), there are "three assumptions (that) greatly simplify the exposition of indifference curve analysis." They are as follows:

"1. All products are continuously divisible into subunits so that a consumer is not constrained by the size of the units in which the item is sold.

2. The consumer's tastes and order of preference among combinations of products is well defined and consistent.

3. The consumer views products are being desirable – having more is always preferred to having less; this means that … useless and nuisance items are disregarded."

We have already on more than one occasion commented critically on the third of these assumptions. Let us now focus on the first two. The first presents a difficulty because human choice is never continuous; instead, it always discrete. The human animal is simply incapable of making infinitesimally small distinctions. The smooth curve assumption in this critique is rather a surrender of economics to the niceties of mathematics; the calculus techniques cannot be employed with discontinuous functions. Vickrey (1964, p. 36) attempts to evade this primordial fact with his statement that "… this assumption does not involve such a drastic departure from reality as might seem at first glance: even though one would never purchase half a shirt or a third of a pair of shoes, if we remember that we are concerned with a flow of consumption over time, there can be a continuous gradation between purchasing a pair of shoes on the average (of) every eight months a purchasing a pair on the average (of) every six months…" But this will not do. Yes, we can generate a smooth*er* indifference curve in this manner; however, an *infinitely* divisible one will still be beyond our reach, if we wish to cleave to even a vestige of Vickrey's "reality." Stigler (1966, p. 50, fn 3), is far more realistic. He states: "Strictly speaking, continuity requires also that the consumer can discriminate between combinations differing by infinitesimal amounts." But this is false upon its face.

Nor is the second of Thompson's assumptions acceptable. To be sure, consistent tastes is a requisite for indifference curve analysis. Transitivity must be characterized as rationality if we are to have indifference curve analysis, but, when we realize that each preference decision is made at a different *time*, it would appear difficult, again if we wish to retain any vestige of "reality" to insist that the individual chooser cannot change his mind, without being considered irrational.

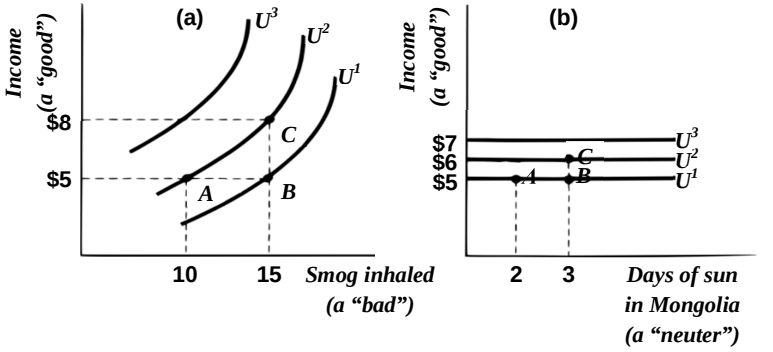
C. The Good

The only entry in this category gets "it" right, as far as it goes. The only reason we do not include it in the excellent category, below, is because it does not go as far as do the latter: all the way to circular indifference curves.

Browning and Zupan

Browning and Zupan (2006) uphold the honor of intermediate texts, for they offer a correct analysis throughout their book. For example, Browning and Zupan (2006, p. 49) clearly and correctly distinguish between economic "goods" and economic "bads." They carry through with this insight in Browning and Zupan (2006, p. 55, figure 3.6a – our diagram 26) where they properly note U3>U2>U1, explaining (2006, p. 54): "If we hold income constant at $50 but increase units of smog – a move from A to B – the person will be worse off (that is, on a lower indifference curve) because smog is a 'bad.'" Nor have these authors forgotten this important lesson many pages later in their text (Browning and Zupan, 2006, p. 143, figure 5.10 – our diagram 27), when they correctly note that U2>U1>U0. In view of the fact that several other authors get this right in some places, and wrong elsewhere, Browning and Zupan are to be singled out for congratulations.

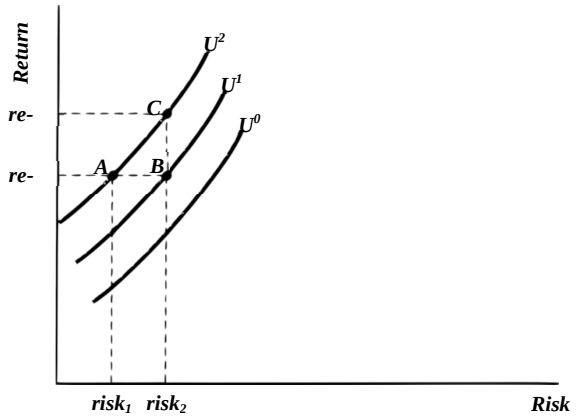
Diagram 26. Indifference Maps for a "Bad" and a "Neuter"



a) Indifference curves have the shape shown here when a "good" is on the vertical axis and a "bad" on the horizontal axis.
b) They have the shape shown here with a "good" on the vertical axis and a "neuter" on the horizontal axis.

Source: Browning and Zupan (2006, p. 55, figure 3.6a).

Diagram 27. The Return-Risk Tradeoff



Since volatility or risk is a bad and expected return is a good, the investor's indifference curves are upward sloping.

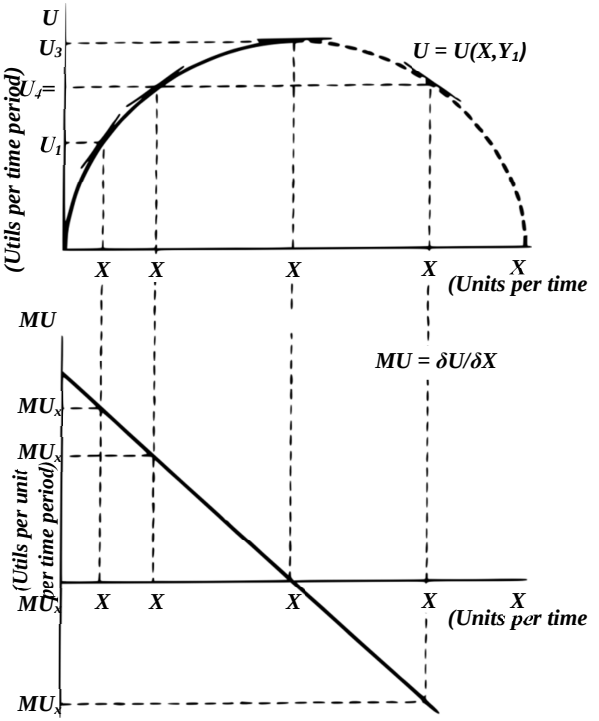
Source: Browning and Zupan (2006, p. 143, figure 5.10).

D. The Excellent

Mathis and Koscianski

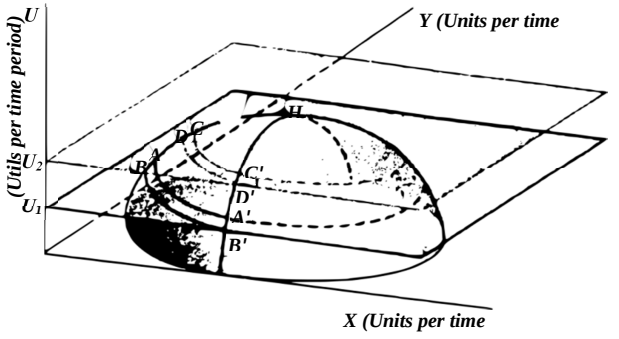
Mathis and Koscianski, 2002 offer incisive insights into the indifference curve. They start off on a high note in Mathis and Koscianski (2002, p. 56, figure 3.8 – our diagram 28) where they depict marginal utility in its negative manifestation. Carrying through, they (2002, p. 59, figures 3.11 – our diagram 29 and 3.12 – our diagram 30; 2002, p. 60, figure 3.13 – our diagram 31) then show the implications of this assumption: a three dimensional diagram (3.11 – our diagram 29), leading to a circular indifference curve in two dimensions (3.12 – our diagram 30). To be sure they accentuate in a solid curve the "usual" part of the indifference curve in both 3.12 – our diagram 30 and 3.13 – our diagram 31, but they are careful to note in 3.13 – our diagram 31 point H, the satiation point. Yes, they (2002, p. 60) do refer to the fact that "those indifference curves lying further to the 'northeast' in the two-dimensional indifference curve map correspond to increasingly higher levels of utility", but, unlike some mentioned above, they are clear that this refers only to the traditional downward sloping convex part of the circular indifference curve.

Diagram 28. Derivation of Marginal Utility Function from Utility Curve



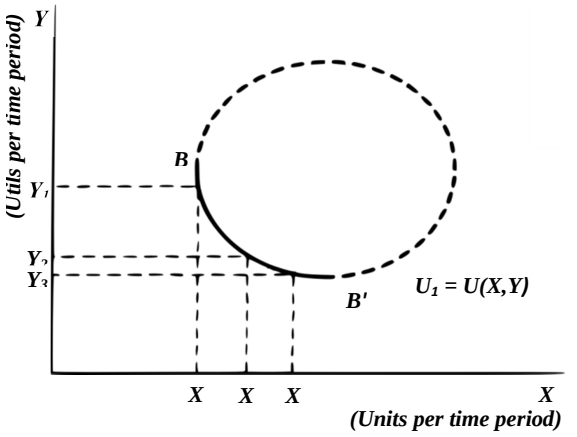
Source: Mathis and Koscianski (2002, p. 56, figure 3.8).

Diagram 29. Indifference Curves of a Three-Dimensional Function



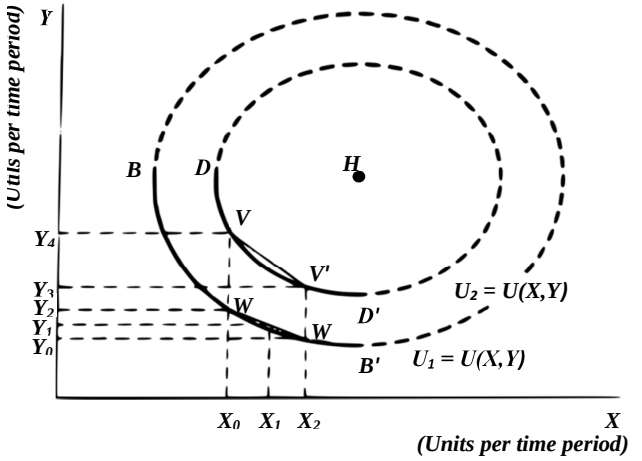
Source: Mathis and Koscianski (2002, p. 59, figures 3.11).

Diagram 30. Indifference Curve Reflecting Satiation



Source: Mathis and Koscianski (2002, p. 59, figure 3.12).

Diagram 31. Indifference Curve Mapping

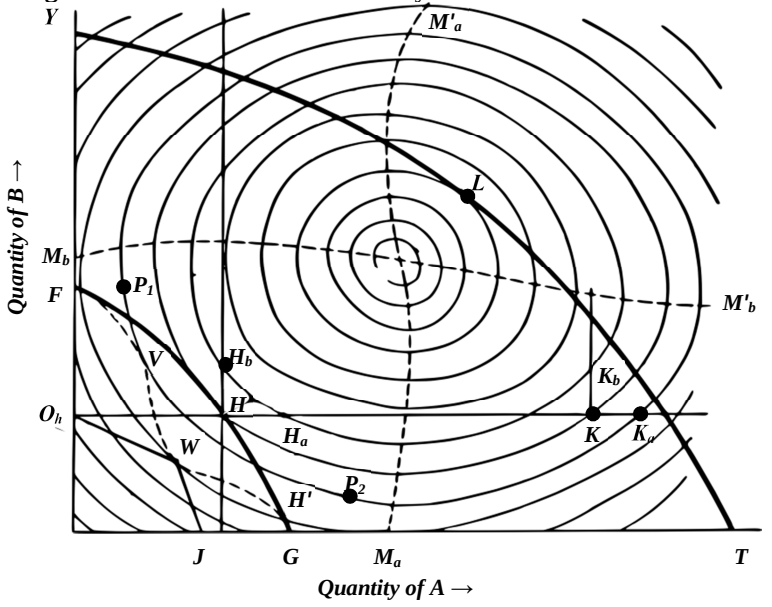


Source: Mathis and Koscianski (2002, p. 56, figure 3.13).

Boulding

Boulding (1966, p. 605, figure 136 – our diagram 32) offers one of the very best indifference curve analyses of any of the entries in our sample.[12] His treatment is masterful. He knows full well that the optimal or satiation point is at M; that if we are restricted to remain inside the budget line (or production possibilities curve SLT) that M is the place for the consumer to locate himself. However, we are for some reason restricted to SLT, then to place ourselves at L is to place ourselves at the highest indifference curve possible. Not for him the fallacious reasoning that there are other indifference curves northeast, or to the right and above L, and that therefore they must be preferable to L. He states (Boulding, 1966, p. 606): "… moving from K to Kb is still moving to a preferred position." This is a magisterial treatment, which stands head and shoulders above any of the others in this compilation.

Diagram 32. An Indifference Curve System



Source: Boulding (1966, p. 605, figure 136).

[12] The fact that this book was published in 1966 (earlier editions appeared in 1941, 1948 and 1955) demonstrates that there has been retrogression, not progress, in the profession of economics, at least in this one case.

Vickrey

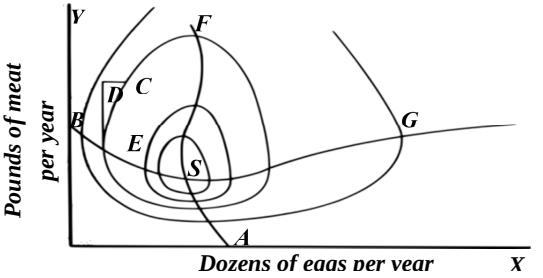
Vickrey (1964, pp. 36-38, figure 6 – our diagram 33) offers a magnificent treatment of the subject matter under discussion. We risk boring the reader with his insights, since we are going to quote this excellent economist[13] at great length:

"… it is possible to depict on the same indifference map situations where the consumer is satiated with one or both of the commodities: i.e., situations where giving the consumer additional amounts of a commodity will yield no increase in satisfaction. Where the consumer is satiated with x but not with y, the indifference curves will be horizontal, and where he is satiated with y but not with x the curves will be vertical. There can also be regions or situations where commodities have become discommodities or nuisances; if one of the two commodities has become a nuisance, the curves will have a positive slope. This is shown in figure 6 (our diagram 33), which may be taken to represent an indifference map of an individual with respect to meat and eggs. In the region OASB both commodities are desirable goods, and this is the normally significant region of the indifference map. Along the line AS the individual has as many eggs as he can make use of (in combination with the various indicated quantities of meat: the more meat he has, obviously, the fewer eggs he can use) and the indifference curves are horizontal; along the line BS the individual is satiated with meat and the curves are vertical. Above BS the excess of meat beings to be a nuisance, and if it is dumped willy-nilly on the consumer's doorstep, he may[14] have to spend some effort in buying it or carting it away in order to avoid a stench. Thus at point C the individual would be willing to pay someone CD of eggs (which are still at that point a desirable commodity to him) for carting away the excess DE of meat. At the satiation point S the individual has exactly that quantity of meat and eggs that will give him the maximum of satisfaction. The curves extend much further above and to the right of S than below and to the left, indicate that on the whole the trouble of getting rid of a given surplus is much less onerous than would be the lack of an equal amount. In the region SFG the curves are again negatively sloped, indicating that the individual would be willing to dispose of more eggs if he could be burdened with less meat, or vice versa."

[13] Full disclosure: Vickrey was a professor at Columbia University when Block was a graduate student there.

[14] Were we Vickrey's editors, we would have changed this "may" to a "will."

Diagram 33. Characteristics of Indifference Maps

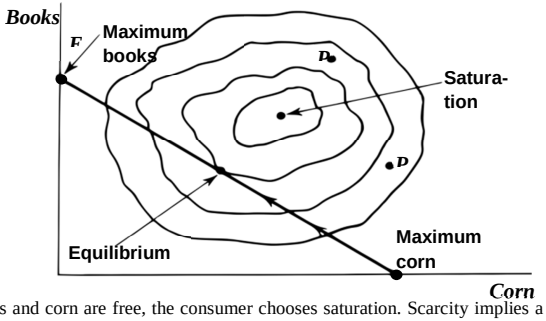


Source: Vickrey (1964, pp. 36-38, figure 6).

McCloskey

McCloskey (1982, p. 27, figure 2.2 – our diagram 34) is a superb treatment of our subject matter. It, too, offers the circular (or elliptical) family of indifference curves. States this author (1982, p. 27, emphasis added): "The rational consumer chooses a point that puts him on the highest attainable contour, *which is the contour as close as possible to saturation."* Nor does he undermine this vital insight, as does Varian (2006) by violating it, when push comes to shove, as he does in our own diagram 7. Very much to the contrary, in the follow up example (McCloskey (1982, p. 31, figure 2.5 – our diagram 35) carries through consistently, and states: "Given the budget line, choose the highest indifference curve that the budge line touches," and in this context, too, it is that budget line that lies closest to the satiation point.

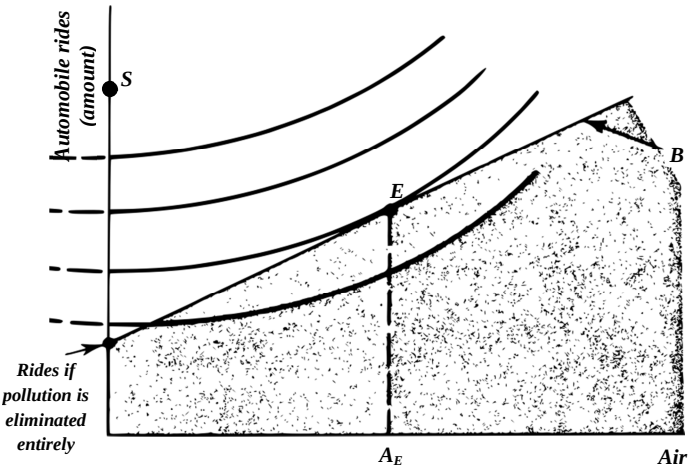
Diagram 34. Tastes Are a Hill



In a world where books and corn are free, the consumer chooses saturation. Scarcity implies a budget constraint that makes saturation unattainable. The rational consumer chooses a point that puts him on the highest attainable contour, which is the contour as close as possible to saturation.

Source: McCloskey (1982, p. 27, figure 2.2).

Diagram 35. The Choice Between a Bad that is Costly to Remove and a Good



The rule for choosing between a good and a bad is identical to between two goods. Given the budget line, choose the highest point the budget line touches.

Source: McCloskey (1982, p. 31, figure 2.5).

As can be seen, the entries in this category, with the exception of Mathis and Koscianski (2002), are venerable ones. That is, Boulding (1966) and Vickrey (1964) were published some 50 years ago. Even the McCloskey (1982) is no spring chicken. Have we lost something as a profession in the last half century? Based upon what is to follow, it would be difficult to avoid this conclusion.

Conclusions

What contribution does this paper make, over and above the one made by Mathis and Koscianski, Boulding, Vickrey and McCloskey? As far as we are concerned, these four authors hit the nail squarely on the head. However, they did not take to task a whole host of other authors who erred in this regard. That is the only contribution of the present effort. Had Mathis and Koscianski, Boulding, Vickrey or McCloskey carried through in this critique, the present paper would have been unnecessary.

We have been at least implicitly assuming the validity of indifference curves, and have in this paper only attempted to “get them right”. However, there are Austrian School critiques of the indifference curves per se, used

correctly or not, which may be of interest to the reader; for example, Barnett (2013); Block (1999, 2003, 2005, 2007); Hoppe (2005); Hulsmann (1999); Machaj (2007); Mises (1998); Rothbard (1997a, 1997b).

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