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A Syntactical Analysis of Lewis's Triviality Result

Abstract. The first part of the paper contains a probabilistic axiomatic extension of the conditional system **WV**, here named $\mathbf{WV}\boldsymbol{Pr}$. This system is extended with the axiom ($Pr4$): $PrA = 1 \supset \Box A$. The resulting system, named $\mathbf{WV}\boldsymbol{Pr}^*$, is proved to be consistent and non-trivial, in the sense that it does not contain the wff (Triv): $A \equiv \Box A$. Extending $\mathbf{WV}\boldsymbol{Pr}^*$ with the so-called Generalized Stalnaker's Thesis (GST) yields the (first) Lewis's Triviality Result (LTriv) in the form $(\Diamond(A \wedge B) \wedge \Diamond(A \wedge \neg B)) \supset PrB|A = PrB$. In §4 it is shown that a consequence of this theorem is the thesis (CT1): $\neg A \supset (A > B \supset A \dashv B)$. It is then proven that (CT1) subjoined to the conditional system $\mathbf{WV}\boldsymbol{Pr}^*$ yields the collapse formula (Triv). The final result is that $\mathbf{WV}\boldsymbol{Pr}^*$+(GST) is equivalent to $\mathbf{WV}\boldsymbol{Pr}^*$+(Triv). In the last section a discussion is opened about the intuitive and philosophical plausibility of axiom ($Pr4$) and its role in the derivation of (Triv).

Keywords: conditionals; conditional probability; Stalnaker's Thesis; triviality; collapse of modalities

§1. In what follows A, B, C, ... will be (meta)variables for wffs, $\neg$, $\supset$, $\wedge$, $\vee$, $\equiv$ will be the usual symbols for truth-functional operators, while the symbol $>$ will be used for the primitive conditional operator. Formation rules are standard. Parentheses will be omitted around the $>$-formulas, the $=$-formulas and where no ambiguity arises. The set of wffs is called WFF.

Auxiliary symbols are:

$$\top \quad := \quad A \vee \neg A \qquad (\mathrm{Def}\,\top)$$
$$\bot \quad := \quad \neg\top \qquad (\mathrm{Def}\,\bot)$$
$$A \asymp B \quad := \quad A > B \wedge B > A \qquad (\mathrm{Def}\asymp)$$
$$A \Supset B \quad := \quad \neg(A > \neg B) \qquad (\mathrm{Def}\Supset)$$

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$$\Box A \quad := \quad \neg A > A \qquad (\text{Def}\,\Box)$$

$$\Diamond A \quad := \quad \neg(A > \neg A) \qquad (\text{Def}\,\Diamond)$$

$$A \dashv B \quad := \quad \Box(A \supset B) \qquad (\text{Def}\dashv)$$

Following the lines of [Nute, 1981, p. 129], the axiom schemata of the weak conditional system **WV** are formulated as follows:

PC: all the tautologies of the truth-functional propositional calculus

and

$$A > A \qquad (\text{ID})$$

$$\neg A > A \supset B > A \qquad (\text{MOD})$$

$$A >< B \supset (A > C \supset B > C) \qquad (\text{CSO})$$

$$(A > B \wedge A \Supset C) \supset (A \wedge C) > B \qquad (\text{CV})$$

$$A > B \supset (A \supset B) \qquad (\text{CMP})$$

For the sake of simplicity, in what follows the word "axiom" will be intended to have the same meaning as "axiom schema".

The rules are:

MP: from $\vdash A$ and $\vdash A \supset B$ infer $\vdash B$;
RCEC: from $\vdash A \equiv B$ infer $\vdash C > A \supset C > B$;
RCK: for any $n > 0$, from $\vdash (A_1 \wedge \cdots \wedge A_n) \supset B$ infer $\vdash (C > A_1 \wedge \cdots \wedge C > A_n) \supset C > B$.

Subjoining to **WV** the following axiom

$$(A \wedge B) \supset A > B \qquad (\text{CS})$$

the resulting system is called **VC**.

An important **WV**-theorem which will be of use in what follows is *Conditional Contrariety*, i.e.

$$(\Diamond A \wedge A > B) \supset A \Supset B \qquad (\text{CC})$$

Our first aim is to extend the language of **WV** to formulate a set of axioms such as to enable the application of standard Probability Calculus without making use of quantificational language. The axiomatization we propose is adapted from the one introduced in [Maksimović, 1983].[1]

As proved by David K. Lewis in [1973], the monomodal fragment of both **WV** and **VC** is the well-known system **KT**.

[1] For the scientific background of Maksimovic's work see for instance [Ognjanović, Rašković and Marković, 2009/2016].

The beginning step is to define a set, named TERM, of probabilistic terms. Such terms are defined starting from the elementary terms, which are:

(a) names of the members of the set $\mathbb{Q}$ of rational numbers;
(b) any formula of form PrA, where A belongs to WFF and Pr is a monadic operator forming elementary terms from members of WFF.

A recursive definition of TERM is then given as follows:[2]

– $\mathrm{Term}(0) = \{\underline{s} : s \in \mathbb{Q}\} \cup \{PrA : A \in \mathrm{WFF}\}$;
– $\mathrm{Term}(n+1) = \{\mathsf{f}, \mathsf{f} + \mathsf{g}, \underline{s} \bullet \mathsf{g}, -\mathsf{f} : \mathsf{f}, \mathsf{g} \in \mathrm{Term}(n)$ and $s \in \mathbb{Q}\}$;
– $\mathrm{Term} = \bigcup_{n=0}^{\infty} \mathrm{Term}(n)$.[3]

The members of TERM will be denoted in what follows as f, g and h. Their behaviour is governed by a set of axioms modelled on the ones which Maksimović introduces on p. 25 of his work with the aim of providing the properties required for computation ($1 \bullet \mathsf{f} = \mathsf{f}$, $\mathsf{f} \bullet 0 = 0$, *Commutatitivity and Associativity of* $+$, *Distributivity of* $\bullet$ with $+$ and of $+$ with $\bullet$ etc.). We will refer to them as *Computation Axioms*. In the set of computation axioms given by Maksimović there is no room for wffs containing the symbol for division -/-. The reason is that, in order to avoid the problem of division by 0, Maksimović prefers writing $x \bullet y^{-1}$ in place of x/y, relying on the well-known identity $y^{-x} = 1/y^x$. Under such presuppositions in his treatment the conditional probability $PrB|A$ is identical to $Pr(A \wedge B) \bullet PrA^{-1}$. In order to perform this device, he introduces, among other axioms, an axiom stating that if $PrA = 0$, $PrB|A$ is also 0 (so the value of $PrB|A$ is never undetermined). For a reason which will become clearer in §2, however, we choose here to assume more traditionally that division is defined by stipulating that $x = z/y$ is definitionally identical to $x \bullet y = z$, if $y \neq 0$.

At this point we are able to define the notion of a basic probabilistic formula as being any formula of the form $\mathsf{f} \geq 0$. The set of all probabilistic formulas $\mathrm{WFF}^{\mathrm{p}}$ is defined as the smallest set which satisfies the following conditions:

1. it contains all the basic probabilistic formulas;

[2] Any underlined symbol x is understood to be the name of x.

[3] The basic idea behind this definition is that, according to Maksimović, each term can be rewritten into an expression describing the summation $\sum_{i=1}^{n}$ of i terms whose general form is $\mathrm{s}_i \ . \ PrB_i + \mathrm{s}$.

2. it is closed under the following rules of construction: if A and B are probabilistic formulas, then $\neg A$, $A \wedge B$ and $A > B$ are probabilistic formulas.

Symbols of current use are then introduced by definition. More specifically, $f \geq g$ is defined as $f - g \geq 0$, where $f - g$ is $f + (-g)$. The identity relation $f = g$ is defined as $f \geq g \wedge g \geq f$. Then the transitivity of $=$ follows from the transitivity of $\geq$, which belongs to the set of the aforementioned computation axioms.

We shall call a *formula* every member of the sets WFF and WFF$^{\mathrm{p}}$. The formation rules introduced by Maksimović neither admit the mixing of propositional and probabilistic wffs nor the iteration of probabilistic terms and probabilistic formulas. For what will be clear in what follows, iteration is required for our purposes, even if no axiom will be introduced to govern the behaviour of this special kind of wffs.

§2. In his important paper [1986], David K. Lewis defines the notion of a probability function with the help of modal notions such as necessity, equivalence, implication and incompatibility, which he defines in terms of possible worlds.[4] Defining necessity as truth at all worlds (p. 299) he seems to rely on a vague Leibnizian notion of necessity without a reference to a specific modal system. In the present context, however, it should be kept in mind that necessity is defined inside formal systems of classical conditional logics, so it has the properties determined by the properties of the background conditional system.

Assuming that **WV** is the minimal conditional system, and endorsing the standard definition of necessity as $\Box A := \neg A > A$, a formal representation of the axioms devised by Lewis consists in subjoining to **WV** the following axioms written in the extended language formulated in the preceding pages:

$$0 \leq PrA \leq 1 \tag{$Pr1$}$$

$$\neg\Diamond(A \wedge B) \supset Pr(A \vee B) = PrA + PrB \tag{$Pr2$}$$

$$\Box A \supset (PrA = 1) \tag{$Pr3$}$$

REq: if $\vdash A \equiv B$, then $\vdash PrA = PrB$.

[4] "We may think of a probability function P as an assignment of numerical values to all sentences of the language, obeying these standard laws of probability: $1 \geqslant \mathrm{P}(A) \geqslant 0$; if A and B are equivalent, then $\mathrm{P}(A) = \mathrm{P}(B)$; if A and B are incompatible, then $\mathrm{P}(A \vee B) = \mathrm{P}(A) + \mathrm{P}(B)$; if A is necessary, then $\mathrm{P}(A) = 1$" [Lewis, 1976, p. 299].

Lewis introduces the notion of *conditional probability* by definition:

$$PrB|A \quad := \quad Pr(B \wedge A)/PrA \qquad \text{if } PrA \neq 0 \tag{Def-|-}$$

In case $PrA = 0$, $PrB|A$ is undefined, i.e. it may be any rational number r s.t. $0 \leqslant r \leqslant 1$.

The system which is **WV** extended with the Computation Axioms mentioned in §1 and the preceding axioms is named here **WV*Pr***. It is understood that all the inference rules of **WV** hold in **WV*Pr***. An intuitive axiom which establishes a link between conditional probability and truth of conditionals is the so-called Generalized Stalnaker's Thesis [see Stalnaker 1970, p. 75; and 1976, p. 302]:

$$Pr(A \wedge C) \neq 0 \supset PrB|(A \wedge C) = PrA > B|C \tag{GST}$$

From (GST) one easily derives so-called *Stalnaker's Thesis* by taking $\top$ as the value of C:

$$PrA \neq 0 \supset PrB|A = PrA > B \tag{ST}$$

Before discussing what follows from subjoining (GST) to **WV*Pr***, we choose to accept among the axioms the converse of (*Pr*3), i.e.

$$PrA = 1 \supset \Box A \tag{Pr4}$$

or the equivalent wff $\Diamond A \supset PrA \neq 0$.

A discussion about (*Pr*4) will take place in §6. Here we may simply remark that in the context of conditional logic the identification of $\Box A$ with $PrA = 1$ and of $\Diamond A$ with $PrA \neq 0$ is technically convenient. Suffice it to remark that thanks to (*Pr*4) and to replacement of proven equivalents definition (Def-|-) turns out to be equivalent to:[5]

$$\Diamond A \supset PrB|A = Pr(B \wedge A)/PrA \tag{CP$\Diamond$}$$

while (GST) and (ST) become respectively

$$\Diamond(A \wedge C) \supset PrB|(A \wedge C) = PrA > B|C \tag{GST$\Diamond$}$$

$$\Diamond A \supset PrB|A = PrA > B \tag{ST$\Diamond$}$$

Looking at (CP◊), let us remark that an instantiation in (CP◊) is $\Diamond A \supset PrA|A = 1$. From this wff we derive $\Diamond A \supset PrA > A = 1$ via (ST◊). Thanks to (*Pr*4) we obtain $\Diamond A \supset \Box(A > A)$ and, by **KT**, $\Diamond A \supset A > A$.

[5] The same formula may be found in [Bradley and Swartz, 1979, p. 383].

But since also $\neg\Diamond A \supset (A > A)$ is a theorem of **WV**, $A > A$ may be derived as a theorem. This provides an example of how the law $\neg\Diamond A \supset A > B$ (i.e. axiom (MOD)) jointly with ($Pr4$) allows the removal of the restriction due to the premise $PrA \neq 0$ in deriving modal-conditional formulas from probabilistic formulas.

The idea of adopting Maksimović's axiom $PrA = 0 \supset PrB/A = 0$ and removing the clause $PrA \neq 0$ from the definition of *Conditional Probability* and from (ST) is technically interesting. Unfortunately, it has the drawback of being equivalent to $Pr\neg A = 1 \supset Pr\neg B/A = 1$. Then by applying (ST) we would have $\Box\neg A \supset PrA > \neg B = 1$, $\Box\neg A \supset \Box(A > \neg B)$ and then $\Box\neg A \supset \Box\Box\neg A$, i.e. the **S4** axiom: a wff which is not a theorem of **KT**, i.e. of the modal fragment of **VW** and **VC**.

A consequence of (GST) and (ST) is (CS): $(A \wedge B) \supset A > B$. The proof is as follows:

1.	$Pr(A \wedge B) \neq 0 \supset PrB\vert(A \wedge B) = 1$	*Comp. Axiom*
2.	$\Diamond(A \wedge B) \supset PrB\vert(A \wedge B) = 1$	1, ($Pr4$)
3.	$\Diamond(A \wedge B) \supset PrB\vert(A \wedge (A \wedge B)) = 1$	2, REq
4.	$\Diamond(A \wedge B) \supset Pr(A > B\vert A \wedge B) = 1$	3, (GST$\Diamond$)
5.	$\Diamond(A \wedge B) \supset Pr((A \wedge B) > (A > B)) = 1$	4, (ST$\Diamond$)
6.	$\Diamond(A \wedge B) \supset \Box((A \wedge B) \supset A > B)$	5, ($Pr4$)
7.	$\neg\Diamond(A \wedge B) \supset \Box((A \wedge B) \supset A > B)$	**KT**
8.	$\Box((A \wedge B) \supset A > B)$	6, 7
9.	$(A \wedge B) \supset A > B$	8, **KT**

Given that (CS) is the characteristic axiom of **VC**, we have then a proof of the fact that **VC** is a subsystem of $\mathbf{VW}\boldsymbol{Pr}^*$.

From the definition of *Conditional Probability* in the form (CP$\Diamond$) the so-called *Multiplication Rule* follows in the form

$$\Diamond A \supset Pr(A \wedge B) = PrA \bullet PrB|A \tag{MR}$$

while (GST) and (ST) turn out be equivalent to (GST$\Diamond$) and (ST$\Diamond$), respectively. In [1976, p. 304], Stalnaker shows that (ST) may be generalized to (GST), so (ST$\Diamond$) may be generalized to (GST$\Diamond$). This relation between the two formulas will not be considered in what follows.

§3. Any conditional system $\boldsymbol{X}$ which is an extension of **WV** is said to be *trivial* if one of the $\boldsymbol{X}$-theorems is $(A \supset B) \supset A > B$ or $A > B \supset A \multimap B$. An instance of the first wff would be $(\neg B \supset B) \supset \neg B > B$, while an instance of the second would be $\top > B \supset \top \mathrel{-\!\!3} B$. In both cases, given that $\Box B \supset B$ is a **WV**-theorem, a consequence would be the equivalence

$$B \equiv \Box B \tag{Triv}$$

Conversely, it is easy to see that if (Triv) is a $\boldsymbol{X}$-theorem both $(A \supset B) \supset A > B$ and $A > B \supset A \mathrel{-\!\!3} B$ are such. So any system $\boldsymbol{X}$ which extends **WV** is trivial if and only if (Triv) is an $\boldsymbol{X}$-theorem. In what follows we will establish that:

(i) **WV*Pr**** is a consistent system;
(ii) **WV*Pr**** is a non-trivial system.

In order to reach the first result we need to define the notion of a **WV*Pr****-model. In what follows the abbreviation $[A]^i$ stands for the set of worlds j accessible to i at which A is true.

A **WV*Pr****-*model* is a 5-ple $\langle W, f, R, m, V\rangle$, where

(i) W is a non-empty set of possible worlds;
(ii) f is a function mapping a pair $\langle A, i\rangle$, where A is a proposition and i is a world, to sets of worlds such that, for every proposition A and world i, $f(A, i)$ is a subset of $[A]^i$ (intuitively, the set of A-worlds "more approximate" to i);[6]
(iii) R is a binary relation on W;
(iv) if j belongs to $f(A, i)$ then iRj;
(v) m is a measure function recursively defined on both terms and sets of worlds, as follows. It assigns to every elementary term (see p. 419) and to every set of worlds $[A]^i$, $[B]^j$, ... indexed by names of possible worlds i, j, ... a rational number in the interval $[0, 1]$, and for the rest it has the following properties:
  a. $m[A \vee B]^i = (m[A]^i + m[B]^i) - m[A \wedge B]^i$,
  b. $m[\neg A]^i = 1 - m[A]^i$,
  c. $m[A \wedge B]^i \leqslant m[A > B]^i \leqslant m[\neg A \vee B]^i$,
  d. $m[\top]^i = 1$,
  e. $m[\bot]^i = 0$,
  f. $m[A]^i = 1$ only if $[A]^i = [\top]^i$;
(vi) V is an evaluation function from pairs of propositions and worlds to the set $\{\mathrm{t}, \mathrm{f}\}$ that is defined in a standard way as far as truth-functional operators are concerned and furthermore has the following clauses:

[6] The properties of f are the basic properties of so-called *selection functions*, as formulated in [Stalnaker, 1968].

a. if $V(A, i) = 1$ then $i \in f(A, i)$; (*Weak Centering*)
b. $V(A > B, i) = \text{t}$ iff $V(B, j) = \text{t}$ at every j in $f(A, i)$;
c. $V(\mathsf{f}^i \geq 0) = \text{t}$ iff $\mathsf{f}^i \geq 0$, where f is a term and f^i is recursively defined in this way: $s = s$; $(PrA)^i = m[A]^i$; $(\mathsf{f} + \mathsf{g})^i = \mathsf{f}^i + \mathsf{g}^i$; $(\mathsf{f} \bullet \mathsf{g})^i = \mathsf{f}^i \bullet \mathsf{g}^i$; $(-\mathsf{f})^i = -(\mathsf{f}^i)$;
d. $V(PrA \geq 0, i) = \text{t}$ iff $V((PrA)^i \geq 0) = \text{t}$.

The definition of conditional probability yields the following identity:

$$(PrB|A)^i = m[A \wedge B]^i / m[A]^i \text{ if } [A]^i \neq \varnothing \tag{Id}$$

A consequence of (vi.c) and (vi.d) is

(vi.e) $V(PrA \geq 0, i) = \text{t}$ iff $m[A]^i \geq 0$,

and also

(vi.f) $V(PrA = PrB, i) = \text{t}$ iff $m[A]^i = m[B]^i$.

Given the preceding definitions, the notion of validity is defined in this way: a wff A is **WV*Pr****-*valid* iff $V(A, i) = \text{t}$ at every i of every **WV*Pr****-model. Notice that this definition implies that a valid wff must be true for every measure function m whose arguments are indexed by the name of possible worlds i, j, k, ...

We now have the tools to prove the following Soundness Theorem:

THEOREM 1. *Every* **WV*Pr*****-theorem is* **WV*Pr*****-valid.*

SKETCH OF THE PROOF. Standard induction on the length of proofs. All the axioms of **WV** are obviously **WV*Pr****-valid. As far as the probabilistic axioms are concerned, it may be seen that each one of them receives value t. As an example, we restrict ourselves to the proof of the validity of ($Pr2$), ($Pr3$) and ($Pr4$).

Ad ($Pr2$): It is the wff $\neg\Diamond(A \wedge B) \supset Pr(A \vee B) = PrA + PrB$. Suppose that there is a world i of a **WV*Pr****-model such that $\neg\Diamond(A \wedge B)$ has value t at i, which means that $A \wedge B$ has value f at all worlds j R-accessible to i. So $[A \wedge B]^i = \varnothing$. Given that $(PrA)^i = m[A]^i$, $Pr(A \vee B)^i = m[A \vee B]^i$. But $m[A \vee B]^i = m[A]^i + m[B]^i$, by clause (v.a), and by (vi.c), we have $m[A]^i + m[B]^i = (m[A] + m[B])^i$. Then $m[A \vee B]^i = (m[A] + m[B])^i$ and by the recursive definition in (vi.c)), $m[A \vee B]^i = Pr(A \vee B)^i = (Pr[A] + Pr[B])^i$. But by (d) this implies $V(PrA \vee B = Pr[A] + Pr[B], i) = \text{t}$. Thus, for every world $i$of every **WV*Pr****-model, ($Pr2$) has value t.

Ad ($Pr3$): From ($Pr3$) $\Box A \supset PrA = 1$ and ($Pr4$) $PrA = 1 \supset \Box A$ we have the equivalence $\Box A \equiv PrA = 1$. Clause (vi.f) states that $V(PrA = PrB, i) = \mathrm{t}$ iff $m[A]^i = m[B]^i$. So $V(PrA = Pr\top, i) = \mathrm{t}$ iff $m[A]^i = m[\top]^i$. By ($Pr2$), $Pr(A \vee \neg A) = 1$, so $V(PrA = 1, i) = \mathrm{t}$ iff $m[A]^i = m[\top]^i$. But by (v.g), this implies $[A]^i = [\top]^i$, whose meaning is that A is true at all possible worlds related to i at which $\top$ is true. So, for every j s.t. iRj, $V(A, j) = \mathrm{t}$. This means $V(\Box A, i) = \mathrm{t}$. So $V(PrA = 1, i) = \mathrm{t}$ implies $V(\Box A, i) = \mathrm{t}$. Since the converse is easily proved, this means that for every i, $V(\Box A \equiv PrA = 1, i) = \mathrm{t}$.

The inference rules of **WV*Pr**** are the same as those of **WV**, and can be shown to be validity-preserving. Hence all the **WV*Pr***-theorems are **WV*Pr****-valid. ⊣

An obvious corollary of Theorem 1 is the following:

COROLLARY 1. **WV*Pr**** *is a consistent system.*

Thanks to Theorem 1 we are able to prove the following theorem.

THEOREM 2. **WV*Pr**** *is not a trivial system.*

PROOF. Suppose by reductio that $A \supset \Box A$ is a **WV*Pr****-theorem. This wff then has value t at every world of every **WV*Pr****-model. Let us then consider a **WV*Pr****-model M such that $W = \{i, j\}$, iRj, and, for an atomic A, $m[A]^i = 0.5$, $V(A, i) = t$, $V(A, j) = \mathrm{f}$. Thus $V(\Box A, i) = \mathrm{f}$ and the wff $A \supset \Box A$ has value f at i in M. So, by Theorem 1, $A \supset \Box A$ cannot be a **WV*Pr****-theorem. ⊣

Another result which may be proved is that (GST) is not a **WV*Pr****-theorem. A semantic argument could be provided for this, but the result follows from the theorems which will be proved in the next section. In fact, it will be proved that extending **WV*Pr**** with (GST) yields (Triv), which is not a **WV*Pr****-theorem. So, (GST) cannot be a **WV*Pr****-theorem.

§4. Going back to Lewis's paper [1986], it is well-known that this paper proves that, on the basis of Kolmogorov axioms, Stalnaker's Generalized Thesis, (GST), has the consequence that the language cannot provide more than two possible but incompatible propositions.[7]

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[7] This is known as Lewis's First Triviality Result. The Second Triviality Result consists in the following: "Except in a trivial case, there is no way to interpret >

In the light of the equivalence $\Diamond A \equiv PrA \neq 0$ which follows from axioms P3 and P4, Lewis's Triviality Result, here named "LTriv", may be written as follows:

$$(\Diamond(A \wedge B) \wedge \Diamond(A \wedge \neg B)) \supset PrB|A = PrB \tag{LTriv}$$

The syntactical proof of (LTriv) in $\mathbf{WV}\boldsymbol{Pr}*$+(GST) is as follows. For sake of simplicity it makes use of the so-called Conditionalization Rule ($A \vdash B$ implies $\vdash A \supset B$), but it could reconstructed without this device.

Lemma 1. *In* $\mathbf{WV}\boldsymbol{Pr}^*$+(GST), (LTriv) *follows from the hypothesis*

$$\neg\Diamond B \vee \neg\Diamond\neg B$$

Proof. Let us suppose $\neg\Diamond B \vee \neg\Diamond\neg B$. Since $\neg\Diamond B$ implies $\neg\Diamond(A \wedge B)$, from the given hypothesis we have $\neg\Diamond(A\wedge B)\vee\neg\Diamond(A\wedge\neg B)$, so $\neg(\Diamond(A\wedge B)\wedge\Diamond(A\wedge\neg B))$. Then by Scotus's law $(\Diamond(A\wedge B)\wedge\Diamond(A\wedge\neg B)) \supset PrA > B = PrB$ is a consequence of $\neg\Diamond B \vee \neg\Diamond\neg B$. ⊣

Lemma 2. *In* $\mathbf{WV}\boldsymbol{Pr}^*$+(GST), (LTriv) *follows from the hypothesis*

$$\Diamond B \wedge \Diamond\neg B$$

Proof. The result will be proved for (GST◊), which is equivalent to (GST) in $\mathbf{WV}\boldsymbol{Pr}^*$+(GST).

0. $\Diamond B \wedge \Diamond\neg B$ — Hypothesis
1. $\Diamond(A \wedge B) \wedge \Diamond(A \wedge \neg B)$ — Hypothesis
2. $Pr(A \wedge B) \neq 0 \wedge Pr(A \wedge \neg B) \neq 0$ — 1, (Pr4), $\Diamond A \supset PrA \neq 0$
3. $PrA > B|B = PrB|A \wedge B$ — 1, (GST), MP
4. $PrB|A \wedge B = 1$ — $\Diamond(A \wedge B) \vdash PrB|(A \wedge B) = 1$, 1, MP
5. $PrA > B|B = 1$ — 4, 3
6. $PrA > B|\neg B = 0$ — (ST◊), $PrB|(A \wedge \neg B) = 0$, 1
7. $PrA > B = (Pr(A > B|B) \bullet PrB) + (Pr(A > B|\neg B) \bullet Pr\neg B)$
   (Pr2), $\vdash A > B \equiv ((A > B \wedge B) \vee (A > B \wedge \neg B))$, $x = x/y \bullet y$
8. $PrA > B = 1 \bullet PrB + 0 \bullet Pr\neg B$ — 5, 6, 7, 10

uniformly so that (ST) holds throughout a class of probability functions closed under conditionalizing." The first Triviality Result relies on the presupposition that (GST) holds for every probability function, while one could be willing to require that (GST) holds only for a specific class of them, provided that it is closed under conditionalization. The Second Triviality result shows that trivialization may be proved also for this second case. What we call here the Triviality Result is the First Triviality result.

9. $PrA > B = PrB$ 9, Comput. Ax.
10. $(\Diamond(A \wedge B) \wedge \Diamond(A \wedge \neg B)) \supset PrA > B = PrB$ 1,9
11. $(\Diamond(A \wedge B) \wedge \Diamond(A \wedge \neg B)) \supset PrB|A = PrB$ 10, 1, (ST$\Diamond$) ⊣

THEOREM 3. *(LTriv) is a thesis of* **WV*Pr****+(GST).

PROOF. From Lemmas 1 and 2, (LTriv) follows from $\Diamond B \wedge \Diamond\neg B$ and also from $\neg(\Diamond B \wedge \Diamond\neg B)$. So from their disjunction, which is an obvious theorem. ⊣

The following theorems prove that **WV*Pr****+(GST) is equivalent to the trivial system **WV*Pr****+(Triv).

THEOREM 4. *All theses of* **WV*Pr***$^+$(GST) *are theses of* **WV*Pr****+(Triv).

PROOF. (GST) is $Pr(A \wedge C) \neq 0 \supset PrB|(A \wedge C) = PrA > B|C$. A first consequence of (Triv) is $\Diamond C \supset \Box C$. So $\Box C \vee \Box\neg C$. Other two consequences are the following (due to (MOD): $\Box C \supset B > C$):[8]

$$\Box C \supset \Box\Box C \tag{4}$$

$$B > C \ \vee\ B > \neg C \tag{CEM}$$

From such premises we derive the following:

1. If $\Box\neg C$, $Pr(A \wedge C) = 0$, and (GST) follows by Scotus's law.
2. If $\Box C$, by (*Pr*3), $PrC = 1$.

Since $\Box B \vee \Box\neg B$, we have $PrB = 0 \vee PrB = 1$. There are two alternatives:

(a) Suppose $PrB = 0$ and $Pr(A \wedge C) \neq 0$. This means that, by (*Pr*4), we have $\Box\neg B$, and by (4), we get $\Box\Box\neg B$. In **WV**, $\Box\Box\neg B$ implies $\Box(A > \neg B)$. So $PrA > \neg B = 1$ and $PrA > B = 0$. So $PrA > B|C = 0$. But also, given that $PrB = 0$, $PrB|(A \wedge C) = 0$. So, due to $0 = 0$, $PrA > B|C = PrB|(A \wedge C)$ and a fortiori, by $Pr(A \wedge C) \neq 0$, (GST).

(b) Suppose $PrB = 1$, which by (*Pr*4) implies $\Box B$. By (4), $\Box B$ implies $\Box\Box B$ and $\Box(A > B)$. So also $PrA > B = 1$ and $PrA > B|C = 1$.

We have to consider now two subhypotheses.

Suppose $\Box A$. Then $\Box(A \wedge B)$, by **KT**. So $1 = Pr(A \wedge B) = PrA = PrB$. But $\Box A$ and $\Box B$ jointly imply $\Box\Box(A \wedge B)$ and so, by (CS), $\Box(A > B)$ and consequently $PrA > B = 1 = Pr(A > B|C)$. So $PrB|(A \wedge C) = PrA > B|C$, and a fortiori (GST).

[8] Subjoining to **VC** the so-called *Conditional Excluded Middle*, (CEM), the resulting system is Stalnaker's system **C2**.

Suppose $\neg\Box A$. Given that $\Box A \vee \Box\neg A$, then $\neg\Box A$ implies $\Box\neg A$. So also $PrA = 0$ and $Pr(A \wedge C) = 0$. Then (GST) follows by Scotus's law. ⊣

In the proof of the converse of Theorem 4 a key role is played by the proof of the following critical theses:

$$\neg A \supset (A > B \supset A \dashv B) \quad \text{(CT1)}$$

$$\neg B \supset (A > B \supset A \dashv B) \quad \text{(CT2)}$$

In order to prove the converse of Theorem 4 we need the following lemmas.

Lemma 3. (CT1) *follows from the following hypothesis*

$$\neg\Diamond(A \wedge B) \vee \neg\Diamond(A \wedge \neg B) \quad \text{(Hyp1)}$$

Proof. We show that (CT1) is derived from the following two subhypotheses: (1a) $\neg\Diamond(A \wedge B)$ and (1b) $\neg\Diamond(A \wedge \neg B)$. The proof is via conditionalization but may be performed without this device.

1.	$\neg\Diamond(A \wedge B)$	Hyp. (1a)
2.	$\neg\Diamond(A \wedge B) \supset A > \neg B$	$\Box(A \supset \neg B) \vdash_{\mathbf{WV}} A > \neg B$, (Def $\Diamond$)
3.	$A > \neg B$	1, 2, MP
4.	$\Diamond A \supset ((A > \neg B \wedge A > B) \equiv \bot)$	(CC)
5.	$\Diamond A \supset ((A > \neg B \wedge A > B) \supset A \dashv B)$	4, $\bot \vdash B$, PC
6.	$\Diamond A \supset (A > B \supset A \dashv B)$	5, 3, PC, MP
7.	$\neg\Diamond A \supset (A > B \supset A \dashv B)$	$\neg\Diamond A \vdash (A \dashv B)$, PC
8.	$A > B \supset A \dashv B$	6, 7, PC
9.	$\neg A \supset (A > B \supset A \dashv B)$	8, PC
10.	$\neg\Diamond(A \wedge \neg B)$	Hyp. (1b)
11.	$\Box(A \supset B)$	10, **KT**
12.	$\neg A \supset (A > B \supset A \dashv B)$	11,(Def $\dashv$), PC

Hence, by conditionalization and steps 1, 9, 10, 12, we obtain:

$$(\neg\Diamond(A \wedge B) \vee \neg\Diamond(A \wedge \neg B)) \supset (\neg A \supset (A > B \supset A \dashv B))$$ ⊣

Lemma 4. *In* $\mathbf{WV}\boldsymbol{Pr}^*$+(LTriv), (CT1) *follows from the hypothesis*

$$\Diamond(A \wedge B) \wedge \Diamond(A \wedge \neg B) \quad \text{(Hyp2)}$$

Proof. 1. $\Diamond(A \wedge B) \wedge \Diamond(A \wedge \neg B)$ (Hyp2)

2. $\Diamond A \supset (A \dashv \neg B \supset \neg(A > B))$ (CC), $A \dashv \neg B \vdash A > \neg B$

3. $PrB|A = PrB$ 1, **KT**, (LTriv)
4. $(\Diamond(A \wedge (A \wedge B)) \wedge \Diamond(A \wedge \neg(A \wedge B))) \supset PrA > (A \wedge B) = Pr(A \wedge B)$ (LTriv), $A \wedge B$ for B
5. $(\Diamond(A \wedge B) \wedge \Diamond(A \wedge \neg B)) \supset (\Diamond(A \wedge B) \wedge \Diamond\neg(A \wedge B))$ **KT**
6. $(\Diamond(A \wedge B) \wedge \Diamond(A \wedge \neg B)) \supset PrA > B = Pr(A \wedge B)$ 5, 4, PC
7. $(\Diamond(A \wedge B) \wedge \Diamond(A \wedge \neg B)) \supset PrB = Pr(A \wedge B)$ 6, (LTriv)
8. $\Diamond A \supset (A > B \supset \Diamond(A \wedge B))$ 2, PC, (Def $\Diamond$), **KT**
9. $\Diamond A \supset ((A > B \wedge \Diamond(A \wedge \neg B)) \supset PrB = Pr(A \wedge B))$ 8,7, PC
10. $\neg\Diamond A \supset \neg\Diamond(A \wedge \neg B)$ **KT**
11. $\neg\Diamond A \supset ((A > B \wedge \Diamond(A \wedge \neg B)) \supset PrB = Pr(A \wedge B))$ 10, Scotus' Law, PC
12. $(A > B \wedge \Diamond(A \wedge \neg B)) \supset PrB = Pr(A \wedge B)$ 9,11, PC
13. $PrB|A = PrB \supset Pr(A \wedge B) = PrA \bullet PrB$ (MR), 1, $\Diamond(A \wedge B) \supset \Diamond A$
14. $Pr(A \wedge B) = PrA \bullet PrB$ 13, 1, (LTriv), MP
15. $(A > B \wedge \Diamond(A \wedge \neg B)) \supset PrB = PrA \bullet PrB$ 12, 14
16. $(A > B \wedge \Diamond(A \wedge \neg B)) \supset PrB/PrB = PrA \bullet PrB/PrB$ 15, Comput. Ax.
17. $\Diamond B \supset ((A > B \wedge \Diamond(A \wedge \neg B) \supset 1 = PrA)$ 16, $\Diamond B \supset PrB/B = 1$
18. $\Diamond B \supset ((A > B \wedge \Diamond(A \wedge \neg B) \supset \Box A)$ 17, ($Pr4$)
19. $\Diamond B \supset (\neg\Box A \supset (A > B \supset \neg\Diamond(A \wedge \neg B))$ 18, PC
20. $\Diamond B \supset (\neg A \supset (A > B \supset A \dashv B))$ 19, $\neg A \vdash_{\mathbf{KT}} \neg\Box A$
21. $(B \dashv \bot \wedge A > B) \supset A > \bot$ **VW**
22. $\Box\neg B \supset (A > B \supset \neg\Diamond A)$ $\Box\neg B \supset B \dashv \bot$, 21
23. $\neg\Diamond B \supset (\neg A \supset (A > B \supset A \dashv B))$ 22, PC, $\neg\Diamond A \vdash_{\mathbf{KT}} A \dashv B$
24. $\neg A \supset (A > B \supset A \dashv B)$ 20, 23[9] ⊣

LEMMA 5. (CT1) *is a theorem of* **WV*Pr****+(GST).

PROOF. By Lemmas 3 and 4 we have that (CT1) follows from (Hyp1) and (Hyp2), respectively. So (CT1) follows from their disjunction, which is an instance of the PC-theorem $A \vee \neg A$. ⊣

LEMMA 6. (CT2) *is a theorem of* **WV**+(CT1).

PROOF. 1. $\neg A \supset (A > B \supset A \dashv B)$ (CT1)
2. $(\neg A \wedge \neg B) \supset (A > B \supset A \dashv B)$ 1, PC
3. $(\neg B \wedge (A \supset B)) \supset (A > B \supset A \dashv B)$ $\neg B \wedge (A \supset B) \vdash \neg B \wedge \neg A$, 2

[9] This proof is a simplification of the proof of the same theorem which can be found in [Pizzi, 1990]. The paper contains the assertion that (CT1) and (CT2) subjoined to **VC** do not yield a trivial system, but the proof provided by the author is incorrect.

4. $(\neg B \wedge A > B) \supset (\neg B \wedge (A \supset B))$ **WV**
5. $\neg B \supset (A > B \supset A \,\sqsupset\!\!\rightarrow B)$ 4, 3, PC ⊣

LEMMA 7. (Triv) *is a theorem of* **VC**+(CT2).

PROOF. As already stated, the characteristic axiom of **VC** is (CS):

1. $(A \wedge \neg B) \supset ((\neg B \wedge A > B) \vee (\neg B \wedge A > \neg B))$
   (CS): $(A \wedge \neg B) \supset A > \neg B$, PC
2. $(A \wedge \neg B) \supset (A \,\sqsupset\!\!\rightarrow B \vee A \,\sqsupset\!\!\rightarrow \neg B)$ 1, (CT2), PC
3. $(A \wedge B) \supset (A \,\sqsupset\!\!\rightarrow B \vee A \,\sqsupset\!\!\rightarrow \neg B)$ 2, $B/\neg B$
4. $A \supset (A \,\sqsupset\!\!\rightarrow B \vee A \,\sqsupset\!\!\rightarrow \neg B)$ 2, 3, $\vdash A \equiv ((A \wedge B) \vee (A \wedge \neg B))$
5. $\top \supset (\Box B \vee \Box \neg B)$ 4, $\top$ for A, $\Box A \equiv \top \,\sqsupset\!\!\rightarrow A$
6. $\Diamond B \supset \Box B$ 5, MP, (Def $\Diamond$)
7. $B \supset \Box B$ 6, $B \supset \Diamond B$
8. $B \equiv \Box B$ 7, **KT** ⊣

THEOREM 5. (Triv) *is a theorem of* **WV*Pr****+(GST).

PROOF. By Lemma 5, (CT1) is a theorem of **WV*Pr****+(GST). By Lemma 6, since **VC** is a subsystem of **WV*Pr****, (CT2) is also a theorem of **WV*Pr****+(GST). By Lemma 7 (Triv) is derivable in **VC**+(CT2). So (Triv) is derivable in **WV*Pr****+(GST). ⊣

§5. A byproduct of the preceding proof concerns the critical wff (CT1). To begin with, Lemmas 6 and 7 show that if (CT1) is subjoined to **VC** it yields (CT2) and the collapse of modal distinctions. This proves that (CT1) is not a theorem of **VC** (which is a non-trivial conditional system), and a fortiori cannot be a theorem of any system weaker than **VC**.

(CT1) however has a place in the studies on conditionals and deserves special attention. In [Veltman, 1976] the author lays the grounds of the so-called Premise Semantics for conditionals, subsequently developed by Kratzer [see for instance 2012]. Roughly speaking, this theory consists essentially in saying that $A > B$ is true iff B follows from A coinjoined to all possible ways of adding true sentences to the antecedent maintaining consistency. In [1976] Veltman showed that if $A > B$ is defined in this way we reach the equivalence

$$A > B \equiv ((A \supset B) \wedge (\neg A \supset A \,\sqsupset\!\!\rightarrow B)) \qquad \text{(KVT)}$$

A first result of this equivalence is that, given that $A \wedge B$ implies, by PC, $(A \supset B) \wedge (\neg A \supset A \,\sqsupset\!\!\rightarrow B)$. So, by the equivalence (KVT), $A \wedge B$ implies also $A > B$. The wff (CS): $(A \wedge B) \supset A > B$ is then a consequence of (KVT) simply by standard logic. As a consequence, any system containing (KVT) is at least as strong as the conditional system **VC**. Now what can be easily seen is what follows:

THEOREM 6. **VC**+(CT1) *and* **VC**+(KVT) *are equivalent systems.*

PROOF. Suppose that (KVT) is subjoined to **VC** as (KVT) implies (i): $A > B \supset ((A \supset B) \supset (\neg A \supset (A \prec\!\!3\, B))$. Hence, by PC, we have (ii): $(A > B \supset (A \supset B)) \supset (A > B \supset (\neg A \supset (A \prec\!\!3\, B))$. But the antecedent of (ii) is (CMP) of **WV**. So, by MP and PC, we reach (CT1): $\neg A \supset (A > B \supset A \prec\!\!3\, B)$.

Conversely, let us suppose that (CT1) is subjoined to **VC**. By Lemmas 6 and 7, above this has as a consequence $A \equiv \Box A$, for any A. So also $A \supset B \equiv A \prec\!\!3\, B$ and $A > B \equiv A \prec\!\!3\, B$. (KVT) turns out, by replacement of proven equivalents, a consequence of $A \prec\!\!3\, B \equiv (A \prec\!\!3\, B \wedge (\neg A \supset A \prec\!\!3\, B))$, which is an instance of a PC-theorem. ⊣

A consequence of Theorem 6 is that adding (KVT) to **VC** yields the collapse of modal distinctions.

THEOREM 7. **WV**+(KVT) *is equivalent to* **WV**+(Triv).

PROOF. As shown in the proof of Teorem 6, **WV**+(Triv) yields (KVT), by applying $A > B \equiv A \prec\!\!3\, B$ and replacement of proved equivalents to the theorem $A \prec\!\!3\, B \equiv (A \prec\!\!3\, B \wedge (\neg A \supset A \prec\!\!3\, B))$. Conversely: **WV**+(KVT) yields as the theorem (CS). So, by Theorem 6, it contains the system **VC**. So, it has as a theorem (CT1); and so, by Lemma 7, also (CT2) and (Triv). ⊣

An obvious consequence of Theorem 7 is the following:

THEOREM 8. **WV*Pr****+(KVT) *is equivalent to* **WV*Pr****+(GST).

§6. The preceding results raise some questions due to there being in the object-language both probability operators and modal-conditional operators. As already remarked, a critical point of discussion concerns the introduction of axiom (*Pr*4). The semantics introduced in §2 implies that the assignment of value "true" to $PrA = 1$ is made with respect to some possible world i. This intuitively may be interpreted as saying that at i it is true that A has the maximum probability, but it is not obvious that this implies that $\Box A$ is true at i, i.e. that "A is true at all possible worlds accessible to i".

As a matter of fact, what one thinks about ($Pr4$) depends on the background philosophical assumptions about the notions of probability and necessity.[10] There are indeed many reasons to argue that the meaning of $\Box A$ matches a probabilistic interpretation. This for instance is the line followed by N. Rescher at p. 218 of [1963], where he writes:

> (i) A statement of the type $\Box p$ is to be true (i.e., have V-value 1) if and only if $Prp = 1$.
> (ii) A statement of the type $\Box p$ is to be capable of assuming only the probability values 0 and 1.

The statement (ii) introduces an axiom for nested probability statements such as $Pr(Prp = 1) = x$, which may be easily seen to yield the modal system **S5** and does not follow from (i).[11] In [Fattorosi-Barnaba and Amati, 1987] the authors introduce a logic whose primitive symbol is $\Diamond^r$ ($r \in [0, 1]$) to be read as "A has probability higher than r". At p. 383 of their paper the possibility operator $\Diamond$ is identified with $\Diamond^0 A$, i.e. with $PrA \neq 0$. Hence, given that $\Box^r A = \neg\Diamond^r\neg A$ (p. 385), $\Box^0 A$ ($PrA = 1$) is identified with $\Box A$. The same identification may be found in [Holliday and Icard, 2014].

The plausibility of ($Pr4$) may be discussed at length but we shall confine ourselves to two remarks.

The first is based on the consideration that in $PrA = x$ the number x is often considered as a rational number expressing the percentage of A-worlds (i.e. of worlds at which A is true) with respect to the global number of possible worlds. Since $PrA = 1$ is equivalent to $PrA = Pr\top$, the number of A-worlds is coincident with the percentage of possible worlds at which $\top$ is true. If $\neg A$ were true at some possible world, PrA could not be exactly 1. So, if $PrA = 1$, this implies that A is true at all possible worlds. *A fortiori* then A will be true at all possible worlds related by some relation R to some arbitrary possible world i. So, if $\Box A$ is semantically defined in terms of such R, it follows that $\Box A$ is true at any arbitrary i.

According to some different conception of probability, e.g., a statistical one, the preceding argument cannot be developed. If saying that $PrA = 1$ simply means that A is certain in the actual physical world or that A has some kind of physical necessity, ($Pr4$) states the collapse of this weaker operator of necessity on the stronger one.

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[10] See for instance [Eagle, 2015, p. 7].

[11] For an analogous approach see [Montanaro and Bressan, 1981].

A question which will not be discussed here but may be left to other inquiries concerns not the meaning of Axiom (*Pr*4) but its relevance in the derivation of (Triv). In other words, an open problem is to understand whether (Triv) may be derived by adding (GST) to the simple **WV*Pr*** without the essential use of (*Pr*4). In the case of a negative answer, a conjecture is that in place of the modal formula (Triv) we are anyway able to derive as a theorem not $\Box A \vee \Box\neg A$ (which leads to (Triv) in **KT**) but $Pr A = 1 \vee Pr A = 0$, which is not a modal formula but states the full trivialization of the probability calculus.

A second line of possible inquiry concerns the analysis of the concept of approximation to triviality. In [Pizzi, 2019] the author maintains that the system **VC** expresses a partial trivialization of modal notions to axiom (CS). It should also be remarked that (CT1) is equivalent to $(\neg A \wedge A > B) \equiv (\neg A \wedge A \prec\!\!3\; B)$, where $\neg A \wedge A > B$ is an explicit conditional in the sense of [Pizzi, 2019], while (CS) is equivalent to $(A \wedge (A \supset B)) \equiv (A \wedge A > B)$, where $A \wedge A > B$ is also an explicit conditional. The above equivalences then may be described as stating the collapse of an explicit conditional into the homologous explicit strict or material conditional, so that it is not surprising that (CT1) subjoined to **VC** yields (Triv). In this connection it would be interesting to know if subjoining (CT1) to **WV** or to some system weaker than **VC** yields (Triv) or some different formula expressing an approximation to triviality.

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