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## Logical Nihilism and the Logic of ‘prem’

**Abstract.** As the final component of a chain of reasoning intended to take us all the way to logical nihilism, [Russell \(2018\)](#) presents the atomic sentence ‘prem’ which is supposed to be true when featuring as premise in an argument and false when featuring as conclusion in an argument. Such a sentence requires a non-reflexive logic and an endnote by [Russell \(2018\)](#) could easily leave the reader with the impression that going non-reflexive suffices for logical nihilism. This paper shows how one can obtain non-reflexive logics in which ‘prem’ behaves as stipulated by [Russell \(2018\)](#) but which nonetheless has valid inferences supporting uniform substitution of any formula for propositional variables such as modus tollens and modus ponens.

**Keywords:** logical nihilism; non-reflexive logic; non-transitive logic; dual valuations; modus ponens

### 1. Introduction

Logical nihilism is the thesis that there are no logical laws. Following [Russell \(2018\)](#), we can understand logical nihilism as the claim that there are no valid inferential schemas. In a language with propositional variables, this claim corresponds to the claim that there is no valid inference such that any formula can be uniformly substituted for propositional variables that are possibly occurring in the inference. While some readers may prefer a Fregean notion of a logical law and thus consider only sentential schemas such as  $\varphi \rightarrow \varphi$  as candidates for logical laws (where  $\rightarrow$  is a conditional connective of the language under consideration), we shall here, following [Russell \(2018\)](#), also consider inferential schemas such as  $\varphi \rightarrow \psi, \varphi \vDash \psi$  as candidates for logical laws, where  $\vDash$  is an expression of the metalanguage representing “entails”.

A non-reflexive logic is a logic in which not every formula entails itself, i.e. for which the inferential schema  $\varphi \vDash \varphi$  does not hold. This inferential schema is referred to by [Russell \(2018\)](#) as the law of identity. While this is perhaps not the immediate candidate for a logical law to question when things get tough, there is some recent research on the use of a non-reflexive logic to accommodate self-referential definitions in order to block the set-theoretic and truth-theoretic paradoxes. Examples include ([Gilmore, 1986](#)), ([Greenough, 2001](#)), ([Schroeder-Heister, 2016](#)), ([French, 2016](#)), ([Fjellstad, 2017](#)), ([Nicolai and Rossi, 2018](#)) and ([Murzi and Rossi, 2017](#)). The non-reflexive logics presented in the literature to that purpose share the feature that they do not have any valid inferences that support uniform substitution of any formulas for propositional variables. In effect, then, adopting such a logic would amount to embracing logical nihilism in the above sense. Some might even see it as an argument against that approach to blocking the set-theoretic and the truth-theoretic paradoxes.

That a non-reflexive logic represents logical nihilism is also an idea that shows up the discussions of ([Russell, 2018](#)) and ([Dicher, 2020](#)). As an illustration of a strategy for creating counterexamples to any logical law whatsoever, [Russell \(2018\)](#) presents a sentence labelled PREM with the following characteristics:

[L]et ‘prem’ be an atomic sentence whose value is true when it features in the premises of an argument and false when it features in the conclusion (or in any other linguistic context). ([Russell, 2018](#), p. 316)

PREM is intended as a counterexample to the law of identity, that every formula entails itself. If validity is (necessary) truth-preservation from the premises to the conclusion, then surely a formula which is true when evaluated as premise but false when evaluated as conclusion suffices as a counterexample to the law of identity. In an endnote directly attached the presentation of PREM, ([Russell, 2018](#)) can be interpreted as asking whether PREM takes us all the way to logical nihilism:

[Does] this really get us all the way to nihilism? Strictly speaking, that depends on what logical constants are in the language. In particular, if you have  $\top$  as a 0-place truthfunctor that is always interpreted as true, then you will have some arguments with  $\top$  and things like  $\top \vee \top$  as a conclusion. These will be valid regardless of the interpretation, so with these logical constants you won’t get all the way to nihilism. I don’t highlight that very much here because it seems to me that a logic with

only these laws may as well be logical nihilism, in that everything that seems bad about the one seems bad about the other.

(Russell, 2018, n15)

Understood as a remark concerning PREM, Russell’s (2018) point seems to be that we can certainly add truth-constants to a non-reflexive logic in order to obtain valid inferences that support uniform substitution, but that such inferences are not particularly useful. Indeed, the endnote concludes with that such inferences are not “useful for doing metatheory. Or assessing proofs in arithmetic.” After all, the inferences do not have any place-holders for the propositions which we are interested in reasoning about.

The aim of this paper is to show that a non-reflexive logic does not represent the straightforward path to logical nihilism suggested by Russell’s (2018) endnote despite the fact that the non-reflexive logics explored in the literature on set-theoretic and truth-theoretic paradoxes are de facto instances of logical nihilism.

The strategy will be as follows. While this paper agrees with (Russell, 2018) that it will depend on what logical constants there is in the language, there is no reason to limit our attention to  $\top$  and  $\perp$ . Instead, we can easily tweak the interpretation of familiar logical constants such as  $\neg$  and  $\rightarrow$  to thereby obtain non-reflexive logics for PREM with what we can describe as useful valid inferences that support the uniform substitution of any formula for propositional variables. In particular, the paper will proceed as follows. After elaborating on some further aspects with logical nihilism in section 2, the paper presents in section 3 the semantics for TONK proposed by Fjellstad (2015) as adapted to PREM. It then shows first in section 4 how to tweak the interpretation of negation to obtain a non-reflexive logic for PREM in which modus tollens holds, and then in section 5 how to tweak the interpretation of the conditional to obtain a non-reflexive logic for PREM in which modus ponens holds. Finally, the paper observes that the simple strategy to obtain a logic for PREM in which modus ponens holds is not transferable to a non-reflexive logic with transparent truth without significant sacrifices.

This paper should thus not be read as attempting to block Russell’s (2018) path to nihilism which consists in questioning logical laws through the introduction of constants to capture various semantic or logical phenomena. Instead, it should be read as a defence of non-reflexive logics which consists in showing that they do not require us to give up

the idea of there being valid inferences supporting uniform substitution of any formula for propositional variables which are useful for reasoning about our favourite propositions. Failure of the law of identity is not a *shortcut* to logical nihilism.

## 2. Simple logical nihilism

A logic  $L$  based on a language  $\mathcal{L}$  can be explicated as collection of pairs  $\langle \Gamma, A \rangle$  where  $A$  is a formula of  $\mathcal{L}$  and  $\Gamma$  is a set of formulas of  $\mathcal{L}$ . Based on Cook's (2012) analysis of a logical truth, it seems fair to say that a logic  $L$  defined for a language  $\mathcal{L}$  which includes propositional variables is empty if and only if for every  $\langle \Gamma, A \rangle \in L$  there is an inference  $\langle \Gamma(B/p), A(B/p) \rangle$  obtained by uniformly substituting a formula  $B$  for a propositional variable  $p$  in every formula of  $\Gamma$  and in  $A$  such that  $\langle \Gamma(B/p), A(B/p) \rangle \notin L$ . With logical nihilism as the view according to which there are no valid inferential schemas, a logic can be said to represent logical nihilism just in case it is empty. As far as Russell's (2018) endnote goes, however it is not sufficient for a logic to be non-empty. Instead, Russell (2018) seems to require more. For example, the inference in question cannot be valid solely in virtue of constants such as  $\top$  and  $\perp$ . Assume for example that  $\langle \{\perp\}, \top \rangle \in L$ . It certainly satisfies uniform substitution because there are no propositional variables to replace. On the other hand, it is quite useless as an inference precisely because there is no place-holder for the propositions we would like to reason about. Consider in the same vein the inference  $\langle \{\perp\}, p \wedge \neg p \rangle$ . While this inference allows us to replace the propositional variable with our favourite propositions, there is still a sense in which this inference does us no good with respect to reasoning about our favourite propositions. To avoid logical nihilism then, a logic should not only be not empty, but also satisfy inferential schemas that are useful for reasoning in the way intended by Russell (2018, n15), i.e. "useful for doing metatheory [o]r assessing proofs in arithmetic." No matter how this notion of usefulness should be made more precise, I think it is fair to treat modus ponens and modus tollens as paradigmatic examples thereof.

To rescue non-reflexive logics from logical nihilism then, we need a non-reflexive logic in which PREM behaves as stipulated by Russell (2018), but which nonetheless contains an inference containing propositional variables for which uniform substitution holds and which is useful

in the above sense. With modus ponens and modus tollens being the paradigmatic examples, we shall concentrate on them.

### 3. From tonk to prem

In the concluding remarks, [Russell \(2018\)](#) notes that the counterexample to the law of identity makes

one curious about regularities in truth-preservation over sentences whose truth-value can change in the course of an argument — an under-explored topic, and perhaps one that has been under-explored because of the fear of logical nihilism. ([Russell, 2018](#), p. 321)

Luckily for our purposes, the proposal that the truth-value of a sentence could change in the course of an argument is not completely unexplored in the contemporary literature. Without any fear of logical nihilism, [Fjellstad \(2015\)](#) presents a semantics for Prior’s connective TONK which delivers a non-reflexive and non-transitive logic based on precisely that idea through the definition of two bivalent valuation functions  $V_p$  and  $V_c$  with a definition of  $A$ -TONK- $B$  such that  $A$ -TONK- $B$  can be true on  $V_p$  but false on  $V_c$  (and vice versa). Entailment is now defined as truth-preservation from  $V_p$  to  $V_c$ . In particular, the following clauses are used for TONK,  $\neg$  and  $\rightarrow$  by [Fjellstad \(2015\)](#):<sup>1</sup>

- $V_p(A \rightarrow B) = 1$  iff  $V_c(A) = 0$  or  $V_p(B) = 1$
- $V_c(A \rightarrow B) = 1$  iff  $V_p(A) = 0$  or  $V_c(B) = 1$
- $V_p(\neg A) = 1$  iff  $V_c(A) = 0$
- $V_c(\neg A) = 1$  iff  $V_p(A) = 0$
- $V_p(A\text{-TONK-}B) = 1$  iff  $V_p(B) = 1$
- $V_c(A\text{-TONK-}B) = 1$  iff  $V_c(A) = 1$

In addition, it is required that for every propositional variable  $P$ ,  $V_p(P) = V_c(P)$ . Let a pair  $\langle V_p, V_c \rangle$  satisfying these conditions be a *dual valuation for TONK*. Entailment is now defined as follows:

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<sup>1</sup> We thus take the set of formulas to consist of those obtained using those connectives from a set of propositional variables. While we will replace TONK with PREM and later on also add CONC, we shall keep the rest of the language fixed and thus not consider other standard connectives than a negation and a conditional. Our focus on these two connectives is justified through the fact that they are the ones that occur in modus ponens and modus tollens.

$\Gamma \models_1 B$  if and only if for every *dual valuation* for TONK  $\langle V_p, V_c \rangle$ ,  
if for every  $A \in \Gamma, V_p(A) = 1$  then for  $V_c(B) = 1$ .

It is natural to look at this framework in the search of models for PREM with an initial proposal being to simply replace the clauses for TONK with the requirement that every valuation is such that  $V_p(\text{PREM}) = 1$  and  $V_c(\text{PREM}) = 0$ . We shall moreover drop the restriction on propositional variables since it induces failure of uniform substitution; the result of substituting PREM for  $p$  in for example the valid inference  $\langle \{p\}, p \rangle$  is not valid. Let this be a *preliminary dual valuation* for PREM.

As it turns out,  $\neg$  and  $\rightarrow$  in *preliminary dual valuations* for PREM are isomorphic to *preliminary tetravaluations* for PREM defined as functions  $V$  from the formulas to the set  $\{t, p, c, f\}$  such that  $V(\text{PREM}) = p$  and in which  $\neg$  and  $\rightarrow$  satisfy the following tables:

$A \rightarrow B$	t	p	c	f
t	t	p	c	f
p	t	p	t	p
c	t	t	c	c
f	t	t	t	t

$\neg A$	$A$
t	f
p	p
c	c
f	t

These tables are easily recognised as the material conditional and negation for FDE as presented by [Belnap \(1977\)](#) and [Omori and Wansing \(2017\)](#). For entailment, however, we replace the above definiens with the following for tetravalent valuations:

every preliminary tetravalent valuation for PREM  $V$  is such that  
if for every  $A \in \Gamma, V(A) \in \{t, p\}$  then  $V(B) \in \{t, c\}$ .

The proof that they are equivalent is trivial considering the fact that we can treat the dual valuations as tetravaluations along the following lines:

- $V_p(A) = 1$  and  $V_c(A) = 1$  iff  $V(A) = t$
- $V_p(A) = 1$  and  $V_c(A) = 0$  iff  $V(A) = p$
- $V_p(A) = 0$  and  $V_c(A) = 1$  iff  $V(A) = c$
- $V_p(A) = 0$  and  $V_c(A) = 0$  iff  $V(A) = f$

The reader will thus be spared the tedious details.

While this framework is certainly a good starting point since it permits a change in context from premise to conclusion, it will deliver us logical nihilism in the sense discussed by [Russell \(2018\)](#). This is as expected since the trivalent valuations obtained by dropping  $c$ , the result of which are valuations known as strong Kleene, is the typical tool for

obtaining theories of transparent truth and naive validity based on a non-reflexive logic, see for example (Fjellstad, 2017), but also (French, 2016), (Nicolai and Rossi, 2018) and (Murzi and Rossi, 2017). After all, one virtue with that approach is that one can use standard rules from a bilateral sequent calculus for classical logic to define  $\rightarrow$  and  $\neg$ . This is also more or less the framework considered for PREM by Dicher (2020), thus following the lead of Russell (2018) who employs FDE for other purposes in the section where PREM is introduced. Let’s do better.

#### 4. A negational twist

In the preliminary dual valuations for PREM the truth or falsity of  $\neg$ -PREM when featuring as conclusion depends on the truth or falsity of PREM as premise. However, why shouldn’t it rather be the case that the truth or falsity of  $\neg$ -PREM when featuring as conclusion depends on the truth or falsity of PREM as conclusion? After all, we can certainly also make sense of the proposal that  $\neg$ -PREM is always true as conclusion since PREM is always false as conclusion.<sup>2</sup>

Since the discussion by Russell (2018) does not provide any direct guidance on the issue of negation with regard to PREM, we might as well adopt the following clauses for  $\neg$  with regard to dual valuations:

- $V_p(\neg A) = 1$  iff  $V_p(A) = 0$
- $V_c(\neg A) = 1$  iff  $V_c(A) = 0$

These clauses articulate the idea that formulas of the form  $\neg A$  are evaluated with regard to the *same* context, i.e. that the value of  $\neg A$  as premise depends on the value of  $A$  as premise as opposed to value of  $A$  as conclusion. The corresponding table for  $\neg$  in tetravaluations is the following:<sup>3</sup>

$\neg A$	$A$
t	f
p	c
c	p
f	t

Let us call the resulting collections of valuations in which  $\neg$  is defined as proposed in this section and  $\rightarrow$  is defined as in the previous section

<sup>2</sup> As suggested to me by Bogdan Dicher.

<sup>3</sup> This table actually corresponds to that recently used by Button (2016) and originally proposed by Church (1944).

*negational dual valuations for PREM and negational tetravaluations for PREM respectively.*

With the help of the new valuations we can now define  $\Gamma \models_2 B$  as either

- for every negational tetravaluation for PREM  $V$ , if for any  $A \in \Gamma$ ,  $V(A) \in \{t, p\}$  then  $V(B) \in \{t, c\}$

or

- for every negational dual valuation for PREM  $\langle V_p, V_c \rangle$ , if for any  $A \in \Gamma$ ,  $V_p(A) = 1$  then  $V_c(B) = 1$ .

as definiens.

To make reasoning about what is valid in this logic easier it is useful to have a sound and complete sequent calculus. In order to obtain rules that allow for the kind of straightforward compositional construction of complex formulas that we are used to in the case of a standard bilateral sequent calculus for classical logic such as that presented by [Negri and von Plato \(2001\)](#), we shall here rely on quadrilateral as opposed to bilateral sequents, i.e. we shall use sequents with *four* as opposed to *two* positions for formulas. A bilateral sequent written for example as  $\Gamma \Rightarrow \Delta$  typically represents a pair  $\langle \Gamma, \Delta \rangle$  of (possibly empty) multisets of formulas, where the first element of the pair is represented by the first position (i.e. the position left of  $\Rightarrow$ ), and the second element of the pair is represented by the second position (i.e. the position right of  $\Rightarrow$ ). Our quadrilateral sequents of the form

$$\Gamma \mid \Gamma' \Rightarrow \Delta \mid \Delta'$$

on the other hand, shall be taken to represent a quadruple of (possibly empty) multisets of formulas  $\langle \Gamma, \Gamma', \Delta, \Delta' \rangle$ . The result is slightly more complex sequents (since four is greater than two), but in the trade-off we obtain very straightforward compositional rules that facilitate root-first proof search.

The intended interpretation for quadrilateral sequents of the form  $\Gamma \mid \Gamma' \Rightarrow \Delta \mid \Delta'$  is that such a sequent is valid just in case there is no negational tetravaluation such that

for every formula  $A \in \Gamma$ ,  $V(A) \in \{c, f\}$ , for every formula  $A \in \Gamma'$ ,  $V(A) \in \{t, p\}$ , for every formula  $A \in \Delta$ ,  $V(A) \in \{p, f\}$  and for every formula  $A \in \Delta'$ ,  $V(A) \in \{t, c\}$ .



Equivalently, such a sequent is valid just in case there is no negational dual valuation such that

for every formula  $A \in \Gamma$ ,  $V_p(A) = 0$ , for every formula  $A \in \Gamma'$ ,  $V_p(A) = 1$ , for every formula  $A \in \Delta$ ,  $V_c(A) = 0$  and for every formula  $A \in \Delta'$ ,  $V_c(A) = 1$ .

It follows from this interpretation of a quadrilateral sequent that, if the sequent calculus is sound and complete, then

$\Gamma \vDash_2 B$  if and only if the sequent  $|\Gamma \Rightarrow B|$  is derivable.

That is, the sequent calculus will be designed such that  $\Gamma$  implies  $A$  just in case we can derive the sequent such that the left-most position is empty and  $\Gamma$  is the position left of  $\Rightarrow$  and the multiset containing one copy of  $B$  is the position right of  $\Rightarrow$  and the right-most position is empty. From the perspective of the desired definition of validity, the left-most and the right-most positions play a supporting role. As a consequence of their addition and the above definition of the validity of a sequent, it follows that there will in our case be valid sequents that do not represent valid inferences of  $\vDash_2$ . Examples of such sequents include the following:

PREM | PREM  $\Rightarrow$  |      PREM |  $\Rightarrow$  | PREM

Of course, a quick inspection of the sequents and the below rules will reveal that sequents representing valid inferences can be derived from these. Nevertheless, all this might seem odd for the reader who is used to thinking that every object constructed with a proof in a deductive system for a particular logic must represent a valid inference of that logic. However, such a reader should keep in mind that this sequent calculus is merely intended a mathematical tool to simplify reasoning about what is valid in a particular logic. For some brief remarks on this author’s philosophical perspective on sequent calculi; see (Fjellstad, 2020).

As usual for sequent calculi, the calculus will consist of initial(ly derivable) sequents and rules with zero or more sequents as premises and one sequent as conclusion. Both initial sequents and the rules are presented in the usual schematic form. Derivations are trees where the leafs are (instances of the presented) initial sequents and zero premise rules and each non-leaf node is obtained from one or more nodes with a(n instance of a presented) rule such that the root of each subtree is itself a derivable sequent. The root of the tree is thus the sequent for which

the tree is a derivation. For a general introduction to sequent calculus as a proof-theoretic framework; see (Negri and von Plato, 2001).

Our calculus consists of initial sequents for propositional variables and zero-premise rules for PREM of the following form:

$$\begin{array}{ll} \Gamma, P \mid P, \Gamma' \Rightarrow \Delta \mid \Delta' & \Gamma \mid \Gamma' \Rightarrow \Delta, P \mid P, \Delta' \\ \text{PREM}, \Gamma \mid \Gamma' \Rightarrow \Delta \mid \Delta' & \Gamma \mid \Gamma' \Rightarrow \Delta \mid \Delta', \text{PREM} \end{array}$$

And the following rules for  $\neg$  and  $\rightarrow$ :

$$\begin{array}{c} \frac{\Gamma \mid \Gamma' \Rightarrow \Delta, A \mid \Delta' \quad \Gamma \mid B, \Gamma' \Rightarrow \Delta \mid \Delta'}{\Gamma \mid A \rightarrow B, \Gamma' \Rightarrow \Delta \mid \Delta'} \\ \frac{\Gamma \mid A, \Gamma' \Rightarrow \Delta, B \mid \Delta'}{\Gamma \mid \Gamma' \Rightarrow \Delta, A \rightarrow B \mid \Delta'} \\ \frac{A, \Gamma \mid \Gamma' \Rightarrow \Delta \mid \Delta' \quad \Gamma \mid \Gamma' \Rightarrow \Delta \mid \Delta', B}{\Gamma \mid \Gamma' \Rightarrow \Delta \mid \Delta', A \rightarrow B} \\ \frac{B, \Gamma \mid \Gamma' \Rightarrow \Delta \mid \Delta', A}{A \rightarrow B, \Gamma \mid \Gamma' \Rightarrow \Delta \mid \Delta'} \\ \frac{A, \Gamma \mid \Gamma' \Rightarrow \Delta \mid \Delta'}{\Gamma \mid \Gamma', \neg A \Rightarrow \Delta \mid \Delta'} \quad \frac{\Gamma \mid \Gamma', A \Rightarrow \Delta \mid \Delta'}{\neg A, \Gamma \mid \Gamma' \Rightarrow \Delta \mid \Delta'} \\ \frac{\Gamma \mid \Gamma' \Rightarrow \Delta, A \mid \Delta'}{\Gamma \mid \Gamma' \Rightarrow \Delta \mid \Delta', \neg A} \quad \frac{\Gamma \mid \Gamma' \Rightarrow \Delta \mid \Delta', A}{\Gamma \mid \Gamma' \Rightarrow \neg A, \Delta \mid \Delta'} \end{array}$$

The simplicity of the rules with regard to compositionality is due to the use of quadrilateral as opposed to bilateral sequents. The reader sceptical of this claim is free to attempt to develop an equally elegant sound and complete bilateral sequent calculus for the same logic without treating the negation of complex formulas as a defined rather than a primitive symbol.

That the calculus is sound and complete should be obvious considering the proofs in for example (Fjellstad, 2015, 2017). From soundness and completeness we can conclude that  $\Gamma \vDash_2 A$  if and only if  $\mid \Gamma \Rightarrow A \mid$  is derivable.

Modus tollens is now derived as follows for two propositional variables  $P_1$  and  $P_2$ :

$$\frac{\frac{P_1 \mid P_1 \Rightarrow \mid}{\mid P_1, \neg P_1 \Rightarrow \mid} \quad \frac{\mid \Rightarrow P_2 \mid P_2}{\mid \Rightarrow \neg P_2, P_2 \mid}}{\mid \neg P_1, P_2 \rightarrow P_1 \Rightarrow \neg P_2 \mid}$$

Moreover, it follows by induction on the complexity of a formula that sequents of the following forms are derivable for every formula  $A$ :

$$\Gamma, A \mid A, \Gamma' \Rightarrow \Delta \mid \Delta' \qquad \Gamma \mid \Gamma' \Rightarrow \Delta, A \mid A, \Delta'$$

It follows that the sequent obtained by uniformly replacing every occurrence of either  $P_1$  or  $P_2$  with any formula  $A$  in the sequent  $\mid \neg P_1, P_2 \rightarrow P_1 \Rightarrow \neg P_2 \mid$  is derivable. Modus tollens holds thus for every formula.

It would therefore be rather intolerant to treat this logic as representing logical nihilism despite it being non-reflexive since  $\text{PREM} \not\models_2 \text{PREM}$ . Of course, we have only delivered modus tollens, and is it straightforward to find a countermodel to modus ponens: let  $V_p(P_1) = 1, V_c(P_1) = 0, V_p(P_2) = 0$  and  $V_c(P_2) = 0$ . It follows that  $V_p(P_1 \rightarrow P_2) = 1$ . Similarly with for example double negation elimination.

A perhaps more serious issue is that the clauses for negation ensure that  $V_p(\neg\text{PREM})=0$  and  $V_c(\neg\text{PREM})=1$  which in turn means that  $\models_2$  will be non-transitive; it holds that  $A \models_2 \neg\text{PREM}$  and  $\neg\text{PREM} \models_2 B$  for arbitrary formulas  $A$  and  $B$ . Now, while [Russell \(2018\)](#) didn’t provide any guidelines about negation, it would seem unreasonable that a sentence which was only supposed to induce failure of reflexivity also, because of the negational twist, induces a failure of transitivity. Instead, that role seems to fall upon the overlooked cousin of PREM, namely the constant CONC which is supposed to always be true as conclusion but false as premise. With the negational twist,  $\neg\text{PREM}$  behaves as CONC. While there is some conceptual prudence in that equivocation, it seems unfair not only towards PREM, but especially towards CONC.

### 5. Reconsidering the conditional

With CONC in the picture as a constant which is supposed to satisfy the requirement that  $V_p(\text{CONC}) = 0$  and  $V_c(\text{CONC}) = 1$  in the case of dual valuations and the requirement that  $V(\text{CONC}) = c$  in the case of tetravaluations, and under the assumption that the negation of one shouldn’t be equivalent to the other, it seems better to leave negation as it was and rather reconsider the conditional as a path towards valid inferences supporting uniform substitution of any formula for a propositional variable.

The literature on many-valued logics offer plenty of tetravalued conditionals. In addition to the FDE conditional, we have for example

Lukasiewicz's (1970) conditional, Avron's (1991) conditional and Sutcliffe, Pelletier and Hazen's (2018) conditional. However, rather than going through each of these to check their suitability, we can also simply ask what we want from the conditional in the first place. For example, if we want modus ponens to hold, then it must be the case that

$$(MPD) \quad \text{if } V_p(A \rightarrow B) = 1, \text{ then } V_p(A) = 0 \text{ or } V_c(B) = 1.$$

Since all we require is a single valid inference supporting uniform substitution, and philosophers in general are quite fond of modus ponens, we shall stick with this as requirement.

With regard to tetravaluations, the corresponding requirement is that

$$(MPT) \quad \text{if } V(A \rightarrow B) \in \{t, p\} \text{ and } V(A) \in \{t, p\}, \text{ then } V(B) \in \{c, t\}.$$

With this being equivalent to that if  $V(A) \in \{t, p\}$  and  $V(B) \in \{p, f\}$  then  $V(A \rightarrow B) \in \{c, f\}$ , it follows that any table of the form

$A \rightarrow B$	t	p	c	f
t		c/f		c/f
p		c/f		c/f
c				
f				

will do the trick. Incidentally, it turns out that none of the conditionals proposed by Lukasiewicz (1970), Avron (1991) and Sutcliffe, Pelletier and Hazen (2018) satisfy this requirement. This shouldn't surprise us since they were not designed for the purpose of obtaining modus ponens in a logic for which the law of identity fails. On the other hand, we have plenty of alternatives just waiting to be generated from the above template. All we need to do is to fill out the blanks in such a way that it delivers other desirable properties of a conditional. There are certain limitations to which properties one can obtain, of course, and the reader is referred to (Égré et al., 2000) for some negative results.

From the perspective of the quadrilateral sequents presented in the previous section, it suffices to note that the above requirement is also equivalent to the following:

$$(*) \quad \text{if } V(A \rightarrow B) \in \{t, p\}, \text{ then } V(B) \in \{c, t\} \text{ or } V(A) \in \{c, f\}.$$

This in turn, means that any conditional satisfying our requirement will be such that the following rule is admissible in a quadrilateral sequent

calculus which is sound and complete under the interpretation suggested in the previous section:

$$\frac{A, \Gamma \mid \Gamma' \Rightarrow \Delta \mid \Delta' \quad \Gamma \mid \Gamma' \Rightarrow \Delta \mid \Delta', B}{\Gamma \mid A \rightarrow B, \Gamma' \Rightarrow \Delta \mid \Delta'}$$

With the admissibility of the sequents

$$A, \Gamma \mid A, \Gamma' \Rightarrow \Delta \mid \Delta' \quad \Gamma \mid \Gamma' \Rightarrow \Delta, B \mid \Delta', B$$

it is straightforward to derive modus ponens. On the other hand, with  $V(\text{PREM}) = \text{p}$ ,  $V(\text{CONC}) = \text{c}$  and entailment defined as in the previous sections, the logic is still non-reflexive because of PREM and non-transitive because of CONC.

A perhaps important caveat with any conditional satisfying this requirement is that the value  $p$  can no longer be used as a safe harbour for the Curry sentence when truth is transparent in the sense that  $V(\text{Tr}(t)) = V(A)$  whenever the closed term  $t$  functions as a name for the formula  $A$  and  $\text{Tr}$  is the truth predicate. Assume that  $\kappa$  is a term functioning as a name for the formula  $\text{Tr}(\kappa) \rightarrow \perp$  where  $V(\perp) = \text{f}$ . The condition on the conditional implies now that if  $V(\text{Tr}(\kappa) \rightarrow \perp) = \text{p}$  then  $V(\text{Tr}(\kappa)) \in \{\text{c}, \text{f}\}$ .

Luckily, however, the aim of this paper is not to rescue non-reflexive logics for transparent truth from logical nihilism. Instead, the aim was to clarify whether Russell’s (2018) PREM and non-reflexive logics in general invite logical nihilism or not. To that end this paper has illustrated how a logic for PREM need not deliver logical nihilism in the sense of (Russell, 2018) since a logic for PREM can certainly contain inferences with which we can reason about our favourite propositions. It’s just a matter of being a bit creative.

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