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Chisholm’s Modal Paradox(es) and Counterpart Theory

Abstract. Lewis’s [1968] counterpart theory (LCT for short), motivated by his modal realism, made its appearance within a year of Chisholm’s modal paradox [1967]. We are not modal realists, but we argue that a satisfactory resolution to the paradox calls for a counterpart-theoretic (CT-)semantics. We make our case by showing that the Chandler–Salmon strategy of denying the S4 axiom $[\diamond\diamond\psi \rightarrow \diamond\psi]$ is inadequate to resolve the paradox — we take on Salmon’s attempts to defend that strategy against objections from Lewis and Williamson. We then consider three substantially different CT-approaches: Lewis’s LCT, Forbes’s (FCT), including his fuzzy version, and Ramachandran’s (RCT). We argue that the best approach is a mish-mash of FCT and RCT.

Keywords: Chisholm’s paradox; counterpart theory; Lewis; Salmon; Forbes; Williamson

1. Aims

Counterpart theory made its appearance in [Lewis, 1968], within a year of Chisholm’s [1967] modal paradox. Fifty years on, this paper takes a fresh look at both in the course of making a case for a counterpart-theoretic (CT-)solution to the paradox that addresses points raised in more recent discussions.

Lewis’s theory is motivated by his modal realism; he takes the truth of a *de re* modal statement, S, to be explained by non-modal facts in other possible world involving counterparts of the objects mentioned in S. We are not modal realists, and, so, do not buy the supposed explanatory virtues of counterpart theory. However, we side with, e.g., Stalnaker [1986] and, more recently, Sider [2001], in thinking that Chisholm’s para-

dox provides motivation for a CT-resolution even so¹ — though, from our perspective, this is in order to *represent* our modal views rather than *explain* their truth.

We make our case by developing two lines of objection to the main non-counterpart-theoretic solution on the market — namely, the Chandler–Salmon strategy of denying the S4-axiom: $\diamond\diamond\psi \rightarrow \diamond\psi$ [see Chandler, 1976; Salmon, 1981] — in response to Salmon’s [1986; 1989; 1993] attempted rebuttals. Two closely-related paradoxes are introduced along the way. Our considerations suggest that a unified solution to these will have to invoke counterpart theory. We examine three kinds of CT-strategies with a view to settling on the approach which does least damage to our modal intuitions.

2. Chisholm’s paradox and the Chandler–Salmon solution

2.1. The paradox and S4

Chisholm’s [1967] modal paradox, as presented here,² arises from a compelling principle concerning composite artefacts:

The Moderate Toleration Principle (MTP): Necessarily, any (large) composite object (such as e.g. a ship) might have originally been composed of a slightly different set of parts (i.e. the same-but-for-one-or-two parts) but could not have been composed of a very different set of parts.

Suppose a ship α is composed of a set of n planks, P_0 , in the actual world w_0 . Let ‘ P_kx ’ mean ‘ x is a ship that is qualitatively identical in design and composition to α as it is in w_0 , and comprises all but k of the planks α is made of in w_0 .’

Since α is P_0 in w_0 , by MTP α is P_1 in some other world w_1 ; but, then, by MTP again, that ship, the one in w_1 , supposedly α , is P_2 in yet another world w_2 ; and by MTP again, that ship, the one in w_2 , supposedly α , is P_3 in a world w_3 . And so on. Eventually, we reach the

¹ This is not their sole motivation, but the others will not concern us here. Kripke [1980, p. 51, fn. 18] also floats the possibility of a CT-approach to handle questions raised by Chisholm’s paradox.

² Salmon [1981] and Williamson [1990] present more refined versions which avoid certain complications; but our crude version will do for now.

conclusion that α is P_n in some world w_n , i.e. that α is composed of an entirely different collection of planks in world w_n , which would appear to directly contradict the latter part of MTP.

Thus, the paradox is standardly taken to be this: MTP requires that the following set of sentences be consistent, whereas our reasoning above, given succour by Kripkean S5-semantics for quantified modal logic (QML) (KS5 for short), renders it inconsistent [see, e.g., Forbes, 1983, 1984; Salmon, 1981, 1989]:

$$\Gamma = \{P_0\alpha, \neg\Diamond P_n\alpha, \Box(P_k\alpha \rightarrow \Diamond P_{k+1}\alpha) \mid 0 < k \leq n\}.$$

We can think of the $[\Box(P_k\alpha \rightarrow \Diamond P_{k+1}\alpha)]$ as *toleration premises*. However, we prefer to represent the paradox with the more general toleration premises $[\Box(x)(P_kx \rightarrow \Diamond P_{k+1}x)]$ which MTP explicitly avows. Thus, our focus will be on the (in)consistency of

$$\Gamma_1 = \{P_0\alpha, \neg\Diamond P_n\alpha, \Box(x)(P_kx \rightarrow \Diamond P_{k+1}x) \mid 0 \leq k < n\}.$$

MTP apparently demands that it is consistent, whereas a natural line of reasoning and KS5 decree otherwise.

Of course, one might take the KS5-inconsistency as grounds for maintaining that the very composition of a (composite) object, such as a ship, when it comes into being is *essential* to it: so that e.g. α 's original composition could not have been even slightly different.³ But we find the moderate toleration principle more compelling than such absolute essentialism; so, we seek a solution to the paradox which accommodates MTP.

Chandler [1976] and Salmon [1981, 1986] render Γ_1 consistent by proposing a non-transitive accessibility relation between worlds, which, on Kripke-semantics, is tantamount to rejecting the S4-axiom $[\Diamond\Diamond\psi \rightarrow \Diamond\psi]$. So, e.g., what is possible relative to world w_1 need not be possible relative to the actual world w_0 ; thus, although α 's being P_2 at world w_2 entails the truth of $[\Diamond\Diamond P_2\alpha]$ at w_0 , it does not entail that $[\Diamond P_2\alpha]$ is true at w_0 . We'll call this the Chandler–Salmon (CS-)solution.⁴

That rejecting S4 renders Γ_1 consistent is indisputable; but we think two lines of objection demonstrate that this strategy is inadequate.

³ This would be a variety of *mereological essentialism*, but not the variety Chisholm [1973] argues for, which applies to more fundamental composite objects; he does not regard things like *tables* and *ships* as *primary* objects.

⁴ Salmon [1986, p. 82] also allows vagueness in the accessibility relation, so that e.g. it can be indeterminate whether one world is accessible (possible) relative to another. But the points we will be making here are unaffected by this.

Strengthened Chisholm's paradox. The first objection centres on the fact that the CS-solution still allows that $[\diamond^n P_n \alpha]$ holds, where ' \diamond^n ' stands for n iterations of ' \diamond '. Indeed, Salmon [1993, pp. 156–157] takes it to be an interesting consequence of his strategy that $[\diamond^n P_n \alpha]$ follows, even though $[\diamond P_n \alpha]$ does not, from $[P_0 \alpha]$ if one allows, as he does, 'infinite necessitations' of the toleration principles $\Box(x)(P_k x \rightarrow \diamond P_{k+1} \alpha)$ — that is, $[\Box^m(x)(P_k x \rightarrow \diamond P_{k+1} \alpha)]$ for every m . But, allowing $[\diamond^n P_n \alpha]$ is surely to concede that α is P_n in a world, e.g. w_n , that is *possible in some sense*, albeit, a world which, according to the S4-strategy, is *inaccessible from (impossible relative to)* the actual world. Lewis puts the point as follows:

[...] by what right do we ignore worlds that are deemed inaccessible? Accessible or not, they're still worlds. We still believe in them. Why don't they count? [Lewis, 1986, p. 246]

Salmon [1989] addresses this objection, but we argue he fails to refute it. As we too deny modal realism, we have no quarrel with Salmon's conception of possible worlds as:

[...] certain sorts of (in some sense) maximal abstract entities according to which certain things (facts, states of affairs) obtain and certain other such things which do not obtain. [Salmon, 1989, p. 5]

This conception allows for (metaphysically, logically, nomologically) *impossible* worlds; so, we agree with his point against Lewis that the mere fact there is a world, w , *according to which* $[\alpha \text{ is } P_n]$ holds does not thereby render it metaphysically possible.

However, Salmon glosses over a distinction one might wish to make, between, as we might put it, *relative* impossibility and impossibility *tout court*, which we can explicate in terms of a non-relative notion of *realizability*. There are worlds, in Salmon's sense, according to which ' $2 + 2 = 6$ ' holds, but these, one may hold, are not *realizable* — not *actualizable* even from a God's-eye perspective — given the nature of the arithmetical terms therein. Likewise, there are worlds according to which humans are also elephants; but these, one may hold, are not realizable either, given the natures (essences?) of humans and elephants. Such worlds, i.e. unrealizable ones, are impossible *tout court*, not merely contingently impossible.

Now, it strikes us that some advocates of MTP advocate it on *essentialist* grounds; the thought that α *could not have been* P_n , for them,

reflects the view that $[\alpha \text{ is } P_n]$ is not just relatively impossible, but impossible tout court, in the same way that [humans are elephants] is. Such essentialists will deny $[\Diamond^n P_n \alpha]$ as well as $[\Diamond P_n \alpha]$. So, these MTP-supporters at any rate will not be appeased by the CS-strategy, since this, as Salmon owns, dictates that $[\Diamond^n P_n \alpha]$ holds given infinite necessitations of the toleration principles.

So, we contend that the CS-strategy, by Salmon's own lights, is inadequate insofar as it does not speak to motivations for MTP that also motivate $[\neg \Diamond^n P_n \alpha]$ — the strategy falls short in failing to render the following set of QML-sentences consistent:

$$\Gamma_2 = \{P_0 \alpha, \neg \Diamond^n P_n \alpha, \Box^n (x)(P_k x \rightarrow \Diamond P_{k+1} x) \mid 0 \leq k < n\}.$$

If it is not yet obvious that counterpart theory is the way to go, our second objection to the CS-strategy, based on another variation on Chisholm's paradox, should make this clear.

2.2. Williamson's paradox

This objection emerges from [Williamson \[1990\]](#), who rejects the CS-solution on the grounds that there are similar paradoxes:

[...] in which the series of worlds [in our initial example: worlds w_0 , w_1 , ..., and w_n , where $[P_i \alpha]$ holds at world w_i] is viewed not from one end [e.g. from the perspective of w_0] but from the standpoint of a world outside the series, from which all its member are equally possible.

[[Williamson, 1990](#), p. 126], our insertions

Here is a simplistic variation in keeping with our ship example to illustrate his point. Suppose a person, X, in charge of building a ship with a unique design, decides, on the advice of his deranged astrologer, on the following procedure to select the collection planks that will be used. Each plank in the warehouse is labelled; X then gets a computer to make a list of $n+1$ collections of planks, $Q_0 - Q_n$, which could be used to build the ship, where collection Q_k has k planks that are not in Q_0 ; X then gets a programme running on the computer which randomly selects a number between 0 and n , inclusive. All X has to do is press the return key to display the current selection. Unfortunately, just as X is about to press the key, there is a power cut and the computer switches off. So, no ship of this unique design is actually made.

Let us name each merely possible ship that would have been constructed had there been no power cut, and number k had been selected by the computer, β_k . Now, by the moderate toleration principle MTP, ships β_0 and β_n , being originally constructed from entirely different planks, are distinct ships: $\beta_0 \neq \beta_n$. But, seeing as any ship β_{k+1} differs from ship β_k only in being originally constructed with a collection of planks one plank different from the collection that β_k was built from, MTP would appear to license the conclusion $[\beta_k = \beta_{k+1}]$. So, we get all the following identity statements coming out true: $[\beta_0 = \beta_1]$; $[\beta_1 = \beta_2]$; \dots ; and $[\beta_{n-1} = \beta_n]$. But, by transitivity, we get $[\beta_0 = \beta_n]$, contradicting our hypothesis about those ships. So, we have a very similar paradox to Chisholm's, but neither S4 nor any non-modal analogue of it seems pertinent here.

One significant dis-analogy, noted by Salmon [1993, p. 159] is that Williamson's version involves *cross-world* identities. Since none of these ships exist in the actual world, one may well question whether *any* of the above identity statements are true in the actual world. However, Williamson's objection can be recast in terms of *possible* identities: $[\Diamond\beta_0 = \beta_1]$; $[\Diamond\beta_1 = \beta_2]$; \dots ; and $[\Diamond\beta_{n-1} = \beta_n]$. On Kripkean QML-semantics these possibilities are not compatible with the hypothesis $[\neg\Diamond\beta_0 = \beta_n]$. For, the Kripkean treatment of identity ensures the validity of the following weak-necessity of identity principle, even if S4 is denied:⁵

$$\Diamond a = b \rightarrow \Box(\exists x(x = a \vee x = b) \rightarrow a = b), \quad (\text{WN})$$

i.e., if it is possible that $a = b$ then necessarily, if a or b exists, $a = b$.

We reject Williamson's [1990] own resolution of this paradox because it effectively amounts to a rejection of moderate toleration: no member of the sequence of identities of the form $[\beta_m = \beta_{m+1}]$ (or $[\Diamond\beta_m = \beta_{m+1}]$) is determinately true or determinately false on his view [see, e.g., Williamson, 1990, p. 133]. Salmon would agree; he says, "[Williamson's] solution to Chisholm's paradox thus involves embracing a fairly intolerant form of mereological essentialism" [Salmon, 1993, p. 158].

Salmon [1993, p. 161 ff.] tackles this paradox by way of the following strategy (which we adapt for our example). Suppose that one of these

⁵ On Kripkean QML-semantics, treating constants as unbound variables, $[a = b]$ is true at some world on a model, M , iff $\text{Ref}(a) = \text{Ref}(b)$ in M ; but, then, $[(\exists x(x = a) \rightarrow a = b)]$ will come out true at every world on M .

merely possible ships is such that the worlds where that ship is built (on the basis of this selection process) are closer to actuality than any world where one of the other ships is built. For the sake of argument (given the limitations of our example), let us suppose it is ship β_0 . What our toleration principle MTP gives succour to is the truth of the first few members in Williamson's series, e.g., $[\beta_0 = \beta_1]$ and $[\beta_1 = \beta_2]$, because these ships are 'close enough to actuality' to render the identities possible, that is, *possible relative to actuality*. Some member(s) of the series, however—Salmon reckons—will be false, or neither determinately true nor determinately false. So, Salmon contends, the conjunction of the *determinately true* members of Williamson's series do *not* entail the conclusion $[\beta_0 = \beta_n]$ as Williamson's reductio requires.

An obvious problem with Salmon's strategy is that it ignores Williamson's hypothesis that all the possibilities are *equally possible*: the worlds where the various ships are built are *equally close* to the actual world. If so, it would seem reasonable to allow that if one member of the series is true, then all are. The trouble is, Salmon has given no convincing reason for questioning Williamson's 'equally possible' hypothesis.

But the more telling problem is this. Even if one grants that Salmon has succeeded in casting doubt on the claim that all the members of the series are (determinately) true, his strategy still dictates that $[\diamond^k \beta_{k-1} = \beta_k]$ is determinately true for each k , $0 < k < n$. But, in that case, $[\diamond^n \beta_{k-1} = \beta_k]$ also comes out true for all such k . Our initial objection now resurfaces: even if S4 is denied, the Kripkean treatment of identity ensures the validity of a stronger weak-necessity of identity principle:

$$\diamond^n a = b \rightarrow \Box(\exists x(x = a \vee x = b) \rightarrow a = b) \quad (\text{WN}^n)$$

i.e., if it is possibleⁿ that $a = b$ then necessarily, if a or b exists, $a = b$. So, Salmon's strategy against Williamson still commits him to the consistency of the following set of QML-sentences, *contra* Kripkean QML-semantics:

$$\Gamma_3 = \{ \neg \diamond \beta_0 = \beta_n, \diamond^n \beta_k = \beta_{k+1} \mid 0 \leq k < n \}.$$

Salmon [1986] has argued that Chisholm's paradox is *not* a paradox about *identity*—his main grounds being that the premises generating the paradox do not explicitly invoke identity. He makes the same point in his response to Williamson [Salmon, 1993, p. 159]. But, if our rejoinders to Salmon are correct, it is precisely the treatment of identity in Kripke-QML-semantics—and *not* S4, as Salmon maintains—which is behind

Williamson's paradox. In light of this, we should be open at least to an over-arching solution to our paradoxes that plays on the interpretation of identity.

Moreover, we should be prepared for a solution that countenances contingent identity, in some sense at least. For, presumably the members of Γ_3 can only be jointly true if there are worlds w and w' and possible ships β_j , and β_k such that (i) and (ii) below hold:

- (i) at w : $\beta_j = \beta_{j+1}$,
- (ii) at w' : $(\beta_{j+1} = \beta_k) \wedge (\beta_j \neq \beta_k)$

But (ii) entails:

- (iii) at w' : $\beta_j \neq \beta_{j+1}$

So,

- (iv) β_j and β_{j+1} are only contingently identical at world w .

Cue *counterpart theory* — the obvious way of accommodating contingent identity. Our goal in the remainder of the paper is to identify a CT-framework that allows for a solution to our paradoxes at minimum cost to our modal intuitions. Along the way, we'll revisit Lewis's [1968] original theory (LCT), Forbes's [1982] so-called canonical counterpart theory (FCT), including FCT with Forbes's [1983; 1984] fuzzy semantics — his preferred means of tackling Chisholm's paradox — and, Ramachandran's [1989; 2008] 'narrow-scope' counterpart theory (RCT).

3. LCT and solution 1 (denies counterpart transitivity and S4)

Lewis [1968] provides a procedure for translating any QML-sentence, ψ , into a sentence of LCT, which is basically QML with special predicates ' Cxy ' (x is a counterpart of y), ' Wx ' (x is a world), and ' Ixy ' (x is in y), and special postulates guaranteeing:

- (a) that every object (that is not a world) exists in exactly one world;
- (b) that every object is its own counterpart.

A QML-argument is *valid* if and only if its LCT-translation is valid in standard predicate logic (PL). Here is the translation scheme:

LCT TRANSLATION SCHEME

- LT1 $\text{Tr}(\psi) = \psi^{w^*}$ (ψ holds in the actual world, w^*) followed by a recursive definition of φ^u (φ holds at world u);
- LT2a φ^u is φ , if φ is atomic;
- LT2b $(\neg\varphi)^u$ is $\neg\varphi^u$;
- LT2c $(\varphi \vee \gamma)^u$ is $\varphi^u \vee \gamma^u$;
- LT2d $(\forall t\varphi)^u$ is $\forall t(\text{It}u \rightarrow \varphi^u)$;
- LT2e $(\Box\varphi t_1 \dots t_n)^u$ is:
 $\forall v\forall t'_1 \dots \forall t'_n ((Wv \wedge \text{It}'_1 v \wedge \text{Ct}'_1 t_1 \wedge \dots \wedge \text{It}'_n v \wedge \text{Ct}'_n t_n) \rightarrow \varphi^{t'_1 \dots t'_n})$.

This semantics is intended for closed QML-sentences, but we will take the liberty of interpreting constants as the LCT-rules would treat free variables; and, henceforth, to make translations a bit simpler, we will use the variables ' u ', ' v ' and ' w ', sometimes with numerical subscripts, to range over worlds, and ' x ', ' y ' and ' z ', sometimes with numerical subscripts, to range over possible individuals.

In a nutshell, Lewis's approach takes a QML-sentence $[\varphi(a_1, \dots, a_n)]$ to be *possibly* true if and only if $[\varphi(b_1, \dots, b_n)]$ is true at some world w , where both (C) and (E) below hold:

- (C) $\text{Ref}(b_k)$ is a *counterpart* of $\text{Ref}(a_k)$ for each k .
 (E) $\text{Ref}(b_k)$ exists in w for each k .

Thus, the LCT-interpretation of (1) below is given by (L₁):

$$\diamond \neg \text{F}a \tag{1}$$

$$\exists w \exists x (\text{I}xw \wedge \text{C}xa \wedge \neg \text{F}x) \tag{L_1}$$

LCT affords a consistent interpretation of MTP. Consider the way the paradox was initially set up:

- [...] by MTP α is P_1 in some other world w_1 ; but, then, by MTP again, *that ship*, the one in w_1 , which is α by hypothesis, is P_2 in yet another world w_2 [...].

The CT-interpretation is that the first token of the demonstrative '*that ship*' in the above passage picks out a *counterpart*, x_1 , of α in w_1 ; and it is *that ship*, x_1 , which has a counterpart, x_2 , in w_2 which is P_2 ; and it is *that ship*, x_2 , which has a counterpart, x_3 , in w_3 which is P_3 ; and so on. Crucially, x_2 need not be a counterpart of α if the counterpart relation is not transitive: thus, what is (im)possible for one ship need

not be (im)possible for a counterpart of that ship. So, although this line of reasoning commits us to a world w_n in which a counterpart of a counterpart... of a counterpart of α is P_n , we are not thereby committed to $[\diamond P_n \alpha]$, i.e. to α having a counterpart that is P_n .

Thus, $[\diamond \diamond Ft \rightarrow \diamond Ft]$, an instance of the S4-axiom $[\diamond \diamond \psi \rightarrow \diamond \psi]$, is LCT-invalid, and

$$\Gamma_1 = \{P_0 \alpha, \neg \diamond P_n \alpha, \Box(x)(P_k x \rightarrow \diamond P_{k+1} x) \mid 0 \leq k < n\}$$

comes out LCT-consistent. So, we have a putative solution to Chisholm's original paradox. Two points we wish to highlight — anticipating our discussion of Forbes in Section 4 — are, first, that

$$\Gamma = \{P_0 \alpha, \neg \diamond P_n \alpha, \Box(P_k \alpha \rightarrow \diamond P_{k+1} \alpha) \mid 0 \leq k < n\}$$

also comes out LCT-consistent, and, second, that we are not here denying that every world is possible relative to (accessible from) every other world. In standard QML-semantics, the S4 and S5 axioms are valid if the accessibility relation *between worlds* is an equivalence relation. But LCT takes every world to be accessible from every other; it is, rather, the nature of counterpart-relation (whether it is reflexive, symmetric or transitive) which determines what kind of system we have — e.g. we get S4 if it is transitive, S5 if it is an equivalence relation, etc.

Denying counterpart-transitivity also straightforwardly resolves what we are calling Williamson's paradox, because it renders:

$$\Gamma_3 = \{\neg \diamond \beta_0 = \beta_n, \diamond^n \beta_k = \beta_{k+1} \mid 0 \leq k < n\}$$

(where, recall, the β_k are all unactual but possible ships) LCT-consistent. Briefly: for some j , β_{j+1} may now have a counterpart that is not a counterpart of β_j .

But, our CT-interpretation of MTP makes $[\diamond^n P_n \alpha]$ true in LCT, thereby rendering

$$\Gamma_2 = \{P_0 \alpha, \neg \diamond^n P_n \alpha, \Box^n(x)(P_k x \rightarrow \diamond P_{k+1} x) \mid 0 \leq k < n\}$$

LCT-inconsistent. So, LCT does not yield a solution to the strengthened Chisholm's paradox.

There are two further shortcomings we wish to note, as further motivation for the approach we are going to recommend. The first is the familiar fact that in LCT ' \Box ' represents *weak necessity*: crudely, a *de*

re statement is necessarily (possibly) true just in case it is true in every (some) world *where all the objects mentioned therein exist*. Thus, the following sentence comes out LCT-valid:

$$\forall x \Box \exists y (y = x) \quad (\text{W}\Box)$$

i.e., everything necessarily exists.

The LCT-translation (with added underlining>) is:

$$\forall x (\text{I}xw^* \rightarrow \underline{\forall w \forall z ((\text{I}zw \wedge \text{C}zx) \rightarrow \exists y (\text{I}yw \wedge y = z))})$$

The underlined sentence is trivially true. While there may be call for such a weak-necessity operator, there is surely need too for an operator which captures *strong necessity*, signifying truth *in all possible worlds, period*; weak necessity can be captured by a strong-necessity operator if required.

The second, little discussed, shortcoming, highlighted by Woollaston [1994] and Schwarz [2012], is the LCT-*invalidity* of the following QML-theorem:⁶

$$\Diamond \varphi \rightarrow \Diamond (\varphi \vee \gamma) \quad (\text{K})$$

For a counterexample model: take w^* , the actual world, and v to be the only worlds; $\text{dom}(w^*) = a, b$; $\text{dom}(v) = d$; $\text{C}da$, that is, d is a counterpart of a ; and $\text{Ext}(\text{F}) = d$. (Note: the extension of a predicate in LCT is constant, not relativized to worlds.) On this model, the following instance of (K) is false (' \underline{a} ' and ' \underline{b} ' are names of a and b , respectively):

$$\Diamond \text{F}\underline{a} \rightarrow \Diamond (\text{F}\underline{a} \vee \text{F}\underline{b}) \quad (2)$$

$$\exists w \exists x (\text{I}xw \wedge \text{C}x\underline{a} \wedge \text{F}x) \rightarrow$$

$$\exists w \exists y \exists z ((\text{I}yw \wedge \text{C}y\underline{a} \wedge \text{I}zw \wedge \text{C}z\underline{b}) \wedge (\text{F}y \vee \text{F}z)) \quad (\text{L2})$$

The antecedent is true because a has a counterpart in a world (namely, world v) which is F ; but the consequent is false because there is no world on this model which contains counterparts of both a and b and where one of them is F . The invalidity of (K) is surely an undesirable, if not intolerable, result.

We turn now to Forbes's favoured solution to Chisholm's paradox.

⁶ Actually, this is not quite right, since (K) is an *open* sentence whereas LCT is intended only for closed sentences. However, it is surely a negative feature of LCT that it cannot be extended to accommodate names without jeopardising (K).

4. Fuzzy FCT and solution 2 (preserves counterpart transitivity and S5)

In FCT, all n -place QML-predicates except ‘=’ are translated as $n + 1$ -place predicates, the last place being taken by world-terms; e.g. $[Fx]$ becomes $[Frx]$ (x is F at world w); the counterpart-relation becomes a three-place relation, $Cxyz$ (x is a counterpart of y at world z); and, crucially, Forbes stipulates that for any object a , and any world w that does not contain a counterpart of a , a is its own, and sole, counterpart at w : $Caaw$; so, every object has a counterpart *at* every world, albeit not *in* every world.

Here is a QML-FCT translation scheme derived from Forbes’s proposed evaluation rules [Forbes, 1982, pp. 35 and 37], ignoring the *actuality* operator (‘ u ’, ‘ v ’ and ‘ w ’ range over worlds as before):

FCT TRANSLATION SCHEME

- FT1 $\text{Tr}(\psi) = \psi^{w^*}$ (ψ holds in the actual world, w^*)
followed by a recursive definition of φ^u (φ holds at world u);
- FT2a φ^u , where φ is an atomic sentence $Ft_1 \dots t_n$, except when F is ‘=’
is: $Ft_1 \dots t_n u$;
- FT2b $(\neg\varphi)^u$ is $\neg\varphi^u$;
- FT2c $(\varphi \vee \gamma)^u$ is $\varphi^u \vee \gamma^u$;
- FT2d $(\forall t\varphi)^u$ is $\forall t(Itu \rightarrow \varphi^u)$;
- FT2e $(\Box\varphi t_1 \dots t_n)^u$, where the t_k are not governed by any modal operator in φ , is: $\forall v\forall t'_1 \dots \forall t'_n((Ct'_1 t_1 v \wedge \dots \wedge Ct'_n t_n v) \rightarrow \varphi^{t'_1 \dots t'_n v})$;
- FT2f $(\Box\varphi)^u$, where every term token in φ lies within the scope of a modal operator, is: $\forall w(\varphi^w)$.

The FCT-translation of (1), for illustration, is (F₁):

$$\begin{aligned} & \diamond\neg Fa && (1) \\ & \exists w\exists x(Cxaw \wedge \neg Fxw) && (F_1) \end{aligned}$$

Forbes [1983, 1984] represents Chisholm’s paradox as posing the problem of accommodating Γ (rather than our Γ_1):

$$\Gamma = \{P_0\alpha, \neg\diamond P_n\alpha, \Box(P_k\alpha \rightarrow \diamond P_{k+1}\alpha) \mid 0 \leq k < n\}.$$

But he condemns solutions which rest on denying S5, or the transitivity of the accessibility relation between worlds. He points to a close parallel

between Chisholm's paradox and the Sorites: in S5, $[\Box(A \rightarrow \Diamond B)]$ is equivalent to $[(\Diamond A \rightarrow \Diamond B)]$; so Γ can be recast as Γ^* :

$$\Gamma^* = \{P_0\alpha, \neg\Diamond P_n\alpha, \Diamond P_k\alpha \rightarrow \Diamond P_{k+1}\alpha \mid 0 \leq k < n\}.$$

This recasting highlights the parallel between the two paradoxes and, thereby, makes it:

[...] much less clear that the problem arises because of some fallacious modal inference since there is no modal logic in the standard Sorites; so a solution of Chisholm's paradox which focusses on the accessibility relation between worlds runs the risk of not directly addressing the heart of the matter. [Forbes, 1984, pp. 172–173]

As we noted in Section 3, LCT accommodates Γ by denying counterpart-transitivity and, thereby, S5, but *not* the equivalence of the world-accessibility relation. It is the denial of S5 Forbes is ultimately challenging — since the standard Sorites does not invoke modal logic — so he is also challenging the denial of counterpart-transitivity as a solution too.

Forbes reckons that the two paradoxes should have similar solutions, and, given our remarks above, these solutions should not rest on denying the equivalence of the counterpart or world-accessibility relations. To this end, he proposes the following fuzzy FCT-semantics, that is, one which invokes *degrees of truth* — any departures are irrelevant to the objections we'll be making. For any FCT-sentences φ and γ :

- $\text{Deg}(\varphi) \in [0, 1]$ — the degree to which φ is true takes a real value between 0 and 1; crudely, in the case φ is an atomic sentence, this reflects the extent to which the individuals referred to in φ satisfy the predicate: 0 for complete non-satisfaction, and 1 for complete satisfaction;
- $\text{Deg}(\neg\varphi) = 1 - \text{Deg}(\varphi)$;
- $\text{Deg}(\varphi \wedge \gamma) = \min\{\text{Deg}(\varphi), \text{Deg}(\gamma)\}$;
- $\text{Deg}(\varphi \vee \gamma) = \max\{\text{Deg}(\varphi), \text{Deg}(\gamma)\}$;
- $\text{Deg}(\varphi \rightarrow \gamma) = 1 - (\text{Deg}(\varphi) - \text{Deg}(\gamma))$, if $\text{Deg}(\varphi) > \text{Deg}(\gamma)$; 1 otherwise;
- $\text{Deg}(\exists x\varphi(x)) = \max\{\text{Deg}(\varphi(c)) : c \text{ is a constant}\}$ (it is assumed that every possible object x has a name, i.e., that for any possible x , $\text{Ref}(c) = x$, for some constant c);
- $\text{Deg}(\forall x\varphi(x)) = \min\{\text{Deg}(\varphi(c))\}$.

A rule (or inference) is *valid* if

[...] its conclusion in any application never takes a degree of truth lower than the greatest lower bound of the premises to which it is applied.
[Forbes, 1984, p. 175]

Chisholm’s paradox is supposedly resolved as follows. The degree of truth of a toleration conditional $[\diamond P_k \alpha \rightarrow \diamond P_{k+1} \alpha]$ is the degree of truth of its CT-translation, which on Forbes’s [1982] canonical counterpart theory is this:

$$\exists u \exists x (Cx\alpha u \wedge P_k x u) \rightarrow \exists v \exists y (Cy\alpha v \wedge P_{k+1} y v) \quad (\text{F1})$$

Since counterpart-hood is grounded on *similarity*, which obviously admits of degrees, and any counterpart x of ship α which is P_k is going to be *more similar* to α than any counterpart y of ship α which is P_{k+1} , the antecedent of (F1) will always be *true* — have a higher degree of truth — than its consequent, and, hence, so too will the conditional itself. Therefore, given the definition of validity, *every* application of *modus ponens* in the paradoxical reasoning, to get $[\diamond P_{k+1} \alpha]$ from $[\diamond P_k \alpha]$, is invalid — or, as Forbes says, “commits the ‘fallacy of detachment’” [Forbes, 1984, p. 175]. On the face of it, then, the reasoning would seem to be blocked at the very first step: even the inference from $[\diamond P_0 \alpha]$ to $[\diamond P_1 \alpha]$ is, on this view, invalid (i.e. illegitimate). And the same strategy would seem to block the inference to $[\diamond \diamond P_2 \alpha]$ too, and, thus the line of reasoning which eventually leads to $[\diamond^n P_n \alpha]$.

Not so. The fact is, Forbes’s solution pivots on his special evaluation rule for the conditional ‘ \rightarrow ’: $[\varphi \rightarrow \gamma]$ is not treated as equivalent to the material conditional $[\varphi \supset \gamma]$, i.e. the disjunction $[\neg \varphi \vee \gamma]$. However, he offers no reasons for thinking that upholders of Γ mean anything other than ‘ \supset ’ when they use ‘ \rightarrow ’. The lacuna in Forbes’s solution, then, is precisely that it does not deliver the consistency of Γ , or, what is more pertinent for our purposes, of Γ_1 , where the ‘ \rightarrow ’ is read as ‘ \supset ’:

$$\Gamma'_1 = \{P_0 \alpha, \neg \diamond P_n \alpha, \Box(x)(P_k x \supset \diamond P_{k+1} x) \mid 0 \leq k < n\}.$$

Here is why. Let x be an object in a world u that is P_k .

1. $\text{Deg}(P_k x u) = 1$ (Forbes’s strategy for resolving Chisholm’s paradox only assumes that counterpart-hood admits of degrees).
2. So, for any sentence γ , $\text{Deg}(P_k x u \supset \gamma) = \text{Deg}(\neg P_k x u \vee \gamma) = \max\{1 - \text{Deg}(P_k x u), \text{Deg}(\gamma)\} = \max\{0, \text{Deg}(\gamma)\} = \text{Deg}(\gamma)$.

3. Hence, for any γ , P_kxu , $(P_kxu \supset \gamma) \therefore \gamma$ comes out a *valid* inference by Forbes's criterion for validity. For, the conclusion, γ , simply could not have a degree of truth that is lower than the greatest lower bound of the premises, since, as we noted in step 2 above, this is nothing other than $\text{Deg}(\gamma)$.
4. So, in particular, $[P_kxu, P_kxu \supset \exists v\exists y(Cyxv \wedge P_{k+1}yv)] \therefore \exists v\exists y(Cyxv \wedge P_{k+1}yv)$ comes out valid.
5. Hence, from our initial premise $[P_0\alpha w_*]$ (α is P_0 in the actual world) we can validly infer $[P_1b_1w_1]$ holds for some counterpart of α , b_1 , at some world world w_1 ; and, then, that $[P_2b_2w_2]$ holds for some a counterpart of b_1 , b_2 , at some world w_2 ; and so on.
6. Assuming counterpart-transitivity, we can thereby validly derive $[\Diamond P_n\alpha]$; in which case, Γ_1 is not consistent.

So, until Forbes provides a case for *not* understanding the toleration principles at play as material conditionals, his fuzzy strategy fails to resolve Chisholm's paradox. (Let us in any case stipulate that henceforth, ' \rightarrow ' should be understood as the material conditional.) And denying counterpart-transitivity would rather defeat his purpose, since there would, in that case, be no need to resort to fuzzy semantics — as we shall now see.

5. FCT, RCT and solution 3 (denies counterpart transitivity but not S5)

Let us signal where we are going. FCT resolves our paradoxes without jettisoning S5, and secures the validity of (K), i.e. $[\Diamond\varphi \rightarrow \Diamond(\varphi \vee \gamma)]$. However, the same is true of a very different CT-approach exemplified by Ramachandran [1989, 2008] and Schwarz [2012]. The two approaches have different, not insignificant, shortcomings; but these are avoided by way of a simple mish-mash of the FCT- and RCT-strategies. This is the theory we will finally recommend.

And henceforth, we are going to assume the counterpart relation is 'many-one': many objects from one world may have a common counterpart at another, but any object can have *at most* one counterpart at any world. Lewis argues that this is implausible, since there evidently could be (are) possible worlds containing identical twins which closely resemble, and are equally similar to, you [Lewis, 1968, p. 29]. But, as

we owned at the outset, our appeal to counterparts is not motivated by Lewisian modal realism; we take it to be motivated by the paradoxes under consideration; and these, we contend, demand merely a many-one counterpart theory. So, we hereby stipulate that something is a counterpart of an object a at world w just in case it is *sufficiently similar to a , and more similar to a than any other object in w .*

The FCT-validity of the S4 and S5 axioms, $[\diamond\diamond\psi \rightarrow \diamond\psi]$ and $[\diamond\psi \rightarrow \square\diamond\psi]$, follows trivially from the FCT translation rule FT2f:

$(\square\varphi)^u$, where every term-token in φ lies within the scope of a modal operator, is: $\forall w(\varphi^w)$

$[\diamond\diamond\psi]$ and $[\square\diamond\psi]$ come out equivalent, and equivalent to $[\diamond\psi]$. This is so regardless of the nature of the counterpart relation. Hence, denying transitivity renders Γ_1 , Γ_2 , and Γ_3 consistent, so all our paradoxes are resolved in one go. And FCT, unlike LCT, rightly secures (K) because it guarantees that every object has a counterpart at every world, thereby eschewing (E) — Lewis’s requirement that a counterpart at world w of an object must exist in w . For example, (2) now gets translated as (F2):

$$\diamond F\underline{a} \rightarrow \diamond(F\underline{a} \vee F\underline{b}) \quad (2)$$

$$\exists w \exists x (C x \underline{a} w \wedge F x w) \rightarrow \exists w \exists y \exists z ((C y \underline{a} w \wedge F y w) \vee (C z \underline{b} w \wedge F z w)) \quad (F2)$$

(F2) is a theorem in FCT, precisely because a has a counterpart at every world.

But, we can achieve the same result by very different means. Ramachandran’s [1989] RCT keeps to Lewis’s 2-place counterpart relation, and n -place predicates remain n -place in translation. What differs is the QML-LCT translation scheme. Our assumption that counterpart-hood is many one allows a simple specification.

RCT TRANSLATION SCHEME

DEFN. 1. A term, t , in a QML-sentence is *modally free* if t is either a constant, or a variable-token that is governed by a modal operator which has narrower scope than the quantifier which binds it.

DEFN. 2. For any QML-sentence, ψ , $[\psi]_P$ is a *preliminary translation*, the result of replacing every atomic constituent, ϕ , of ψ that has modally free term-tokens t_1, \dots , and t_n with:

$$\exists t'_1 \dots \exists t'_n (C t'_1 t_1 \wedge \dots \wedge C t'_n t_n \wedge \phi[t'_k/t_k])$$

where $\lceil \phi[t'_k/t_k] \rceil$ is the result of replacing each token of $\lceil t_k \rceil$ in ϕ with $\lceil t'_k \rceil$.

For any QML-sentence, ψ , the RCT-translation is the result of applying familiar recursive rules to the formula $([\psi]_P)^{w^*}$ (read: $[\psi]_P$ holds at the actual world, w^*):

- (A) φ^u , where ϕ is an atomic QML-sentence, is simply ϕ ;
- (\neg) $(\neg\varphi)^u$ is $\neg\varphi^u$;
- (\vee) $(\varphi \vee \gamma)^u$ is $\varphi^u \vee \gamma^u$;
- (\wedge) $(\varphi \wedge \gamma)^u$ is $\varphi^u \wedge \gamma^u$;
- (\forall) $(\forall x\varphi)^u$ is $\forall x(Ixu \rightarrow \varphi^u)$;
- (\exists) $(\exists x\varphi)^u$ is $\exists x(Ixu \wedge \varphi^u)$;
- (\Box) $(\Box\varphi)^u$ is $\forall v\varphi^v$;⁷
- (\Diamond) $(\Diamond\varphi)^u$ is $\exists v\varphi^v$.

We can think of RCT as narrow-scope counterpart theory since the counterpart quantifiers (CQs) in the RCT-translation of a QML-sentence, ψ , will have narrower scope than any of the connectives in ψ . For example, here are the various translations of $[\Diamond\neg Fa]$:

[LCT]	$\exists w\exists x(Ixw \wedge Cxa \wedge \neg Fx)$	(L ₁)
[FCT]	$\exists w\exists x(Cxaw \wedge \neg Fxw)$	(F ₁)
[RCT]	$\exists w\neg\exists x(Ixw \wedge Cxa \wedge Fx)$	(R ₁)

Note, the CQ ' $\exists x$ ' has narrower scope than the negation in (R1). It is this feature, the narrow scope of CQs, which ensures the RCT-validity of (K).

Here is the RCT-translation of (2):

$$\begin{aligned} & \Diamond F\underline{a} \rightarrow \Diamond(F\underline{a} \vee F\underline{b}) & (2) \\ & \exists w\exists x(Ixw \wedge Cx\underline{a} \wedge Fx) \rightarrow \\ & \quad (\exists w\exists y(Iyw \wedge Cy\underline{a} \wedge Fy) \vee \exists w\exists z(Izw \wedge Cz\underline{b} \wedge Fz)) & (R_2) \end{aligned}$$

Clearly, any world satisfying the antecedent trivially satisfies the consequent.

⁷ This departs from LCT and FCT, where the counterpart relation is only introduced in the translation (evaluation) of sentences governed by modal operators—see LT2e and FT2e.

And the rules (\Box) and (\Diamond) make $[\Diamond\Diamond\psi]$ and $[\Diamond\psi \rightarrow \Box\Diamond\psi]$ trivially valid, as in FCT; so, we get the same solution to our paradoxes: deny counterpart-transitivity.⁸

We have, then, two contrasting CT-approaches that serve our purpose as far as the paradoxes are concerned. But they each have significant failings. A substantive shortcoming of RCT, highlighted by Forbes [1990, p. 169], is that it follows LCT in enforcing (E), or, as he calls it, the *Falsehood Principle*, so that for any n -place predicate, F , the QML-sentence

$$\Box(Ft_1 \dots t_n \rightarrow \exists x_1 \dots \exists x_n(x_1 = t_1 \wedge \dots \wedge x_n = t_n))$$

is valid, whereas it is invalid in Kripke semantics for QML. Forbes sides with Kripke; he says:

[...] whether or not the Falsehood Principle is correct is a metaphysical question which logic should not foreclose. [Forbes, 1990, p. 169]

FCT, on the other hand, has a different substantive failing, pointed out by Ramachandran [1989, p. 133]. While it does render

$$\forall x\Box\exists y(y = x) \tag{W\Box}$$

invalid, the vital necessity-axiom:

$$\Box\psi \rightarrow \psi \tag{N\Box}$$

also comes out invalid. For a counterexample-model take w^* , the actual world, and v to be the only worlds; $\text{dom}(w^*) = a$; $\text{dom}(v) = b$; $Cabw^*$ (a is a counterpart of b at w^*); $\text{Ext}(F, w^*) = \{<a>\}$; $\text{Ext}(F, v) = \{\}$. The following instance of (N \Box), $[\Box F\underline{b} \rightarrow F\underline{b}]$ (where ' \underline{b} ' is a name for b), is false on this model. The FCT-translation is $[\forall w\forall x(Cx\underline{b}w \rightarrow Fxw) \rightarrow Fbw^*]$. The antecedent comes out true on the model because b 's counterpart in any world is F in that world, while the consequent comes out false because b itself is not F at w^* .

⁸ Schwarz [2012] provides an elegant and versatile CT-semantics that also secures (K), but we do not consider it as providing a *third* way of doing so. Briefly, and simplifying much, the evaluation rule for ' \Box ' [Schwarz, 2012, p. 16] is captured as follows. For any world w , $\psi(b_1, \dots, b_n)$ is a w -image of sentence $\psi(a_1, \dots, a_n)$ if, for each k , $\text{Ref}(b_k)$ is a counterpart of $\text{Ref}(a_k)$ that exists in $\text{dom}(w)$. $[\Box\psi]$ is true if for any world w , every w -image of ψ (which we'll abbreviate as ' $w_i(\psi)$ ') is true. It follows that for any φ and γ , $w_i(\varphi \vee \gamma) = [w_i(\varphi) \vee w_i(\gamma)]$; and capturing that as a sentence of predicate logic will reveal counterpart quantifiers that do not govern the disjunction. So, this is in effect a variant of the narrow-scope approach.

Forbes [1990, p. 168] suggests a technical fix which involves introducing what we have been calling narrow-scope CQs. His proposal foreshadows the following mish-mash of FCT and RCT, FRCT. FRCT retains FCT's three-place counterpart relation, and the postulate that for any object a , and any world w that does not contain a counterpart of a , a is its own, and sole, counterpart at w . But it follows the RCT-strategy of taking CQs to govern just atomic sentences.

FRCT TRANSLATION SCHEME. This differs from RCT's translation scheme in two respects.

Firstly, the translation rule for atomic sentences, (A), is replaced by:

(A*) φ^u , where φ is an atomic sentence $Ft_1 \dots t_n$, except when F is '=' is: $Ft_1 \dots t_n u$.

And, secondly, the evaluation rule for (\exists) is replaced by:

($\exists a$) $(\exists x\varphi)^u$ is $\exists x(Ixw \wedge \psi^u)$;

unless the ' $\exists x$ ' was introduced in the preliminary translation, in which case, the rule to use is:

($\exists b$) $(\exists x\varphi)^u$ is $\exists x(\psi^u)$.

This modification to RCT does not affect the solution to our paradoxes or the validity of (K). We can check $[\Box F\underline{b} \rightarrow F\underline{b}]$ now comes out valid. The preliminary translation, $[\Box F\underline{b} \rightarrow F\underline{b}]_P$, given by RCT's Defn. 2, is:

$$\Box \exists x(Cx\underline{b} \wedge Fx) \rightarrow \exists x(Cx\underline{b} \wedge Fx)$$

Applying RCT's remaining rules, using (A*) in place of (A), we get (simplifying a bit):

$$\forall w \forall x(Ixw \wedge Cx\underline{b}w \wedge Fxw) \rightarrow \exists x(Ixw^* \wedge Cx\underline{b}w^* \wedge Fxw^*)$$

which is clearly valid – the consequent is just an instance of the antecedent.

And the Falsehood Principle comes out invalid, as required; consider, e.g., the FRCT-translation of $[\Box(Fa \rightarrow \exists x(x = a))]$:

$$\forall w(\exists x(Cxaw \wedge Fxw) \rightarrow \exists y(Iyw \wedge Cyaw \wedge x = y))$$

is obviously invalid, since, given Forbes's postulates, a may have a counterpart at a world *without* having a counterpart that exists in that world.

What of the Sorites? Of course, we do not deny that the parallels with Chisholm's paradox point towards similar solutions. But we do not have a worked-out position. Instead, we'll settle for a few brief (unsupported) remarks indicating our present thoughts. First of all, we think our objection to Forbes's fuzzy solution to Chisholm's carries over to his solution to the Sorites, since it too pivots on his special treatment of ' \rightarrow '. This solution evidently leaves the material-conditional version of the Sorites, so to speak, untouched. Secondly, we are in any case dubious about the appeal to degrees of truth; *tallness*, *baldness* and other vague properties admit of degrees, but this does not commit us to degrees of *truth*, and certainly not to the view that degrees of truth must be invoked to resolve both paradoxes. Rather, we suspect that the common factor will merely be the invocation of a non-transitive similarity (or counterpart) relation. This is work in progress.

Salmon [1989, p. 148] claims that Chisholm's paradox *demonstrates* the invalidity of S4 modal reasoning. We have argued that denying S4 does not in fact resolve it — for, a strengthened version and Williamson's variation remain. We contend that the paradoxes we have considered rather *demonstrate* the need for a CT-semantics for QML. Such a semantics should not, however, be regarded as providing the *truth-conditions* for QML-statements as, e.g., Lewis [1986] maintains. Rather, the appeal to counterparts is required, we contend, to *represent* certain modal facts, such as those underlying our acceptance of MTP for instance, *for the purposes of logic alone* — to explain the correctness or otherwise of modal inferences.

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