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Logic of Social Ontology and Łoś's Operator

Abstract. In 1947 Jerzy Łoś proposed a positional logic based on the realization operator. We follow his work and present it in the context of fundamental challenges of sociology such as the complexity of social reality and reflexivity of social agents. The paper is an outline of the general concept, as it opens a discussion and sets ground for future elaborations. In this paper, we are considering the concept according to which the expressions put forward by Łoś's system might be indexed not only by spatial and temporal variables, but also by social contexts. And as such Łoś's system might be a significant improvement, a valuable addition for social simulations and computational sociology, which use multi-agent systems and agent-based modeling. We consider how Łoś's operator might be useful for these disciplines, as it gives a chance to combine of formalization with the humanistic coefficient, which represents the issues of complexity and reflexivity of social agents.

Keywords: agent based systems; computational sociology; Jerzy Łoś; logic for social sciences; multi-agent systems; positional logic; realization operator; social ontology; social phenomena; sociology; social sciences

1. Aim of the paper

Seventy years ago, Jerzy Łoś proposed the first positional logic that included a temporal parameter of physical events which would be useful — in his opinion — for natural science [18]. Łoś's idea has been largely forgotten; however, some interesting research and in fact some developments of his idea have occurred in different contexts, which are presented

in further parts of this article. It must be underlined that no logic inspired by Łoś's positional logic was dedicated to the problems of reasoning about social phenomena. In Łoś's own works this logic was intended for natural science and it was further applied to philosophical problems.¹ Below we refer to some motivations and intentions of Jerzy Łoś:

An analysis of inductive reasoning that leads to setting causal relations within natural sciences is a starting point in Łoś's theory. Let us consider a simple example of inductive reasoning:

It flashes and thunders,
 again it flashes and thunders,
it still flashes and thunders,
 if it flashes, then it thunders.

Łoś noticed, that empirical causal sentences, that are conditions or conclusions in inductive reasoning are containing a moment of time after-effect. A sentence "it flashes and thunders" does not mean that it flashes and thunders at the same time, but that it flashes and in a moment it thunders. When it flashes, than usually it does not simultaneously thunder. One can say that when it flashes, than it thunders and does not thunder. The conclusion is paradoxical. The source of this paradox is a lack of time and space coordinates. When we take these coordinates into account, then we can say that when it flashes in place s and-in-time t_1 , then it thunders in place s in certain time t_2 , which is later than t_1 [...]. [12, p. 39]

The aim of our work is to present a new, broader perspective on an application of positional logic; in this case an application to social sciences issues. Positional logic should cope with two problems that are typical of social sciences, specifically for sociology. The first one is an ontological problem of social systems' complexity. However, it should be underlined here that this issue is not distinctive or exclusive for sociology; in fact all empirical sciences deal with it. Nonetheless, the complexity within social sciences is related to imperceptible variables, such as subjective opinions and convictions about the world. This leads to the second problem, a methodological one: how can social systems be researched? There are two, quite extreme options here, which together create a continuum. The first one is to follow the path of so-called social physics, which concentrates exclusively on grasping objective human behaviors and activities, without any focus on their convictions,

¹ For example, for considerations on time and modalities see [10].

knowledge, etc. Such a perspective enables one to sustain the standards that are typical of natural science, but it largely ignores an important social context. Another option is to focus on the *world of meanings* which individuals and collectives operate in. This means going towards the humanities and hermeneutics, which offer limited possibilities for making generalizations and accumulating knowledge. Our proposal is an attempt to build a bridge between these two options. It seems that the grammatical constructions typical of positional logic make it possible to express social contexts which are complex in their nature. This complexity is built upon many elements concurrently influencing each other (e.g., individuals, social roles, cultural patterns, social positions) and the reflexivity of individuals who are able to evaluate their own activities and in order to change their subjective convictions into an objective behavior.

The first option described above is related to the possibility of analyzing social systems by multi-agent systems (MAS). Within the MAS analysis, the concept of an agent is broadened from an individual to different social levels such as institutions, organizations, social groups [28]. At each level, agents are active, they judge their own situation, take actions while evaluating their own interests and subjectively monitoring their social context. At the same time in the MAS analysis there is a question of submitting individual agency to collective agents. A society is not a simple aggregate of individual characteristics. Unintended macrosocial consequences of microsocial actions are a constantly present part of every social system. In MAS computer studies, researchers discovered that behavior, actions taken by computer agents are also hard to predict and are not just an aggregate of features of specific agents in a system. MAS is the approach that tries to elaborate certain solutions for reality of open systems, where agents/participants are heterogenous, express limited trust and have conflicting interests. As M. Dastani and co-authors show [3], MAS studies understand the need to apply such sociological categories like norms, roles, power structure into formalization and simulations.

An analogous idea — the need to create simulations, models of social action — has arisen within sociology. The aim of such simulations was to gain a better understanding of transitions from the micro to the macro level. Another one was to capture reproducible social mechanisms. Simulations were developed within computational social science [16] with empirical data as the basis of analysis. On the other hand, they were used by researchers of agent-based modeling, agent-based simulations

and artificial societies such as Joshua Epstein [4], or Michael W. Macy [20]. These researchers have overlooked the issue of agents' rootedness in the multidimensional world of meanings and social contexts.

Our article — while relating to the above studies, ideas and challenges — sets an agenda for further application of positional logic to social studies, specifically, to sociology. It demonstrates that Jerzy Łoś's idea together with the operator of realization and modification of some approaches in positional logic, can be transmitted into the reality of social contexts and social simulations.

2. Łoś's operator of realization

In 1947, Łoś published the work on temporal logic “Podstawy analizy metodologicznej kanonów Milla” [18] (Foundations of methodological analysis of Mill's canons) and, a year later, an article about epistemic logic entitled “Logiki wielowartościowe a formalizacja funkcji intensjonalnych” [19] (Multi-valued logics and formalization of intensional functions).²

Łoś's works were published in Polish, but short reviews by Henryk Hiż [8] and Roman Suszko [33] made them accessible to a wider audience. Although his work on temporal logic influenced the creation of this separate domain in logic, and work on epistemic logic was one of the first ones to appear, Łoś's accomplishments unfortunately were quickly forgotten within English-speaking academic world (see [37]).

In both of his works Łoś used an original grammar construction for expressing relations between sentences and their context. It was called the operator of realization \mathcal{R} .³ If α is a term and p is a proposition, then $\mathcal{R}_\alpha p$ is also a proposition. The operator of realization \mathcal{R} connects names

² These were truly trailblazing works, as they were probably the first publications dedicated to temporal logic and epistemic logic. Although later these logics were developed with a possible world semantics, there is strong evidence that Łoś's approach can be developed independently and that in many aspects his approach gives even greater possibilities for modelling intensional properties; see [12].

³ Łoś did not use the grand letter R for his operator. In the work on temporal logic [18] he used the upper-case letter U and in the work on epistemic logic [19] the letter L. The letter R was introduced later by Nicholas Rescher (see [12, pp. 31–32]). Regardless of these previous applications, we have decided to use a notation with the letter $\mathcal{R}_\alpha(A)$, where α is a name and A is a sentence. This notation is proposed as a universal one for all applications in the paper [9].

with sentences and creates new sentences. Since a is sometimes called a position, the logics with Łoś's operator are called positional logics.

In the article “Logiki wielowartościowe a formalizacja funkcji intensionalnych” [19] (Multi-valued logics and formalization of intensional functions) Łoś applied the realization operator to model the knowledge of a subject. So the sentence “ $\mathcal{R}_a(p)$ ” represents a fact that an agent a asserts/knows that p . With an axiomatic system, Łoś proposed a very idealistic concept of a rational subject of knowledge (see [17]). The connection between a subject and judgement can be approached differently when interpreted as a less classical kind of knowledge or simply a propositional attitude of an agent a .⁴

In Łoś's work on temporal logic, position a in a sentence $\mathcal{R}_a(p)$ is interpreted as a temporal object — a point in time or moment at which a sentence p is true. We will go back to this work and its approach, as it inspires us to apply the operator \mathcal{R} in a broader context, without omitting any knowledge or propositional attitudes of an agent. Before doing so, we would like to examine other philosophical and logical interpretations of the operator of realization.

Sentences that are in the range of operator \mathcal{R} can be interpreted in different ways; the interpretation depends on how we understand the denotation of the individual a in the expression $\mathcal{R}_a(p)$. This denotation is always a kind of context in which proposition p is referred to (for example, p can hold, be true, be known, be part of the set of beliefs etc). In the literature, such contexts as the following ones have been proposed as:

- temporal: moments or some kind of intervals
- spatial: points or certain parts of space
- epistemic: minds of agents
- mathematical: as solutions to some equations [26, 27, 12].

Let us consider such expressions as:

- (0) $\mathcal{R}_{2018}(\text{It rains})$
- (1) $\mathcal{R}_{\text{Toruń}}(\text{It rains})$
- (2) $\mathcal{R}_{\text{Jan}}(\text{It rains})$
- (3) $\mathcal{R}_8(3 + 5 = x)$.

Expression (0) says that sentence *It rains* is realized in a temporal context denoted by date 2018. Expression (1) says that sentence *It rains*

⁴ Such an approach is present in the work by Marcin Tkaczyk [36], where a denotation function can ascribe sentences to positions regardless of their structures.

is realized in a spatial context denoted by the name of place Toruń. Expression (2) says that sentence *It rains* is realized in the epistemic context denoted by the name Jan, or to put in more philosophically, is the subject of a certain propositional attitude of Jan (knowledge, belief, doubt, etc.). Finally, expression (3) says that sentence $3 + 5 = x$ is valid in the arithmetical context where $x = 8$. All these examples show how flexible the operator of realization is and how many meanings and applications it has when we interpret the relations of the name a and the sentence p in such an expression $\mathcal{R}_a(p)$ as:

- the statement that is denoted by sentence p , in a context denoted by name a , has a certain property.

This property can be a logical value, subject of someone's propositional attitude or possibly another property. For example, in [11] an interpretation of the operator \mathcal{R} was proposed according to which the realization of a sentence p in a position a means that there exists such position b that it stays in a binary relation with a and p is true at b .

However, we must admit that the most often interpretation of position/context is a temporal one, as a moment or an interval of time, see [27, 10, 13]. It is a natural context of use of the realization operator — because of Łoś's groundbreaking work on temporal logic [18].

Let us accept for further considerations that the operator \mathcal{R} relates a sentence to a context denoted by a name. Therefore, a position in positional logic is a kind of context. From our point of view, social contexts are specifically interesting.

The realization operator is closely related to investigation of indexicals initiated by Bar Hillel in the book [2] published in 1971. Also in this aspect Łoś remained ahead of others for decades. David Kaplan in [14], Richard Montagues in [23], Dana Scott in [30] and Robert Stalnaker [32] proposed logical systems simulating the way contexts act. A review of how to get a proposition $c(P)$ expressing content of the sentence P uttered in context c could be found in [21]. The problem is that a content might be so complex that — as Bar Hillel has argued — a satisfactory definition of context is unlikely to be given. Anyway, we can agree that the context of an utterance is determined by the circumstances of utterance. By knowing them, we know the context.

Łoś's realization operator is applied to the special cases of a context. However, one could hardly agree that they are similar to pragmatic contexts. Further in this paper, we consider the realization operator based

on social contexts that are quite far from the notion of context known from pragmatics. For this reason, we only note some similarities between both concepts.

3. Logics of the \mathcal{R} -operator

The logic **MR** is the simplest positional logic extracted from Łoś's temporal logic [9]. Its alphabet is built from sentential letters **Var**, positional letters **Pl**, the classical propositional connectives: \neg , \wedge , \vee , \rightarrow , \leftrightarrow , the operator of realization \mathcal{R} and brackets $)$, $($. In this logic pure formulas of Classical Propositional Language have the function of quasi-formulas, i.e. their aim is to build correct formulas. Let us assume that set **For_{CPL}** contains only the sentential letters **Var** and such expressions as: ϕ , $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$, $(\phi \leftrightarrow \psi)$, where ϕ and ψ belong to **For_{CPL}**. On the other hand, the set of correct formulas **For_{MR}**, so formulas of **MR**, contains only such expressions as: $\mathcal{R}_\alpha(\phi)$, where $\alpha \in \text{Pl}$, $\phi \in \text{For}_{\text{CPL}}$, and expressions $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$, $(\phi \leftrightarrow \psi)$, where ϕ and ψ belong to **For_{MR}**.

MR logic excludes the nesting of the operator \mathcal{R} in the manner that happens in the case of such sentences as: $\mathcal{R}_a(p \wedge \mathcal{R}_b(q))$ and $R_b(R_c(p) \rightarrow \neg R_b R_c(r \vee \neg q))$, where contexts are within contexts.

In work [9] logic **MR** has been axiomatized with *modus ponens*:

$$A, A \rightarrow B \ / \ B$$

and with four schemas of axioms. The first schema is defined with the condition: every substitution of any tautology **CPL** with formulas **MR** is an axiom **MR**.

(Ax1) $s(A)$, if $A \in \text{Taut}_{\text{CPL}}$ and s is a function of substitution of sentential letters with formulas **MR**.

For any formula $A, B \in \text{For}_{\text{CPL}}$ and any positional letter $\alpha \in \text{Pl}$ it is accepted also that:

$$(Ax2) \quad \neg \mathcal{R}_\alpha A \leftrightarrow \mathcal{R}_\alpha \neg A$$

$$(Ax3) \quad (\mathcal{R}_\alpha A \wedge \mathcal{R}_\alpha B) \rightarrow \mathcal{R}_\alpha (A \wedge B).$$

Additionally, if A is a tautology **CPL**, then every formula formed by realization operator and any positional letter $\alpha \in \text{Pl}$ is an axiom:

$$(Ax4) \quad \mathcal{R}_\alpha A, \text{ if } A \in \text{Taut}_{\text{CPL}}.$$

Three kinds of semantics were proposed for such an axiomatized logic. Firstly, there were models $\langle W, d, v \rangle$, where W is a non-empty domain, d is a mapping from Pl to W , and v is a classical valuation of formulas **CPL** at objects from W , i.e., mapping $v: W \times \text{For}_{\text{CPL}} \rightarrow \{0, 1\}$ that fulfills conditions for Boolean connectives (for example, $v(w, \neg A) = 1$ iff $v(w, A) = 0$; $v(w, A \wedge B) = 1$ iff $v(w, A) = 1$ and $v(w, B) = 1$; etc. [9]).

In [12] it was noted that such models are a bit redundant. It is enough to assume models with the valuation $v: W \times \text{For}_{\text{CPL}} \rightarrow \{0, 1\}$ that fulfills classical conditions for those $w \in W$, that for some $\alpha \in \text{Pl}$, $d(\alpha) = w$. So the objects that are not denoted by any terms do not have to behave classically with respect to v . In the same book, an alternative semantics that was based on valuations only was also proposed [12, p. 92].

In the work cited above, the authors propose that positional logic is normal if the operator \mathcal{R} is distributive over all classical connectives, i.e., if for all $A, B \in \text{For}_{\text{MR}}$ its laws are for $* \in \{\wedge, \vee, \rightarrow, \leftrightarrow\}$:

$$\begin{aligned} \mathcal{R}_\alpha(A * B) &\leftrightarrow \mathcal{R}_\alpha(A) * \mathcal{R}_\alpha(B), \\ \neg \mathcal{R}_\alpha(A) &\leftrightarrow \mathcal{R}_\alpha(\neg A). \end{aligned}$$

Logic **MR** is the least normal positional logic. Consequently, it means that each positional logic that respects classical logic in positions as well as out of the operator \mathcal{R} must include logic **MR**. **MR** is also maximal in a sense that one cannot extend it with additional formulas stating something about one position without inconsistency [15].

It is possible that normal positional logics are too strong for the social sciences. However, in our opinion, this is not the case. They can be weakened, for example by the use of many-valued semantics [34], algebraic semantics [36] or other techniques [11]. From a logical point of view both former proposals seem interesting, but many-valuedness in a logic for social sciences may be introduced in a different way than by the use of weaker outer connectives.

An important modification of the positional logic language is to add quantifiers and variables that denote positions to which sentences are referred by \mathcal{R} . Such an extension of **MR** is logic **MRQ**. In its language, there are quantifiers, function constants and predicates. The logic was examined in [12]. In fact, **MRQ** is a combination of First Order Logic and **MR**, because in the logic we can quantify over positions and express different properties of positions with predicates. Clearly, **MRQ** is undecidable as it includes First Order Logic, unless we limit its lan-

guage to monadic predicates. It is worth noting that the nesting of \mathcal{R} is also forbidden in **MRQ**. In particular one can say nothing about relations between positions and cannot quantify in the range of \mathcal{R} . So, in the language of logic **MRQ** there do not appear such expressions like " $\mathcal{R}_\alpha(r(\beta_1, \beta_2))$ " and " $\mathcal{R}_\alpha(\exists_x R_x(A))$ ", where " $r(\beta_1, \beta_2)$ " states that the relation r holds between positions " β_1 " and " β_2 ", whereas " $\exists_x R_x(A)$ " says that there exists a position x such that $A \in \text{For}_{\text{CPL}}$ is realized at x , and both facts happen in position α .

The language of the first positional logic, the system that was designed for the natural sciences, was defined in a different way. In article [18], Jerzy Łoś also did not accepted the nesting of \mathcal{R} . However, he additionally introduced quantifiers over temporal intervals and propositions in the range of \mathcal{R} . In Łoś's language such expressions as " $\exists_x \forall_p \mathcal{R}_x(p)$ " were correct. Let us remember that in the paper, positions are moments in time. Therefore, propositions happen at moments of time. Moreover, Łoś applied the binary functional constant δ that *shifts* a time line. For example, the expression " $\mathcal{R}_{\delta(x, \epsilon)}(p)$ " says that sentence p is true at the moment that appears after the move of time interval ϵ from moment x to some moment denoted by " $\delta(x, \epsilon)$ " (after the length of ϵ starting from x —time for Łoś is representable by the real number line). The other important grammatical feature of Łoś's system is that sentences belonging to For_{CPL} are present in the language. The use of these sentences means that their truth is settled regardless of the context. In the work on epistemic logic [19] Łoś simplified the language (compared to his previous works), but we are not considering this issue here.

Łoś's works were an inspiration to many logicians, including Prior, the founder of tense logic. Other modifications and applications of the realization operator can be found in works by Garson and Rescher [7], Rescher and Uquhart [27] and Rescher [25]. In particular, Prior's work [24], was clearly inspired by Łoś's work. This case, a complex one, but important for the history of temporal logic, was described in [12, pp. 15–16]. It shows that Łoś was the founder of temporal logic.

4. The challenges of reasoning about social phenomena

We would like to propose the broadening of Łoś's concept from physical phenomena to social ones. There are many indicators showing that the

grammatical construction of the operator of realization makes it possible to do so. There are two reasons for constructing such a logic: a cognitive one and a practical one.

The cognitive reason refers to the possibility of making assumptions for formal models of specific social phenomena. Such models should include variables that have up to now been mostly omitted in formalizations and simulations of social processes. This has happened because they were either too complex or the syntax of formal language was not sufficiently flexible. The modeling of social phenomena still faces many barriers. At the cognitive level, sociology struggles with the constant problem of complexity, which lies mainly in nesting individual activities in broader social contexts and in understanding interactions that happen between them. Meanwhile, well-known simulations of social actions and efforts to model social processes have a rather individualistic character — i.e. they deal with agents who act upon simple rules, not with agents who are deeply immersed in a broader social context.

One of the reasons for this situation is that in simulations of social phenomena we can see the domination of the tradition that refers to the frequently cited research by Thomas Schelling [29]. He proposed a model of spatial segregation which shows how complex phenomena are an outcome of rather simple social interactions. Schelling's aim was to show that the spatial segregation in cities can emerge spontaneously, without being driven by pro-discrimination attitudes amongst the citizens. He created a simulation in a form of a field with 208 squares, 13 rows and 16 columns, where some parts were empty, but mostly were taken by agents who were marked with crosses or circles. At the beginning of the simulation, agents were distributed randomly. But, rather quickly, new layouts emerged, where the space was divided between the fields dominated by *circles* or by *crosses*. This happened because Schelling gave agents one simple *conviction*: they want to have at least 1/3 of their neighbours to be agents who are similar to them. When this desire was satisfied by relocating people, it turned out that both groups were separated. With this research Schelling started a whole series of social simulations, which were multi-agent simulations (see [4, 20]). But he also provided a direction for further studies, which was to search for outcomes of agents' activity, which were based on a few simple assumptions. In this way, these agents are far from the real-world agents it would be desirable to represent. This legacy is a source of troubles, especially for researchers who try to use simulations in empirical studies (see [5]).

The passage from individuals' action (actions of people like agents) to collective outcomes is still one of the great challenges of contemporary sociology. This problem is partially visible in the dichotomy between agency and structure. In sociological theories, individual activity—human agency and its individuality—clashes with the impact of social structures which determine this activity. Therefore, sociology explains observable social processes by pointing to repeatable patterns of behavior. Sociology looks for such patterns and interprets them as the key determinant of actions taken by humans. In sociological theory, we find mostly deterministic explanations that still deal with the structure vs agency dualism (see [31]).

However, the sociological concepts which attempt to include a cultural dimension into the individual perspective are still present and alive. A classic example is the humanistic coefficient concept developed by Florian Znaniecki, who postulated the need not to limit researchers' observation only to their own direct experience of the data, but to reconstruct the experience of the people who are the subject of the research [39]. The humanistic coefficient concept assumes that individuals think about the consequences of their actions at the same time as performing them; they make generalizations about their goals and aims and they make their experience more objective in their own consciousness. This means that individual experience is treated as a collective one, as a commonly shared experience. Znaniecki's concept is on the one hand a kind of methodological postulate, but on the other is an attempt to find a passage from micro to macrosociological phenomena. This postulate is visible, although not directly expressed in most qualitative research, but it is absent from attempts to model social reality.

The concept of agency is another attempt to understand society in humanistic categories, specifically to see active individuals as a part of morphogenetic processes of an emerging society. In this approach, an individual is mostly a social actor playing a specific social role. As an actor, however, one has some degree of freedom and possibilities to interpret socially imposed solutions. As Margaret Archer [1] puts it, it is a matter of a capability to reflect on one's own actions, which is the most important feature of humans. Reflexivity allows people to think about their own and others' activity, and to evaluate and make changes in collective action. Social change processes, especially new institutional solutions, are, in Archer's opinion, an effect of structural work where

reflections, thoughts and actions are accumulated and at the same and are an inspiration or an engine for further changes [1].

Simulations of social processes have not been able to include a concept of agency and have not taken into account the humanistic coefficient postulate. In the methodology of qualitative research, the idea of deep insight into individual interpretation is, of course, present and is used to reconstruct common social patterns. However, quantitative methods which are a basis for computational sociology [16], are not sensitive at all to the problem of reflexivity. An application of Łoś's operator of realization and its logic makes it possible to cope with the complexity of social process. This also — in our opinion — enables us to accommodate the postulate of humanistic coefficient and makes the possible the empirical use of agency theory.

The practical reasons (for the whole concept of forming a special logic for sociology) are related to the possibilities of building an ontology on the basis of empirical data (for example in order to solve specific social issues). This concept is in a way a classical one. When there are formalized theories, one can reason about some directly unobserved processes and can make assumptions on how they will proceed. This is exactly the main aim of all simulations and models of social systems. So, our proposal can be applied to such areas of sociology as applied sociology, policy sociology, or clinical sociology which all are oriented to using knowledge for practical solutions [6]. Any sociological intervention needs solid foundations. Such foundations should be driven by simulations that are as close to the social reality as possible, and that demonstrate not only how things are, but also how they will proceed.

One of the most important challenges that we see is also setting an agenda for future studies. Firstly, it is not enough to create a logic that will only describe people's behavior (with individuals presented as agents in a certain time and space). There is a need to create a logic which includes broader contexts such as culture (e.g., specific values), communities (e.g., forms of social control), institutions (e.g., informal rules), etc. Such a logic should also be able to describe passages between these contexts. This is the issue of *social complexity*, where many types of social relations and entities have to be taken into account. Secondly, the biggest challenge for a new logic is to combine the humanistic coefficient with formalization. In other words, it is the problem of how to grasp not only a behavior but also a set of beliefs as separate variables. This is

the issue of the *humanistic coefficient*. It seems that both of these issues can be represented in positional logic.

5. Social phenomena in the context of \mathcal{R} -operator

In the object language with the \mathcal{R} -operator it is possible to talk about sentences and points of their realization. However, social processes take place not only in time but also (similarly to physical processes in Łoś's logic) in more complex contexts. Even the physical interpretation of Łoś requires that the position in the realization operator is composed of a time and space parameter. It can look like this " $\mathcal{R}_{t, \langle x, y, z \rangle}(p)$ ", where t , $\langle x, y, z \rangle$ is a time-space context, an event is described by a sentence p , while t is a time context, and $\langle x, y, z \rangle$ are three dimensions of physical space. When positions have set places under the operator \mathcal{R} , it can be presented as " $\mathcal{R}_{t, x, y, z}(p)$ ".

As mentioned before, social complexity has two dimensions. One is the quantity of components, the other is the process of the interlocking of these components considered from the humanistic perspective. There is a certain similarity of this proposal to pragmatic contexts of statement interpretation. Dana Scott in his [30] proposed understanding of the pragmatic context of a statement interpretation as an n -ary ordered set $\langle w, t, \langle x, y, x \rangle, a, \dots \rangle$, where w is a possible world, t is time, $\langle x, y, x \rangle$ is a place which interpretation refers to, while a, \dots is a set of other parameters which are necessary for a given utterance to become a logical proposition that is equipped with a certain logical value (for example who says, to whom etc.). It is an attempt to formalize a notion of context, which helps to pass from an utterance like *It rains* to a sentence that has a logical value.

In order to express complex social contexts, and with a mechanical, quantitative understanding of complexity, one must assume that positions in the scope of the operator \mathcal{R} are similar to Scott's determinants of a pragmatic context. The statement " $\mathcal{R}_{x_1, \dots, x_n}(p)$ " means that a phenomenon described by a sentence p has happened in the context of:

- institution x_1
- organization x_2
- social group x_3
- place x_4
- time x_5

- position in a group x_6
- social role x_7
- interaction x_8
- individual x_9
- culture x_{10}
- \vdots
- variable x_n — ready to be interpreted.

With this, in a single description we get the rules of the complexity of a phenomenon. If social theories were expressed in positional logic, contexts could be determined and variables x_1, \dots, x_n could reflect social-world properties, where an event described by the operator of realization takes place.

On the other hand the nesting of the operator of realization allows the inclusion of agential aspects of social phenomena, which is the second dimension of complexity. Hence, in our formal language, objective complexity can be expressed (by showing an aspect of knowledge, beliefs, position in a group etc.), but in the same moment the humanistic coefficient is considered and the aspect of interpretations of social phenomena by their participants is added.

For example, the way participants in a stock exchange perceive its condition affects its further condition and performance. Similar remarks apply to banks or any other financial institution. A description of social phenomena is accurate when not only participants (individuals, groups, institutions) are described, but also when their beliefs about contexts are included. Therefore, we deal with social phenomena that happen within other phenomena (additionally we have a certain feedback loop). This problem was well described (although without any formalization) by Robert Merton in the self-fulfilling prophecy concept. Merton's idea is rooted in the sociological category of *the definition of the situation* proposed by the cooperator of Znaniecki, William Thomas, who wrote that "If men define situation as real, they are real in their consequences" [38]. Hence it is possible for false subjective beliefs of individuals to turn into objective truths. Merton shows that in a story of bank bankruptcy: false assumptions about reality affect people's activity, which turns these assumptions into truthful ones. Assuming that a bank will collapse, we withdraw our money and at the same time we lower the bank capital and speed up its real bankruptcy [22].

Therefore it seems that an iteration of operator \mathcal{R} is required, for its multiple applications with nesting. For example, consider the expression:

$$\mathcal{R}_{s,b}(\mathcal{R}_{o,sh,b'}(\mathcal{R}_s(p) \wedge R_{s,sh}(\neg p)))$$

that, for example, can be read as follows:

- on stock exchange s there is belief b that owner o of shares sh thinks (has got belief b') that stock exchange s will go down (p), but shares sh on stock exchange s will not go down ($\neg p$),

when we accordingly interpret positions s , b , o , sh , b' .

On the other hand the utterance:

$$\mathcal{R}_{s,b}(\mathcal{R}_{o,sh,b'}(\mathcal{R}_s(p) \wedge R_{s,sh}(\neg p))) \wedge \mathcal{R}_{s',b''}(\mathcal{R}_{o,sh,b'''}(\mathcal{R}_s(p) \wedge R_{s,sh}(p)))$$

complements the above with the belief that (the right conjunction argument):

- on stock exchange s' it is thought (belief b'') that owner o of shares sh thinks (belief b''') that stock exchange s will go down (p) and shares sh on stock exchange s will go down (p), too,

where, additionally, we accordingly interpret positions s' , b'' , b''' ,

What if we would like to express that the last utterance (belief b'''') belongs to owner o' in the context of stock exchange s' ? Our language allows that. It could look like this:

$$\mathcal{R}_{o',s,b''''}(\mathcal{R}_{s,b}(\mathcal{R}_{o,sh,b'}(\mathcal{R}_s(p) \wedge R_{s,sh}(\neg p)))) \wedge \mathcal{R}_{s',b''}(\mathcal{R}_{o,sh,b'''}(\mathcal{R}_s(p) \wedge R_{s,sh}(p))).$$

The iteration of contexts, especially nesting, is a basic tool for solving the problem of humanistic coefficient in the formalization of this type of presumption. Each sentence describing social complexity can be expressed in a certain social context. Although the operator \mathcal{R} can be nested finitely many times – expressions of positional logic dedicated to social science are a limited string of symbols – there is no limit to the levels of nesting. In the language that we proposed, any social phenomenon can be considered from a broader context.

The problem of nesting and iteration of the operator \mathcal{R} can be found in the literature about positional logic. Most often, it is presented in the perspective of the temporal interpretation of the operator of realization \mathcal{R} in the renowned works [26, 27]. There, the semantics of certain positional logics identifies positions as numbers. So, iterations can be

reduced to arithmetic operations. For example, when a sentence has two moments of time “ $\mathcal{R}_{t_1}\mathcal{R}_{t_2}(A)$ ” with an assumption that t_1 and t_2 denote numbers from a set closed under addition $+$, we can reduce the above utterance to the expression “ $\mathcal{R}_{t_1+t_2}(A)$ ”. For example, it can mean that, if: *in two days (t_1) there is that in three days (t_2) there will be that A , so in five days ($t_1 + t_2$) there will be that A .*

The context has got a temporal aspect; an important one but not the only and most significant one. Therefore, we need more general interpretations of the nesting of contexts than arithmetical ones. In [12] there is a review of approaches to nesting \mathcal{R} and proposals for new solutions.

Generalizations related to complex, social contexts—such as positions $\langle x_1, \dots, x_n \rangle$, where all or some parameters x_i , $1 \leq i \leq n$, can appear in other nestings—have never been considered before. However this requires further studies and logical examinations.

6. Perspective of future research

The broadening of positional logic language with positions for complex contexts, as well as their iterations, makes it possible to describe social systems with their ontological (*mechanical*) and humanistic complexity. This will be of use to theories that seek to describe complex social phenomena. It will permit more accurate modelling of contexts in which many agents participate in collective action.

However, it is possible that in order to investigate such systems, it might be necessary to introduce non-classical logic mechanisms such as non-classical reasoning.

It is worthwhile to keep a classical understanding of connectives, at least outside of the \mathcal{R} operator’s range. Simultaneously, sentences about occurring phenomena can be uncertain, i.e. neither completely true nor completely false but possessed of some degree of truth (or certainty). Therefore, we could introduce the notion of a degree of certainty of a phenomenon occurrence. Let $v_0, \dots, v_i, \dots, v_m$ mean an order of certainty; when v_0 means the phenomenon does not occur at all, v_m means it surely occurs. The symbol v_i , where $0 < i < m$, means that the phenomenon occurs in a certain degree (of truth or certainty) between the classical values.

As a consequence of our deliberations we can introduce to the language of our logic expressions “ $\mathcal{R}_{x_1, \dots, x_n, v_i}(A)$ ”, where x_1, \dots, x_n are social contexts where a phenomenon occurs and is described by a sentence A , while v_i ($1 \leq i \leq m$) denotes the logical value v_i of a sentence “ $\mathcal{R}_{x_1, \dots, x_n}(A)$ ”. With this we get a complex logic. It is classical at the object-level, but multi-valued or fuzzy at the level of nested positions.

Another important tool for describing social phenomena is probability. It is quite similar to many-valuedness, although it is not the same. In the probability approach, we also assign numbers from the interval $[0, 1]$ to phenomena. However, probability and many-valuedness differ on a level of meaning. Nevertheless the logic that is considered here allows us to include a position for probability measurement in the range of operator \mathcal{R} . With this we get a logic with a probabilistic interpretation of social phenomena, but it requires some changes.

To sum up, in this article we are setting an agenda. It seems that the potential to use Łoś's logic for modeling social processes is extensive. The possibility of expressing the complexity of social reality, its multi-dimensionality and correlations between agents is crucial for the general attractiveness of such a formalization, especially for future developments and improvements of agent based systems in sociology and the MAS analysis, which constantly seek ways to implement the reflexivity of agents.

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