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A Realistic View on Normative Conflicts

Abstract. Kulicki and Trypuz (2016) introduced three systems of multivalued deontic action logic to handle normative conflicts. The first system suggests a pessimistic view on normative conflicts, according to which any conflicting option represents something forbidden; the second system suggests an optimistic view, according to which any conflicting option represents something obligatory; finally, the third system suggests a neutral view, according to which any conflicting option represents something that is neither obligatory nor forbidden. The aim of the present paper is to propose a fourth system in this family, which comes with a realistic view on normative conflicts: a normative conflict remains unsolved unless it is generated by two or more normative sources that can be compared. In accordance with this, we will provide a more refined formal framework for the family of systems at issue, which allows for explicit reference to sources of norms. Conflict resolution is thus a consequence of a codified hierarchy of normative sources.

Keywords: action; deontic logic; deontic value; multivalued logic; normative conflict

1. Introduction

Normative conflicts are situations in which an agent is required to perform two or more actions that are mutually incompatible.[1] Normative

[1] For a more refined notion of normative conflict (see Hill, 1987). In this article we will not distinguish normative conflicts from normative dilemmas (cf. Dyrkolbotn et al., 2016, and Brink, 1994, where such distinction is employed); nor will we distinguish between the properties of being normative and being moral. The notion of being normative is broader than that of being moral, and the notion of conflict is broader

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conflicts are widespread in ordinary life.[2] They often stem from different normative sources (as in the tragic story of Antigone in Sophocles' drama), but there are also normative conflicts stemming from a single normative source (as it is when you promise to meet two friends at the same time in different places, or when a self-driving car arrives at a situation when causing harm to either passengers or pedestrians is unavoidable). Some of these conflicts can be 'resolved', by deriving *all-things-considered* duties from *prima facie* duties on a preferential basis (i.e., under the assumption that some *prima facie* duties are of higher importance than others), but this is not in general the case. Even if a conflict cannot be resolved, it is important to detect its presence, so that the (interpretation of the) normative sources at issue can be revised.

Normative conflicts can be expressed in the language of *Standard Deontic Logic* (**SDL**) as formulas of kind $O\phi \wedge F\psi$ or $P\phi \wedge F\psi$, where:

- ϕ and ψ represent two propositions such that either (I) ϕ logically entails ψ or (II) ψ logically entails ϕ;
- O, P and F are, respectively, operators of obligation, permission and forbiddance.

Actually, since in **SDL** the operators P and F are definable in terms of the operator O and boolean negation, one can rewrite the two types of normative conflict as $O\phi \wedge O\neg\psi$ and $\neg O\neg\phi \wedge O\neg\psi$. Given the logical relation between ϕ and ψ, the second schema immediately leads to a contradiction. Furthermore, in systems as strong as **SDL**, the first schema leads to a contradiction as well, since, for every formula ϕ, $O\phi \rightarrow \neg O\neg\phi$ is derivable; therefore, one gets $\neg O\neg\phi \wedge O\neg\psi$ also in this case. In this work we will focus on conflicts involving an obligation and a prohibition.

In order to avoid that normative conflicts always collapse to contradictions, many alternative deontic logics have been devised, such as adaptive deontic logic (see Beirlaen, Straßer, and Meheus, 2013; Goble, 2014), paraconsistent deontic logic (see da Costa and Carnielli, 1986),

than the notion of dilemma, so the term *normative conflict* is more general than others.

[2] Despite this, philosophers sometimes question their existence, see, for instance, (Brink, 1994). However, Brink employs a stronger notion of normative conflict (normative conflict as a conflict of all-things-considered moral obligations which produces paradoxes in ethical theory). We are more sympathetic to the opposite approach, which acknowledges the existence of normative conflicts, as suggested in (Sinnott-Armstrong, 1984).

defeasible deontic logic (see Nute, 1997), and so forth; a detailed survey can be found in (Goble, 2013).

Recently, Kulicki and Trypuz (2016) introduced three systems of multivalued deontic action logic (see also Kulicki and Trypuz, 2019, where a Prolog implementation is provided), which respectively convey a *pessimistic*, *optimistic* and *neutral* view on normative conflicts. Here a fourth sibling of these systems will be introduced, which conveys a *realistic* view on normative conflicts. The meaning of the labels attached to the four systems will be explained in Section 2. In Section 3, our fourth system will be informally motivated and formally presented. In Section 4, we will outline the soundness and completeness proofs for the new system. Finally, in Section 5, we will provide final remarks aimed at clarifying in which sense the proposed system conveys a realistic view on normative conflicts.

2. Multivalued deontic logics

Kulicki and Trypuz's investigations on systems of multivalued deontic action logic are motivated by the existence of normative conflicts. In (Kulicki and Trypuz, 2016), the focus is mainly on classical examples of moral dilemmas, such as the following scenario described by Sophocles: Antigone's two brothers, Eteocles and Polyneices died fighting against each other in order to take control over the city of Thebes. According to the new ruler of the city, Creon, Eteocles deserves a burial, since he died while defending Thebes, whereas Polyneices does not, since he died while attacking Thebes. However, Antigone believes that she should bury her brother Polyneices as well, because this is prescribed by the gods. Antigone is thus under a prohibition to bury Polyneices (by the laws of the city), but at the same time under an obligation to bury him (by the divine laws). Whichever decision Antigone makes, she ends up doing something obligatory from one point of view and forbidden from another. More recent examples of normative conflicts, such as those faced by a self-driving car, are extensively discussed within the same logical framework in (Kulicki and Trypuz, 2019).

The three logical systems proposed in (Kulicki and Trypuz, 2016) to deal with normative conflicts are multivalued in the sense of employing multiple *deontic values*, rather than multiple truth-values. Deontic values are ascribed to actions. The first two systems employ three deontic values, o (obligatory), n (neutral; neither obligatory nor forbidden), and

f (forbidden). The first system proposes a pessimistic view: when combining (aggregating) deontic values o and f, value f overrides value o. The second system suggests an optimistic view: in case of aggregation, value o overrides value f. The last system aims to neutralize the conflicting elements and attempts to get rid of the tragic flavour. A fourth deontic value, c (conflicting; both obligatory and forbidden), is added.[3] In this approach, the aggregation of the deontic values o and f results in the value c. Yet both c and n are regarded as *deontically neutral*, since "[b]oth of them are neither 'purely' obligatory nor 'purely' forbidden, and in that sense are neutral" (see Kulicki and Trypuz, 2016, p. 133). To sum up:

1. The *pessimistic view* says that an action obligatory from one point of view and forbidden from the other is finally regarded as forbidden (so Antigone will do something forbidden);
2. The *optimistic view* says that an action obligatory from one point of view and forbidden from the other is finally regarded as obligatory (so Antigone will do something obligatory);
3. The *neutral view* says that an action obligatory from one point of view and forbidden from another is finally regarded as neither purely obligatory nor purely forbidden, and thus neutral (so Antigone will do something normatively neutral).[4]

The three systems are shown to be sound and complete with respect to a semantics in terms of matrices.[5] We can present the language of these systems, hereafter denoted by $\mathcal{L}$, starting from a countable set $A = \{a_1, a_2, a_3, \ldots\}$ of *atomic action-types*.[6] The set A^* of *well-formed action-types* is defined by the following grammar:

$$\alpha ::= a_i \mid \overline{\alpha} \mid \alpha \sqcap \alpha$$

where $a_i \in A$, $\overline{\alpha}$ is the complement of action-type α ('not doing α') and $\alpha \sqcap \beta$ is the aggregation of action-types α and β ('doing α and

[3] In this paper we use a slightly different notation for deontic values: n stands for the value $\bot$ and c for the value $\top$.

[4] Kulicki and Trypuz use a different caption for the third system: *in dubio quodlibet*.

[5] Actually, there is an axiom which is not valid with respect to the intended semantics (no. 30, on p. 134). Yet it can be corrected easily as shown in (Glavaničová, 2017, p. 258).

[6] See (von Wright, 1951) for the distinction between action-types and action-tokens and the prominent role of the former in deontic reasoning.

β'). Kulicki and Trypuz (2016, p. 128) explain this as α and β being "types of actions coming from different normative systems" and say that $\alpha \sqcap \beta$ refers to "the same action when we express its final deontic status after merging the normative systems". We think that this interpretation is too narrow for the operator $\sqcap$, since one may also form expressions of kind $\alpha \sqcap \alpha$, which make reference to an action coming from a single normative system. Thus, in the subsequent text we will opt for a broader interpretation of $\sqcap$, according to which $\alpha \sqcap \beta$ tells us that actions of types α and β are performed.

The set of well-formed formulas in $\mathcal{L}$ is defined by the grammar below:

$$\phi ::= O\alpha \mid \neg\phi \mid \phi \wedge \phi$$

where $\alpha \in A^*$, O is an operator of obligation (s.t. $O\alpha$ means 'α is obligatory') and $\neg$ and $\wedge$ are the usual boolean connectives for negation and conjunction. Additional boolean operators can be defined in the standard way; moreover, we have $P\alpha$ ('α is permitted'), $F\alpha$ ('α is forbidden') and $N\alpha$ ('α is neutral') as abbreviations for $\neg O\overline{\alpha}$, $O\overline{\alpha}$ and $\neg O\alpha \wedge \neg O\overline{\alpha}$, respectively. Semantically, one can express the intuition at the basis of the three systems proposed by Kulicki and Trypuz with a valuation function DV which maps atomic action-types to values either in the set $\{o, f, n\}$ (first two systems) or in the set $\{o, f, n, c\}$ (third system) and can be extended to an assignment of deontic values to all well-formed action-types such that:

- in the pessimistic system, $DV(\alpha \sqcap \beta) = f$ whenever $DV(\alpha) = o$ and $DV(\beta) = f$;
- in the optimistic system, $DV(\alpha \sqcap \beta) = o$ whenever $DV(\alpha) = o$ and $DV(\beta) = f$;
- in the neutral system, $DV(\alpha \sqcap \beta) = c$ whenever $DV(\alpha) = o$ and $DV(\beta) = f$.

The full description of the way in which deontic values are assigned to complex action-types in the three systems is here not reproduced, for the sake of brevity.

Formulas of $\mathcal{L}$ are evaluated in a bivalent semantics. Let $ATOM = \{O\alpha : \alpha \in A^*\}$ be the set of atomic formulas in $\mathcal{L}$ and V a valuation function assigning to each atomic formula a value in the set $\{0, 1\}$; then, in the first and the second system deontic formulas have the truth-conditions described in Table 1.

Truth-conditions for formulas in the third system will be provided in the next section, when we will present a fourth system in this family.

α	$F\alpha$	$N\alpha$	$O\alpha$	$P\alpha$
f	1	0	0	0
n	0	1	0	1
o	0	0	1	1

Table 1. Truth-conditions for deontic formulas in the first two systems

Our fourth system agrees with the neutral system in assigning deontic value c to the aggregation of two action-types α and β having deontic values o and f; however, we will see that, differently from the neutral system, our fourth system keeps track of the conflict displayed at the syntactic level.

The idea at the basis of the *realistic view* on normative conflicts that will be proposed here is that in ordinary life a conflict between an obligation and a prohibition always remains theoretically unsolved, unless the two come from different sets of norms and a preference relation among these sets of norms is established.

The realistic view does not eliminate the conflicting feature of Antigone's scenario, but it does not result in a *normative explosion* either (where a normative explosion is understood as a derivation of any normative statement whatsoever from two conflicting ones). Even if we can sometimes eliminate the 'guilt' in case of a normative conflict, i.e. claim that an agent should not be considered guilty for her failure to observe all norms involved (e.g., when making responsibility judgments, as it is shown in (Glavaničová and Pascucci, 2019)), we may also have good reasons to preserve the normative conflict and just preclude the normative explosion. Indeed, pointing out that some sets of norms are jointly inconsistent is a useful practice when no general criterion is available to solve certain normative conflicts. For instance, suppose that a computer engineer is testing a software to be used for self-driving cars and that several executions of a program lead to alternative outputs corresponding to mutually incompatible instructions sent to the vehicle in a given scenario; then, the first thing that the engineer should do is to point out that there is some inconsistency in the set of instructions. This is a crucial step even if the engineer is not yet aware of any criterion to solve the specific kind of conflict that occurred in the executions.

If we represent such ideas in a logical system for deontic reasoning, so that an action which is obligatory and forbidden at the same time turns out to be conflicting, then we have a formal tool to show that something went wrong in a scenario where the action at issue was performed. This can motivate attempts to overcome the conflict in analogous scenarios occurring in the future; for instance, it can be a reason to introduce further preference criteria among sets of norms. This is a practical aspect with respect to which our view fundamentally diverges from the neutral view proposed by Kulicki and Trypuz; indeed, if the combination of two normatively conflicting statements is seen as neutral, then there is *no reason for further adjustment of (preferences among) normative sources*. Additionally, under the neutral view there is a conceptual risk that normative conflicts collapse to *normatively irrelevant scenarios*. For instance, Richard may have a walk through the centre of Bayreuth today or not and we can consider his decision as normatively irrelevant, hence neutral, by default. There is nothing wrong with this kind of normative neutrality. On the other hand, if Richard promised his friends to be in the centre of Bayreuth at 3 p.m., and promised his supervisor to be at the university at the same time, being in the city centre is for him both obligatory and forbidden. This situation is different from the former one. It is not neutral: the action in question has conflicting deontic values according to two different normative sources (promises made to friends and promises made to supervisors) and Richard has a motivation to find a way to resolve similar conflicts in the future. In the end it is his fault; he agreed to have two appointments at different places at the same time. In the future, he might think about establishing some criteria of preference among the two sources in order to get a solution to the conflict.

3. The realistic view

There are normative conflicts which arise from different normative sources and normative conflicts which arise from a single normative source. In our opinion, it is fundamental to keep the two sorts of conflicts distinct in logical analysis, since they have different consequences from a practical point of view. Indeed, in the case of a conflict arising from a single normative source s, it is in principle impossible to act in accordance with s; therefore, one has either to revise s by changing some norms in it or to extend s with a rule able to handle internal conflicts, such as a rule obtained by adopting a criterion of preference *among specific norms*. In the case of a conflict arising from two normative sources s_1 and s_2,

α	$\overline{\alpha}$
f	o
n	n
c	c
o	f

$\sqcap$	f	n	c	o
f	f	f	c	c
n	f	n	c	o
c	c	c	c	c
o	c	o	c	o

Table 2. Deontic values of complex actions within a normative source

instead, it is in principle possible to act either in accordance with s_1 or in accordance with s_2, though it is not possible to act in accordance with both; thus, one has to adopt a general criterion of preference *among normative sources.*

In order to capture this intuition, we will here modify the framework in (Kulicki and Trypuz, 2016) by introducing more complex semantic structures including different normative sources.[7] Let $N = \langle S, \prec \rangle$ be called a *normative structure* in which $S = \{s_1, \ldots, s_n\}$ is a finite set of *normative sources* and $\prec$ a strict partial order over S; $\prec$ will be said to be a *preference relation* among normative sources. In the current framework we want deontic values of action-types to be relative to a specific normative source. More precisely, let $DV\colon A \times S \longrightarrow \{o, f, n, c\}$, where A is the set of atomic action-types and S the set of normative sources. According to the present definition of DV, it can be the case that, given $s, s' \in S$ such that $s \neq s'$ and $a \in A$, $DV(a, s) \neq DV(a, s')$. The deontic value of complex action-types *with respect to the same normative source* is computed exactly as in Kulicki and Trypuz's third system. We reproduce the relevant matrices in Table 2.

Notice that the operations $-$ and $\sqcap$ over A^* are not boolean; for instance, they falsify the boolean law $\alpha = (\overline{\beta \sqcap \overline{\alpha}}) \sqcap (\overline{\overline{\beta} \sqcap \overline{\alpha}})$, when $DV(\alpha, s) = f$ and $DV(\beta, s) = c$. In this framework, rather than having a generic operator of obligation O, we introduce a set of *indexed operators of obligation* O_{s_i}, for any $s_i \in S$. Furthermore, we will use operators of kind $O_{s_i \times s_j}$, for any $s_i, s_j \in S$ in order to represent obligations arising from the combination of two normative sources, namely obligations which arise once the norms in the two sources at issue have

[7] Note, however, that Kulicki and Trypuz prefer a simpler notation without the explicit mention of normative sources (see Kulicki and Trypuz, 2019). Except for simplicity, they motivate the choice in terms of its similarity to the usual language of deontic action logic.

$DV(\alpha,\sigma)$	$F_\sigma\alpha$	$N_\sigma\alpha$	$C_\sigma\alpha$	$O_\sigma\alpha$	$P_\sigma\alpha$
f	1	0	0	0	0
n	0	1	0	0	1
c	1	0	1	1	0
o	0	0	0	1	1

Table 3. Evaluation of deontic formulas

been merged. This is done in the spirit of (Jacquette, 1996) and follows some hints provided in (Kulicki and Trypuz, 2016, p. 128). A combination of two normative sources can be taken as a new *complex normative source* and the evaluation of deontic formulas with reference to it agrees with the criteria provided above. Finally, by iterating combinations of normative sources, one gets operators of kind O_σ, where σ is an arbitrary (finite) combination of normative sources, for instance, $\sigma = ((s_i \times s_j) \times s_k) \times (s_l \times s_m)$.

We can stipulate the following definitions:

- $P_\sigma\alpha := \neg O_\sigma\overline{\alpha}$;
- $F_\sigma\alpha := O_\sigma\overline{\alpha}$;
- $N_\sigma\alpha := \neg O_\sigma\alpha \wedge \neg O_\sigma\overline{\alpha}$;
- $C_\sigma\alpha := O_\sigma\alpha \wedge O_\sigma\overline{\alpha}$.

Truth-conditions for deontic formulas making reference to the same normative source are specified in Table 3.

A relevant feature of the enriched framework provided here is the relation between the way in which deontic values are assigned in a complex normative source and the way in which they are assigned in its components. Given that the preference relation $\prec$ constitutes a strict partial order over the set of normative sources, some normative sources are preferable to others and some normative sources are not comparable among them. Given any two normative sources σ and ρ, let $comp(\sigma,\rho)$ be an abbreviation for $(\sigma \prec \rho) \vee (\rho \prec \sigma)$ and $pref(\sigma,\rho)$ denote the normative source which is preferable among σ and ρ, in case $comp(\sigma,\rho)$ holds. Finally, let $DV(\alpha,\prec)/x$ be an abbreviation for "$DV(\alpha, pref(\sigma,\rho))$ if $comp(\sigma,\rho)$, otherwise x". The preference relation interacts with the combination of normative sources as in Table 4.

The axiomatic basis of the logical system used to represent the realistic view (hereafter simply denoted by S) is the following:

A0 any substitution instance of a tautology of Propositional Calculus;

	$DV(\alpha,\sigma)=f$	$DV(\alpha,\sigma)=n$	$DV(\alpha,\sigma)=c$	$DV(\alpha,\sigma)=o$
$DV(\alpha,\rho)=f$	f	$DV(\alpha,\prec)/f$	$DV(\alpha,\prec)/c$	$DV(\alpha,\prec)/c$
$DV(\alpha,\rho)=n$	$DV(\alpha,\prec)/f$	n	$DV(\alpha,\prec)/c$	$DV(\alpha,\prec)/o$
$DV(\alpha,\rho)=c$	$DV(\alpha,\prec)/c$	$DV(\alpha,\prec)/c$	c	$DV(\alpha,\prec)/c$
$DV(\alpha,\rho)=o$	$DV(\alpha,\prec)/c$	$DV(\alpha,\prec)/o$	$DV(\alpha,\prec)/c$	o

Table 4. Merging of deontic values from two normative sources

R0 if $\vdash_S \phi$ and $\vdash_S \phi \to \psi$, then $\vdash_S \psi$;
A1 $O_\sigma \alpha \to (O_\sigma(\alpha \sqcap \beta) \wedge O_\sigma(\beta \sqcap \alpha))$;
A2 $O_\sigma(\alpha \sqcap \beta) \to (O_\sigma \alpha \vee O_\sigma \beta)$;
A3 $O_\sigma \overline{\alpha} \to (O_\sigma(\overline{\alpha \sqcap \beta}) \wedge O_\sigma(\overline{\beta \sqcap \alpha}))$;
A4 $O_\sigma(\overline{\alpha \sqcap \beta}) \to (O_\sigma \overline{\alpha} \vee O_\sigma \overline{\beta})$;
A5 whenever $\sigma \prec \rho$, $(O_\sigma \alpha \equiv O_{\sigma\times\rho}\alpha) \wedge (O_\sigma \alpha \equiv O_{\rho\times\sigma}\alpha)$;
A6 whenever $\neg comp(\sigma,\rho)$, $O_\sigma \alpha \to (O_{\sigma\times\rho}\alpha \wedge O_{\rho\times\sigma}\alpha)$;
A7 whenever $\neg comp(\sigma,\rho)$, $O_{\sigma\times\rho}\alpha \to (O_\sigma \alpha \vee O_\rho \alpha)$;
A8 $O_\sigma \alpha \equiv O_\sigma \overline{\overline{\alpha}}$.

Notice that this axiomatic basis *internalizes some properties of complementation and aggregation*, whereas in (Kulicki and Trypuz, 2016) the relevant properties of these operations are represented as equivalences separated from the deontic axioms (e.g., $\alpha = \overline{\overline{\alpha}}$). An advantage of our approach is that we can provide a uniform axiomatic basis in which no metalinguistic relation (such as the equivalence relation among action types, $=$) is employed.

Before moving to the semantic characterization of the system S, let us briefly analyze Antigone's scenario in terms of it. Burying Polyneices (a) is forbidden according to a normative source s_1 and obligatory according to another normative source s_2; thus, we have $O_{s_1} a \wedge O_{s_2} \overline{a}$. Since the dilemma arises under the assumption that the two normative sources are not comparable, then, by A6, we get $O_{s_1\times s_2} a \wedge O_{s_1\times s_2} \overline{a}$ (as well as $O_{s_2\times s_1} a \wedge O_{s_2\times s_1} \overline{a}$), which witnesses that the conflict remains unsolved.

4. Soundness and completeness of the realistic view

For the soundness part it is easy to check that A0–A8 and R0 are valid schemata and rules with respect to the semantics provided (Tables 1–4). For the completeness result, we adopt the same technique of (Kulicki, 2014).

The first step consists in representing in $\mathcal{L}$ the four deontic values that can be assigned to an action with reference to a normative source. If $x \in \{f, n, c, o\}$, then $\psi_{DV(\alpha,\sigma):x} \in \mathcal{L}$ is the formula codifying the claim that the deontic value assigned by function DV to action-type α with reference to normative source σ is x. We adopt the following definition:

- $\psi_{DV(\alpha,\sigma):f} = O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha$;
- $\psi_{DV(\alpha,\sigma):n} = \neg O_\sigma\alpha \wedge \neg O_\sigma\overline{\alpha}$;
- $\psi_{DV(\alpha,\sigma):c} = O_\sigma\alpha \wedge O_\sigma\overline{\alpha}$;
- $\psi_{DV(\alpha,\sigma):o} = O_\sigma\alpha \wedge \neg O_\sigma\overline{\alpha}$.

Thus, in a normative source σ an action α receives the deontic value 'forbidden' if and only if $O_\sigma\overline{\alpha}$ is true and $O_\sigma\alpha$ is false; in σ α receives the deontic value 'neutral' if and only if both $O_\sigma\overline{\alpha}$ and $O_\sigma\alpha$ are false; in σ α takes the deontic value 'conflicting' if and only if both $O_\sigma\overline{\alpha}$ and $O_\sigma\alpha$ are true; finally, in σ α takes the deontic value 'obligatory' if and only if $O_\sigma\overline{\alpha}$ is false and $O_\sigma\alpha$ is true.

The second step consists in showing that our translation of the deontic values in $\mathcal{L}$ is adequate to mirror the semantics of action complement and action aggregation. In the case of action complement, we need to show that if x and y are, respectively, the deontic values assigned by function DV to action-types α and $\overline{\alpha}$ with reference to normative source σ, then $\vdash_S \psi_{DV(\alpha,\sigma):x} \rightarrow \psi_{DV(\overline{\alpha},\sigma):y}$, namely:

- $(O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha) \rightarrow (O_\sigma\overline{\alpha} \wedge \neg O_\sigma\overline{\overline{\alpha}})$ by A8;
- $(\neg O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha) \rightarrow (\neg O_\sigma\overline{\alpha} \wedge \neg O_\sigma\overline{\overline{\alpha}})$ by A8;
- $(O_\sigma\overline{\alpha} \wedge O_\sigma\alpha) \rightarrow (O_\sigma\overline{\alpha} \wedge O_\sigma\overline{\overline{\alpha}})$ by A8;
- $(\neg O_\sigma\overline{\alpha} \wedge O_\sigma\alpha) \rightarrow (\neg O_\sigma\overline{\alpha} \wedge O_\sigma\overline{\overline{\alpha}})$ by A8.

In the case of action aggregation, we need to show that if x, y and z are, respectively, the deontic values assigned by function DV to action-types α, β and $\alpha \sqcap \beta$ with reference to normative source σ, then $\vdash_S (\psi_{DV(\alpha,\sigma):x} \wedge \psi_{DV(\beta,\sigma):y}) \rightarrow \psi_{DV(\alpha\sqcap\beta,\sigma):z}$, namely:

- $((O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha) \wedge (O_\sigma\overline{\beta} \wedge \neg O_\sigma\beta)) \rightarrow (O_\sigma(\overline{\alpha \sqcap \beta}) \wedge \neg O_\sigma(\alpha \sqcap \beta))$ by A2, A3;
- $((O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha) \wedge (\neg O_\sigma\overline{\beta} \wedge \neg O_\sigma\beta)) \rightarrow (O_\sigma(\overline{\alpha \sqcap \beta}) \wedge \neg O_\sigma(\alpha \sqcap \beta))$ by A2, A3;
- $((O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha) \wedge (O_\sigma\overline{\beta} \wedge O_\sigma\beta)) \rightarrow (O_\sigma(\overline{\alpha \sqcap \beta}) \wedge O_\sigma(\alpha \sqcap \beta))$ by A1, A3;
- $((O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha) \wedge (\neg O_\sigma\overline{\beta} \wedge O_\sigma\beta)) \rightarrow (O_\sigma(\overline{\alpha \sqcap \beta}) \wedge O_\sigma(\alpha \sqcap \beta))$ by A1, A3;
- $((\neg O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha) \wedge (O_\sigma\overline{\beta} \wedge \neg O_\sigma\beta)) \rightarrow (O_\sigma(\overline{\alpha \sqcap \beta}) \wedge \neg O_\sigma(\alpha \sqcap \beta))$ by A2, A3;
- $((\neg O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha) \wedge (\neg O_\sigma\overline{\beta} \wedge \neg O_\sigma\beta)) \rightarrow (\neg O_\sigma(\overline{\alpha \sqcap \beta}) \wedge \neg O_\sigma(\alpha \sqcap \beta))$ by A2, A4;
- $((\neg O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha) \wedge (O_\sigma\overline{\beta} \wedge O_\sigma\beta)) \rightarrow (O_\sigma(\overline{\alpha \sqcap \beta}) \wedge O_\sigma(\alpha \sqcap \beta))$ by A1, A3;
- $((\neg O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha) \wedge (\neg O_\sigma\overline{\beta} \wedge O_\sigma\beta)) \rightarrow (\neg O_\sigma(\overline{\alpha \sqcap \beta}) \wedge O_\sigma(\alpha \sqcap \beta))$ by A1, A4;
- $((O_\sigma\overline{\alpha} \wedge O_\sigma\alpha) \wedge (O_\sigma\overline{\beta} \wedge \neg O_\sigma\beta)) \rightarrow (O_\sigma(\overline{\alpha \sqcap \beta}) \wedge O_\sigma(\alpha \sqcap \beta))$ by A1, A3;

- $((O_\sigma\overline{\alpha} \wedge O_\sigma\alpha) \wedge (\neg O_\sigma\overline{\beta} \wedge \neg O_\sigma\beta)) \to (O_\sigma(\overline{\alpha \sqcap \beta}) \wedge O_\sigma(\alpha \sqcap \beta))$ by A1, A3;
- $((O_\sigma\overline{\alpha} \wedge O_\sigma\alpha) \wedge (O_\sigma\overline{\beta} \wedge O_\sigma\beta)) \to (O_\sigma(\overline{\alpha \sqcap \beta}) \wedge O_\sigma(\alpha \sqcap \beta))$ by A1, A3;
- $((O_\sigma\overline{\alpha} \wedge O_\sigma\alpha) \wedge (\neg O_\sigma\overline{\beta} \wedge O_\sigma\beta)) \to (O_\sigma(\overline{\alpha \sqcap \beta}) \wedge O_\sigma(\alpha \sqcap \beta))$ by A1, A3;
- $((\neg O_\sigma\overline{\alpha} \wedge O_\sigma\alpha) \wedge (O_\sigma\overline{\beta} \wedge \neg O_\sigma\beta)) \to (O_\sigma(\overline{\alpha \sqcap \beta}) \wedge O_\sigma(\alpha \sqcap \beta))$ by A1, A3;
- $((\neg O_\sigma\overline{\alpha} \wedge O_\sigma\alpha) \wedge (\neg O_\sigma\overline{\beta} \wedge \neg O_\sigma\beta)) \to (\neg O_\sigma(\overline{\alpha \sqcap \beta}) \wedge O_\sigma(\alpha \sqcap \beta))$ by A1, A4;
- $((\neg O_\sigma\overline{\alpha} \wedge O_\sigma\alpha) \wedge (O_\sigma\overline{\beta} \wedge O_\sigma\beta)) \to (O_\sigma(\overline{\alpha \sqcap \beta}) \wedge O_\sigma(\alpha \sqcap \beta))$ by A1, A3;
- $((\neg O_\sigma\overline{\alpha} \wedge O_\sigma\alpha) \wedge (\neg O_\sigma\overline{\beta} \wedge O_\sigma\beta)) \to (\neg O_\sigma(\overline{\alpha \sqcap \beta}) \wedge O_\sigma(\alpha \sqcap \beta))$ by A1, A4.

The third step consists in showing that our translation of deontic values in $\mathcal{L}$ is adequate to mirror the semantics of normative source merging ($\times$). We need to show that if x, y and z are, respectively, the deontic values assigned by function DV to action-type α with reference to normative sources σ, ρ and $\sigma\times\rho$, then $\vdash_S (\psi_{DV(\alpha,\sigma):x} \wedge \psi_{DV(\alpha,\rho):y}) \to \psi_{DV(\alpha,\sigma\times\rho):z}$. There are two cases to be considered, depending on whether the two normative sources combined via $\times$ are comparable or not in terms of the preference relation $\prec$. We start with the case in which they are not comparable:

- $((O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha) \wedge (O_\rho\overline{\alpha} \wedge \neg O_\rho\alpha)) \to (O_{\sigma\times\rho}\overline{\alpha} \wedge \neg O_{\sigma\times\rho}\alpha)$ by A6, A7;
- $((O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha) \wedge (\neg O_\rho\overline{\alpha} \wedge \neg O_\rho\alpha)) \to (O_{\sigma\times\rho}\overline{\alpha} \wedge \neg O_{\sigma\times\rho}\alpha)$ by A6, A7;
- $((O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha) \wedge (O_\rho\overline{\alpha} \wedge O_\rho\alpha)) \to (O_{\sigma\times\rho}\overline{\alpha} \wedge O_{\sigma\times\rho}\alpha)$ by A6;
- $((O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha) \wedge (\neg O_\rho\overline{\alpha} \wedge O_\rho\alpha)) \to (O_{\sigma\times\rho}\overline{\alpha} \wedge O_{\sigma\times\rho}\alpha)$ by A6;
- $((\neg O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha) \wedge (O_\rho\overline{\alpha} \wedge \neg O_\rho\alpha)) \to (O_{\sigma\times\rho}\overline{\alpha} \wedge \neg O_{\sigma\times\rho}\alpha)$ by A6, A7;
- $((\neg O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha) \wedge (\neg O_\rho\overline{\alpha} \wedge \neg O_\rho\alpha)) \to (\neg O_{\sigma\times\rho}\overline{\alpha} \wedge \neg O_{\sigma\times\rho}\alpha)$ by A7;
- $((\neg O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha) \wedge (O_\rho\overline{\alpha} \wedge O_\rho\alpha)) \to (O_{\sigma\times\rho}\overline{\alpha} \wedge O_{\sigma\times\rho}\alpha)$ by A6;
- $((\neg O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha) \wedge (\neg O_\rho\overline{\alpha} \wedge O_\rho\alpha)) \to (\neg O_{\sigma\times\rho}\overline{\alpha} \wedge O_{\sigma\times\rho}\alpha)$ by A6, A7;
- $((O_\sigma\overline{\alpha} \wedge O_\sigma\alpha) \wedge (O_\rho\overline{\alpha} \wedge \neg O_\rho\alpha)) \to (O_{\sigma\times\rho}\overline{\alpha} \wedge O_{\sigma\times\rho}\alpha)$ by A6;
- $((O_\sigma\overline{\alpha} \wedge O_\sigma\alpha) \wedge (\neg O_\rho\overline{\alpha} \wedge \neg O_\rho\alpha)) \to (O_{\sigma\times\rho}\overline{\alpha} \wedge O_{\sigma\times\rho}\alpha)$ by A6;
- $((O_\sigma\overline{\alpha} \wedge O_\sigma\alpha) \wedge (O_\rho\overline{\alpha} \wedge O_\rho\alpha)) \to (O_{\sigma\times\rho}\overline{\alpha} \wedge O_{\sigma\times\rho}\alpha)$ by A6;
- $((O_\sigma\overline{\alpha} \wedge O_\sigma\alpha) \wedge (\neg O_\rho\overline{\alpha} \wedge O_\rho\alpha)) \to (O_{\sigma\times\rho}\overline{\alpha} \wedge O_{\sigma\times\rho}\alpha)$ by A6;
- $((\neg O_\sigma\overline{\alpha} \wedge O_\sigma\alpha) \wedge (O_\rho\overline{\alpha} \wedge \neg O_\rho\alpha)) \to (O_{\sigma\times\rho}\overline{\alpha} \wedge O_{\sigma\times\rho}\alpha)$ by A6;
- $((\neg O_\sigma\overline{\alpha} \wedge O_\sigma\alpha) \wedge (\neg O_\rho\overline{\alpha} \wedge \neg O_\rho\alpha)) \to (\neg O_{\sigma\times\rho}\overline{\alpha} \wedge O_{\sigma\times\rho}\alpha)$ by A6, A7;
- $((\neg O_\sigma\overline{\alpha} \wedge O_\sigma\alpha) \wedge (O_\rho\overline{\alpha} \wedge O_\rho\alpha)) \to (O_{\sigma\times\rho}\overline{\alpha} \wedge O_{\sigma\times\rho}\alpha)$ by A6;
- $((\neg O_\sigma\overline{\alpha} \wedge O_\sigma\alpha) \wedge (\neg O_\rho\overline{\alpha} \wedge O_\rho\alpha)) \to (\neg O_{\sigma\times\rho}\overline{\alpha} \wedge O_{\sigma\times\rho}\alpha)$ by A6, A7.

Now we consider the case in which the two normative sources σ and ρ are comparable and we can follow a shorter path to get the intended result. Indeed, as it is illustrated in Table 4, for any action type α we only need to take into account the representation of the deontic value assigned to α by the stronger normative source among σ and ρ. Let us assume that $\sigma \prec \rho$; then, keeping the same definition of the variables x, y and z

used above, we just have to prove that $\vdash_S \psi_{DV(\alpha,\sigma):x} \to \psi_{DV(\alpha,\sigma\times\rho):z}$, namely:

- $(O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha) \to (O_{\sigma\times\rho}\overline{\alpha} \wedge \neg O_{\sigma\times\rho}\alpha)$ by A5;
- $(\neg O_\sigma\overline{\alpha} \wedge \neg O_\sigma\alpha) \to (\neg O_{\sigma\times\rho}\overline{\alpha} \wedge \neg O_{\sigma\times\rho}\alpha)$ by A5;
- $(O_\sigma\overline{\alpha} \wedge O_\sigma\alpha) \to (O_{\sigma\times\rho}\overline{\alpha} \wedge O_{\sigma\times\rho}\alpha)$ by A5;
- $(\neg O_\sigma\overline{\alpha} \wedge O_\sigma\alpha) \to (\neg O_{\sigma\times\rho}\overline{\alpha} \wedge O_{\sigma\times\rho}\alpha)$ by A5.

Therefore, there is a thesis of S corresponding to each line of the matrices in Table 2 and Table 4.

The last step is using our representation of the four deontic vales in $\mathcal{L}$ to show that any formula which receives truth-value 1 under every possible function DV is provable in S. The argument runs as follows: if ϕ takes value 1 under every possible function DV, then, thanks to our simulation within $\mathcal{L}$ of the lines of the matrices in Table 2 and Table 4, ϕ is entailed in S by any formula representing an assignment of deontic values to its action-types.

Let $\psi_{(\alpha,\sigma)} = \psi_{DV(\alpha,\sigma):f} \vee \psi_{DV(\alpha,\sigma):c} \vee \psi_{DV(\alpha,\sigma):n} \vee \psi_{DV(\alpha,\sigma):o}$. It can be easily shown that $\vdash_S \psi_{(\alpha,\sigma)}$. This immediately leads to the fact that, for any finite list of actions $\alpha_1,\ldots,\alpha_n$ and normative sources $\sigma_1,\ldots,\sigma_n$, $\vdash_S \psi_{(\alpha_1,\sigma_1)} \wedge \ldots \wedge \psi_{(\alpha_n,\sigma_n)}$. Thus, if $O_{\sigma_1},\ldots,O_{\sigma_n}$ is the list of deontic operators occuring in ϕ and they respectively have $\alpha_1,\ldots,\alpha_n$ in their scope, then we can infer that $\vdash_S (\psi_{(\alpha_1,\sigma_1)} \wedge \ldots \wedge \psi_{(\alpha_n,\sigma_n)}) \to \phi$, whence $\vdash_S \phi$.

As a final remark, we want to highlight the fact that a comparison among the theorems of our system S and those of any of the three systems proposed in (Kulicki and Trypuz, 2016) can be easily drawn on the basis of our simulation of the matrices in Table 2 and Table 4. For instance, if α takes value c in a normative source σ under a function DV, then we have $O_\sigma\alpha \wedge F_\sigma\alpha$, which can never be the case according to (an adaptation to our framework of) the three systems due to Kulicki and Trypuz.

5. Conclusion

In this paper, we introduced a system of multivalued deontic action logic which represents an alternative to the three systems in (Kulicki and Trypuz, 2016). We provided philosophical motivation to choose our system and characterized it in terms of matrix semantics. In what sense does our system convey a realistic view on normative conflicts? In the sense that it allows for resolving “solvable” conflicts (at the level of the hierarchy among normative sources) and that it does not attempt to “hide” normative conflicts that are not solvable; it rather acknowledges them and allows one to keep track of them. In addition, the system avoids explosion. In other words, our system aims at capturing the way in which we would evaluate normative conflicts in ordinary reasoning: we would resolve normative conflicts that can be resolved; we would simply acknowledge normative conflicts that cannot be resolved; and we would not attempt to derive irrelevant information due to their presence. The system thus offers a plausible evaluation of a given normative scenario. Since insolvable normative conflicts are unwelcome, noting their presence is worthwhile. The very fact that they are noticed motivates future improvement of normative sources. While the neutral view offered by Kulicki and Trypuz can serve well to guide agents facing insolvable conflicts, and to avoid blaming agents that could not do any better, our realistic view can serve well to (critically) evaluate a normative scenario, and to provide insights on how to modify norms.

One can easily observe a connection between the present approach to conflicting oughts and the paraconsistent approach to conflicting information. Imagine that we have a database which contains, among many other propositions, a proposition and its negation. We do not want to infer everything. When asked which truth value is ascribed to the given proposition within the database, the realistic answer is that according to the database, it is true and false at the same time, that is, it has both of these truth values. If asked to assign to it a unique truth value within a four-valued logic, the realistic answer is ‘conflicting’. The approach of the present paper is very similar. When asked which deontic value is ascribed to the action of Antigone burying her brother, the realistic answer is that according to the norms governing Antigone’s behaviour, it is obligatory and forbidden at the same time, that is, it has both of these deontic values. If asked to assign to it a unique deontic value within a four-valued logic, the realistic answer is ‘conflicting’. As indicated above, this does not lift the guilt from the agent’s shoulders. Yet we do not always want to lift this guilt. Yes, facing an action which is obligatory and forbidden at the same time is disturbing, and much less pleasant than facing a neutral action. Neutral actions may be a way to go when ascribing responsibility to people who had to make a decision facing irresolvable normative conflicts not caused by them. However, as the well-known quote says, "if you are going through hell, keep going". The conservation of normative conflicts may motivate further attempts to either resolve them, or to prevent their further occurrences.

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