Logic and Logical Philosophy
Volume 29 (2020), 189–211
DOI: 10.12775/LLP.2019.034

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Mereology with Super-Supplementation Axioms. A Reconstruction of the Unpublished Manuscript of Jan F. Drewnowski

Abstract. We present a study of unpublished fragments of Jan F. Drewnowski's manuscript from the years 1922–1928, which contains his own axiomatics for mereology. The sources are transcribed and two versions of mereology are reconstructed from them. The first one is given by Drewnowski. The second comes from Leśniewski and was known to Drewnowski from Leśniewski's lectures. Drewnowski's version is expressed in the language of ontology enriched with the primitive concept of a (*proper*) *part*, and its key axiom expresses the so-called weak super-supplementation principle, which was named by Drewnowski "the postulate of the existence of subtractions". Leśniewski's axiomatics with the primitive concept of *an ingrediens* contains the axiom expressing the strong super-supplementation principle. In both systems the collective class of objects from the range of a given non-empty concept is defined as the upper bound of that range. From a historical point of view it is interesting to notice that the presented version of Leśniewski's axiomatics has not been published yet. The same applies to Drewnowski's approach. We reconstruct the proof of the equivalence of these two systems. Finally, we discuss questions stemming from their equivalence in frame of elementary mereology formulated in a modern way.

Keywords: mereology; axiomatics for classical mereology; supplementation principles; super-supplementation principles; Stanisław Leśniewski; Jan F. Drewnowski

The subject of our interest is Jan F. Drewnowski's manuscript [3] kept in the family archives of Jacek Drewnowski, Jan F. Drewnowski's son. We have had access to these materials thanks to the kindness of the family. The remarks below focus on fragments containing Drewnowski's own axiomatics for mereology, which he was to present to Leśniewski himself. We do not know whether Leśniewski was acquainted with that version of

Received May 7, 2019. *Revised* August 31, 2019. *Published online* September 28, 2019

his system and we must therefore suppose that Drewnowski's proposal has never been properly considered and discussed in an academic context. This last point also applies to Leśniewski's version of mereology which Drewnowski considered in his manuscript. Both systems will be discussed in relation to the contemporary theory of mereological structures.

1. The contents of the manuscript [3]

The relevant material includes eighteen pages of notes devoted to mereology, containing mainly symbolic expressions and a few commentaries in Polish. In the text we find numerous deletions, corrections, and abbreviations. The first difficulty with reading the text lies in the fact that the manuscript bears three different dates: the academic year 1922/23, the beginning of 1928, and the middle of 1939. It is known that Drewnowski attended Leśniewski's lectures throughout his whole studies at the University of Warsaw; that is from the beginning of the summer semester in 1921 till the beginning of the year 1926.[1] At least some of those notes were made during Leśniewski's classes organized at the University of Warsaw in the academic year 1922/23 (in the heading on page 4 in the manuscript we read: "Numbers of theorems based on lectures from 1922/1923"). At that time Drewnowski attended three courses taught by Leśniewski: "Basics of logistic", "Basics of arithmetics", and "Basics of Euclidean three-dimensional geometry". It is probable that later in the text new numbers of some theses of mereology were introduced in blue and red, and some dates were added: on page 1: I-II.28 (in blue), on page 2: 16/III 28 (in red), on page 4: I-1928 (in pencil). The whole text is preceded by the following note, written on a separate piece of paper:

> Another view on Prof. S. Leśniewski's axiomatics for Mereology. It highlights the role of the assumption of the existence of subtractions. That assumption is present implicite in formulations of various axiomatics provided by S. Leśniewski himself. At that time I submitted the fair copy to Leśniewski himself, but he did not manage to take position on time.

It may have been that the notes went to Leśniewski just before May 1939 (Leśniewski died on May 13, 1939). However, due to the character

[1] A list of classes attended by Drewnowski during his studies can be found in his personal file from the archives of the University of Warsaw [2].

of handwriting used in that additional commentary it seems that it was added much later than 1939. It is difficult today to determine exactly what happened.

Except for the last page, the pages of the manuscript are numbered with Arabic numerals, but the presentation of the contents diverges from the order of the numbered pages.

The content of the notes can be put together in the following order of pages:

1. Pages with numbers 1, 11, 9, 12, 16, 17, 10, 2, 18, 7, 8 contain material with colour markings from 1928. The blue colour was used by the author to highlight his own axiomatics for mereology and a number of theses derivable from it. The red colour was used to highlight Leśniewski's axiomatics and some theses which can be derived from it. The new colour numeration diverges from the original numbering of formulas and it is independent from the numbers of pages. The notes present on the mentioned pages and the introductory commentary to the whole manuscript are interpreted by us in such a way that Drewnowski's intention was to demonstrate the equivalence of both axiomatics.

2. On pages with numbers 3, 4, 5, 6 there are notes from Leśniewski's lectures. They are in part unreadable drafts keeping the numeration of theses from Leśniewski's lectures from the year 1922/23.

3. The pages with numbers 13 and 14 contain a draft version of Drewnowski's axiomatics for mereology with the primitive notion of *subtraction*: $A \varepsilon B - C$ ("A is B not being C", where $B - C$ is the collective class of those B that are not C, provided that A is exterior to C — the definition is found on page 8) and a commentary on it.

4. On page 15 we have draft versions of axiomatics with original contexts: $A \varepsilon BC$ (where BC is the collective class of B being C) and $A \varepsilon B|C$ (where $B|C$ is the collective class of B or C).

We focus our attention on the material from point 1.

2. The transcription of Drewnowski's notes

We take into account the following pages: 1, 11, 9, 12, 16, 10, 2, 7, 8. We skip pages 17, 18 (they contain draft versions of the previous pages), eliminate the crossed-out places, unreadable fragments, and introduce numbering following the use of colours by Drewnowski. The author uses in the text logical symbols taken from Principia Mathematica. An expla-

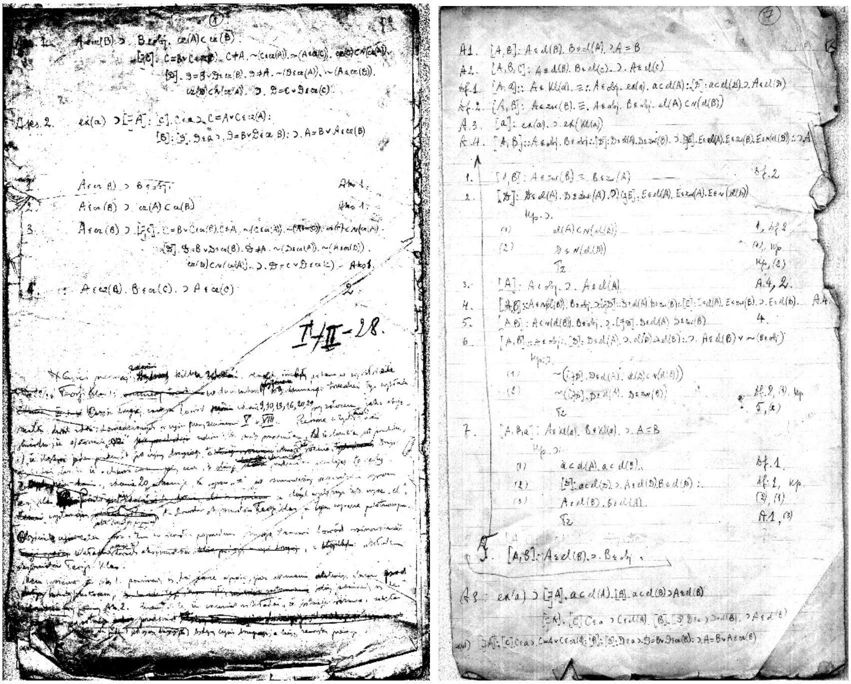

Figure 1. Page 1 – Drewnowski's axiomatics; page 7 – Leśniewski's axiomatics from lectures 1922/23 [3]

nation of the mereological notation introduced to Leśniewski's ontology may be found in [10].

In the formalism we find:

- the variables: 'a', 'b', 'c', ..., 'A', 'B', 'C', ...;
- the two-argument predicate of inherence 'ε' (read as "is");
- the one-argument term operators '`cz`' ("(proper) part" ; "część" in Polish) and '`el`' ("element" or "ingrediens").

Henceforth we will use '`prt`' and '`ing`' instead of '`cz`' and '`el`', respectively. The notion of a *part* in Drewnowski's axiomatics is primitive, and in Leśniewski's system, the notion of an *ingrediens* is primitive.

In the manuscript there are used the following constants defined in Leśniewski's ontology:

- the name constant '`obj`', which is read as "individual object" (later replaced with '`V`');

- the expressions ‘N’ and ‘∩’, which is read as “not” (later replaced with ‘-’) and “and”;
- the expression ‘ex’, which is read as “exists”;
- the symbols ‘=’ of *identity*, ‘≠’ of *non-identity*, ‘⊂’ of *inclusion* and ‘△’ of *intersection*.

The above symbols are introduced in Section 3 by (df V), (df ex), (df -), (df ∩), (df ⊂), (df =), (df ≠) and (df △), respectively. The symbols ‘∩’, ‘ex’, ‘=’, ‘≠’, ‘⊂’ and ‘△’ are later noted in the same way as Drewnowski did. We present the transcription of the analyzed material.

Page 1 – Drewnowski’s axiomatics for mereology:[2]

Aks 1. $A \mathrel{\varepsilon} \mathsf{cz}(B) \supset_. B \mathrel{\varepsilon} \mathsf{obj} \,.\, \mathsf{cz}(A) \subset \mathsf{cz}(B)\,.$
$[\exists C].\ C = B \vee C \mathrel{\varepsilon} \mathsf{cz}(B) \,.\, C \neq A \,.\, \sim(C \mathrel{\varepsilon} \mathsf{cz}(A)) \,.\, \sim(A \mathrel{\varepsilon} \mathsf{cz}(C)) \,.\, \mathsf{cz}(C) \subset \mathrm{N}(\mathsf{cz}(A))\,.$
$[D].\ D = B \vee D \mathrel{\varepsilon} \mathsf{cz}(B) \,.\, D \neq A \,.\, \sim(D \mathrel{\varepsilon} \mathsf{cz}(A)) \,.\, \sim(A \mathrel{\varepsilon} \mathsf{cz}(D)) \,.\, \mathsf{cz}(D) \subset \mathrm{N}(\mathsf{cz}(A)). \supset_.$
$D = C \vee D \mathrel{\varepsilon} \mathsf{cz}(C)$

Aks 2. $\mathsf{ex}(a) \supset [\exists A] : [C].\ C \mathrel{\varepsilon} a \supset_. C = A \vee C \mathrel{\varepsilon} \mathsf{cz}(A) :$
$[B] : [D].\ D \mathrel{\varepsilon} a \supset_. D = B \vee D \mathrel{\varepsilon} \mathsf{cz}(B) : \supset_. A = B \vee A \mathrel{\varepsilon} \mathsf{cz}(B)$

[T]1. $A \mathrel{\varepsilon} \mathsf{cz}(B) \supset B \mathrel{\varepsilon} \mathsf{obj}$ — Aks 1
[T]2. $A \mathrel{\varepsilon} \mathsf{cz}(B) \supset \mathsf{cz}(A) \subset \mathsf{cz}(B)$ — Aks 1
[T]3. $A \mathrel{\varepsilon} \mathsf{cz}(B) \supset [\exists C].\ C = B \vee C \mathrel{\varepsilon} \mathsf{cz}(B) \,.\, C \neq A \,.\, \sim(C \mathrel{\varepsilon} \mathsf{cz}(A)) \,.\, \sim(A \mathrel{\varepsilon} \mathsf{cz}(C))\,.$
$\mathsf{cz}(C) \supset \mathrm{N}(\mathsf{cz}(A)) \,.\, [D].\ D = B \vee D \mathrel{\varepsilon} \mathsf{cz}(B) \,.\, D \neq A\,.$
$\sim(D \mathrel{\varepsilon} \mathsf{cz}(A)) . \sim(A \mathrel{\varepsilon} \mathsf{cz}(D)) \,.\, \mathsf{cz}(D) \subset \mathrm{N}(\mathsf{cz}(A)). \supset_. D = C \vee D \mathrel{\varepsilon} \mathsf{cz}(C)$ — Aks 1
[T]4. $A \mathrel{\varepsilon} \mathsf{cz}(B) \,.\, B \mathrel{\varepsilon} \mathsf{cz}(C). \supset A \mathrel{\varepsilon} \mathsf{cz}(C)$ — [T]2

[Unreadable commentary on axioms]

Page 11

[T]5. $\sim A \mathrel{\varepsilon} \mathsf{cz}(A)$ — [unreadable proof, the thesis T5 follows from Aks 1]
[T]6. $A = B \supset_. \sim(A \mathrel{\varepsilon} \mathsf{cz}(B))$ — [T]5
[T]7. $A \mathrel{\varepsilon} \mathsf{cz}(B) \supset A \neq B$ — [T]6, [T]1
[T]24. $A \mathrel{\varepsilon} \mathsf{cz}(B) \supset [\exists C].\ C \mathrel{\varepsilon} \mathsf{el}(B) \,.\, \mathsf{el}(C) \subset \mathrm{N}(\mathsf{el}(A))\,.$
$[D].\ D \mathrel{\varepsilon} \mathsf{el}(B) \,.\, \mathsf{el}(D) \subset \mathrm{N}(\mathsf{el}(A)) \supset D \mathrel{\varepsilon} \mathsf{el}(C)$ — Df. 1, [T]23

[T24 is also marked in red with no 16a in Leśniewski’s system with which it is being compared (Pg 7–8).]

Page 9

Df. 1. $A \mathrel{\varepsilon} \mathsf{el}(B) \equiv_. A = B \vee A \mathrel{\varepsilon} \mathsf{cz}(B)$
[T]8. $A \neq B \,.\, \sim(A \mathrel{\varepsilon} \mathsf{cz}(B)) \equiv_. A \mathrel{\varepsilon} \mathsf{obj} \,.\, B \mathrel{\varepsilon} \mathsf{obj} \,.\, \sim(A \mathrel{\varepsilon} \mathsf{el}(B))$ — Df. 1
[T]9. $A \mathrel{\varepsilon} \mathsf{el}(B) \supset B \mathrel{\varepsilon} \mathsf{obj}$ — Df. 1, [T]1
[T]10. $A \mathrel{\varepsilon} \mathsf{el}(B) \,.\, B \mathrel{\varepsilon} \mathsf{el}(C) \,.\, \supset A \mathrel{\varepsilon} \mathsf{el}(C)$
Hp. $\supset$:
(1) $B = C \supset A \mathrel{\varepsilon} \mathsf{el}(C)$:

[2] We will sometimes refer to pages using the abbreviation “Pg”.

(2) $B \varepsilon \mathsf{cz}(C) . A = B . \supset A \varepsilon \mathsf{cz}(C) :$
(3) $B \varepsilon \mathsf{cz}(C) . A \varepsilon \mathsf{cz}(B) \supset A \varepsilon \mathsf{cz}(C) :$ [T]4
(4) $B \varepsilon \mathsf{cz}(C) \supset A \varepsilon \mathsf{cz}(C)$. (2), (3), Df. 1, Hp.
Th[3] (1), (4), Df. 1, Hp.

[T]11. $A \varepsilon \mathsf{el}(B) . B \varepsilon \mathsf{cz}(C) \supset A \varepsilon \mathsf{cz}(C)$
Hp. $\supset$
(1) $A = B \supset A \varepsilon \mathsf{cz}(C)$.
(2) $A \varepsilon \mathsf{cz}(B) \supset A \varepsilon \mathsf{cz}(C)$ [T]4
Th (1), (2), Df. 1, Hp.

[T]12. $A \varepsilon \mathsf{cz}(B) . B \varepsilon \mathsf{el}(C) \supset A \varepsilon \mathsf{cz}(C)$.
Hp. $\supset$
(1) $B = C \supset A \varepsilon \mathsf{cz}(C)$
(2) $B \varepsilon \mathsf{cz}(C) \supset A \varepsilon \mathsf{cz}(C)$ [T]4.
Th (1), (2), Df. 1, Hp.

[T]13. $A \varepsilon \mathsf{el}(B) . B \varepsilon \mathsf{el}(A) \supset A = B$
Hp. $\supset_.$
(1) $\sim(A \varepsilon \mathsf{cz}(B)) \supset A =_. B$ Df. 1
(2) $A \varepsilon \mathsf{cz}(B) \supset A \varepsilon \mathsf{cz}(A)$ [T]12, Hp.
(3) $\sim(A \varepsilon \mathsf{cz}(B))$ (2), [T]5
Th (1), (3)

[T]14. $A = B \supset A \varepsilon \mathsf{el}(B)$ Df. 1
[T]15. $A \varepsilon \mathsf{obj} \supset A \varepsilon \mathsf{el}(A)$ [T]14
[T]16. $\mathsf{ex}(a) \supset [\exists A]. a \subset \mathsf{el}(A) . [B]. a \subset \mathsf{el}(B) \supset A \varepsilon \mathsf{el}(B)$ Aks 2, Df. 1

[T16 is marked as no 16b in Leśniewski's system (cf. Pg 7–8).]

Page 12

[T]17. $A \varepsilon \mathsf{cz}(B) \supset A \varepsilon \mathsf{el}(B)$ Df. 1
[T]18. $A \varepsilon \mathsf{cz}(B) \supset A \varepsilon \mathsf{el}(B) . A \neq B$ [T]17, [T]7
[T]19. $A \varepsilon \mathsf{el}(B) . A \neq B . \supset A \varepsilon \mathsf{cz}(B)$ Df. 1
[T]20. $A \varepsilon \mathsf{cz}(B) \equiv_. A \varepsilon \mathsf{el}(B) . A \neq B$ [T]18, [T]19

[T20 is a definition in Leśniewski's system, marked in red.]

[T]21. $A \varepsilon \mathsf{obj} . B \varepsilon \mathsf{obj} . \mathsf{el}(A) \subset \mathrm{N}(\mathsf{el}(B)). \supset$
$\mathsf{cz}(A) \subset \mathrm{N}(\mathsf{cz}(B)). A \neq B . \sim(A \varepsilon \mathsf{cz}(B)) . \sim(B \varepsilon \mathsf{cz}(A))$
Hp. $\supset :$
(1) $[C]. C \varepsilon \mathsf{el}(A) \supset_. \sim(C \varepsilon \mathsf{el}(B)) :$
(2) $[D]. C \varepsilon \mathsf{cz}(A) \supset_.$
(α) $C \varepsilon \mathsf{el}(A)$. [T]17, [T]7
(β) $\sim(C \varepsilon \mathsf{el}(B))$. (1), ($\alpha$)
$\sim(C \varepsilon \mathsf{cz}(B)) :$ [T]17, (β)

[3] In the original text there is used "Tz" from the Polish word "teza" which means "thesis".

(3) $A \varepsilon \mathsf{el}(A) . B \varepsilon \mathsf{el}(B)$ [T]15., Hp.
(4) $\sim(A \varepsilon \mathsf{el}(B)) . \sim(B \varepsilon \mathsf{el}(A))$ Hp., (3)
(5) $\sim(A \varepsilon \mathsf{cz}(B)) . \sim(B \varepsilon \mathsf{cz}(A))$ [T]17, (4)
(6) $\sim(A = B)$ [T]15, (5)
Th. (2), (5), (6), Hp.

[T]22. $A \neq B . \sim(A \varepsilon \mathsf{cz}(B)) . \sim(B \varepsilon \mathsf{cz}(A)) . \mathsf{cz}(A) \subset \mathrm{N}(\mathsf{cz}(B)) . \supset \mathsf{el}(A) \subset \mathrm{N}(\mathsf{el}(B))$
Hp. $\supset$:
(1) $[C]. C \varepsilon \mathsf{cz}(A) \supset_. \sim(C \varepsilon \mathsf{cz}(B))$: Hp.
(2) $[C]. C = A \supset_.$
(α) $C \neq B . \sim(C \varepsilon \mathsf{cz}(B))$. Hp.
$\sim(C \varepsilon \mathsf{el}(B))$ Df. 1, (α)
(3) $[C]. C = B \supset . \sim(C \varepsilon \mathsf{cz}(A))$: Hp.
(4) $[C]. C \varepsilon \mathsf{cz}(A) \supset_.$
(α) $C \neq B$. (3)
(β) $\sim(C \varepsilon \mathsf{cz}(B))$. (1)
$\sim(C \varepsilon \mathsf{el}(B))$: Df. 1, ($\alpha$), ($\beta$)
(5) $[C]. C = A \vee C \varepsilon \mathsf{cz}(A) . \supset \sim(C \varepsilon \mathsf{el}(B))$: (2), (4)
(6) $[C]. C \varepsilon \mathsf{el}(A) \supset_. C \varepsilon \mathrm{N}(\mathsf{el}(B))$: (5), Df. 1
Th. (6)

[T]23. $A \varepsilon \mathsf{obj} . B \varepsilon \mathsf{obj}. \supset: \mathsf{el}(A) \subset \mathrm{N}(\mathsf{el}(B)) \equiv_. A \neq B . \sim(\mathsf{cz}(A) \triangle \mathsf{cz}(B)) .$
$\sim(A \varepsilon \mathsf{cz}(B)) . \sim B \varepsilon \mathsf{cz}(A)$ [derived from T21, T22]

Page 16

[T]24. $A \varepsilon \mathsf{cz}(B) \supset [\exists C]. C \varepsilon \mathsf{el}(B) . \mathsf{el}(C) \subset \mathrm{N}(\mathsf{el}(A))$.
$[D]. D \varepsilon \mathsf{el}(B) . \mathsf{el}(D) \subset \mathrm{N}(\mathsf{el}(A)) \supset D \varepsilon \mathsf{el}(C)$

[T24 is repeated on Pg 11 and it has no 16b there.]

[T]24b. $\mathsf{ex}(a) . a \subset \mathsf{el}(A) . [D]. a \subset \mathsf{el}(D) \supset A \varepsilon \mathsf{el}(D): \supset [C]. C \varepsilon \mathsf{el}(A) \supset$
$\mathsf{el}(C) \subset \mathrm{N}(\mathsf{el}(B)): \supset \mathsf{el}(A) \subset \mathrm{N}(\mathsf{el}(B))$
Hp. $\supset$
(1) $[C, D]. C \varepsilon a . D \varepsilon \mathsf{el}(C). \supset_. D \varepsilon \mathrm{N}(\mathsf{el}(B))$:
(2) $[D]. D \varepsilon \mathsf{el}(a) \supset$
(α) $D \varepsilon \mathsf{el}(D)$ [T]15
$D \varepsilon \mathrm{N}(\mathsf{el}(B))$ (1), (2)
(3) $a \subset \mathrm{N}(\mathsf{el}(B))$ (2)
(4) $a \subset \mathsf{el}(A) \cap \mathrm{N}(\mathsf{el}(B))$ Hp., (3)
(5) $\mathsf{ex}(\mathsf{el}(A) \cap \mathrm{N}(\mathsf{el}(B)))$
$[\exists E]$.
(6) $\mathsf{el}(A) \cap \mathsf{el}(B) \subset \mathrm{N}(\mathsf{el}(E)) . [F]. \mathsf{el}(A) \cap \mathrm{N}(\mathsf{el}(B)) \subset \mathsf{el}(F) \supset E \varepsilon \mathsf{el}(F)$
[T]16, (5)
(7) $\mathsf{el}(A) \cap \mathsf{el}(B) \subset \mathrm{N}(\mathsf{el}(A))$
(8) $\mathsf{el}(A) \cap \mathsf{el}(B) \subset \mathrm{N}(\mathsf{el}(B))$
(9) $E \varepsilon \mathsf{el}(A)$

[T24 is not written in blue and it is not needed for the main proof for the equivalence of the systems under consideration.]

Page 10

[T]25. $(A \neq B\,.\sim(A\,\varepsilon\,\mathsf{cz}(B))\,.\sim(B\,\varepsilon\,\mathsf{cz}(A))\,.\,\mathsf{cz}(A) \subset \mathrm{N}(\mathsf{cz}(B)) \supset [\exists C].\ [C\,\varepsilon\,\mathsf{el}(A)\,.\,\mathsf{el}(C) \subset \mathrm{N}(\mathsf{el}(B))\,.\,[D].\ D\,\varepsilon\,\mathsf{el}(A)\,.\,\mathsf{el}(D) \subset \mathrm{N}(\mathsf{el}(B)) \supset D\,\varepsilon\,\mathsf{el}(C)$

Hp. $\supset$

(1) $A\,\varepsilon\,\mathsf{obj}$

(2) $A\,\varepsilon\,\mathsf{el}(A)$ [T]15

(3) $\mathsf{el}(A) \subset \mathrm{N}(\mathsf{el}(B))$ [T]22., Hp.

(4) $[D].\ D\,\varepsilon\,\mathsf{el}(A)\,.\,\mathsf{el}(D) \subset \mathrm{N}(\mathsf{el}(B)).\supset D\,\varepsilon\,\mathsf{el}(A)$: (2), (3), (4)

Th

[T]26. $A \neq B\,.\sim(A\,\varepsilon\,\mathsf{cz}(B))\,.\sim(B\,\varepsilon\,\mathsf{cz}(A))\,.\,\mathsf{cz}(A) \triangle \mathsf{cz}(B))) \supset [\exists C].\ C\,\varepsilon\,\mathsf{el}(A)\,.\,\mathsf{el}(C) \supset \mathrm{N}(\mathsf{el}(B))\,.\,[D].\ D\,\varepsilon\,\mathsf{el}(A)\,.\,\mathsf{el}(D) \subset \mathrm{N}(el(B)) \supset D\,\varepsilon\,\mathsf{el}(C))$ *Hp.* $\supset$

(1) $\mathsf{ex}(\mathsf{cz}(A) \cap \mathsf{cz}(B))$

$[\exists H].:$

(2) $\mathsf{cz}(A) \cap \mathsf{cz}(B) \subset \mathsf{el}(H)\,.\,[E].\ \mathsf{cz}(A) \cap \mathsf{cz}(B) \subset \mathsf{el}(E) \supset H\,\varepsilon\,\mathsf{el}(E)$ [T]16

(3) $\mathsf{cz}(A) \subset \mathsf{el}(A)\,.\,\mathsf{cz}(B) \supset \mathsf{el}(B)$ [T]17

(4) $\mathsf{cz}(A) \cap \mathsf{cz}(B) \subset \mathsf{el}(A)\,.\,\mathsf{cz}(A) \cap \mathsf{cz}(B) \subset \mathsf{el}(B)$ (3)

(5) $H\,\varepsilon\,\mathsf{el}(A)\,.\,H\,\varepsilon\,\mathsf{el}(B)$ (2), (4)

(6) $\sim(B\,\varepsilon\,\mathsf{el}(A))\,.\sim(A\,\varepsilon\,\mathsf{el}(B))$ Df. 1, Hp.

(7) $H = B \supset B\,\varepsilon\,\mathsf{el}(A)\,.\,H = A \supset A\,\varepsilon\,\mathsf{el}(B)$ (5)

(8) $H \neq B\,.\,H \neq A$ (7), (6)

(9) $H\,\varepsilon\,\mathsf{cz}(A)\,.\,H\,\varepsilon\,\mathsf{cz}(B)$ [T]19, (5), (8)

$[\exists C].$

(10) $C\,\varepsilon\,\mathsf{el}(A)\,.\,\mathsf{el}(C) \subset \mathrm{N}(\mathsf{el}(H))\,.\,[F].\ F\,\varepsilon\,\mathsf{el}(A)\,.\,\mathsf{el}(F) \subset \mathrm{N}(\mathsf{el}(H)) \supset F\,\varepsilon\,\mathsf{el}(C)$ (9), [T]24.

(11) $C = B \supset B\,\varepsilon\,\mathsf{el}(A)$ (10)

(12) $C \neq B$ (11), (6)

(13) $C \neq H\,.\,\mathsf{cz}(C) \subset \mathrm{N}(\mathsf{cz}(H))\,.\sim(C\,\varepsilon\,\mathsf{cz}(H))\,.\sim(H\mathsf{cz}(C))$ [T]21, (10)

(14) $C = A \supset H\,\varepsilon\,\mathsf{cz}(C)$ (9)

(15) $C \neq A$ (14), (13)

(16) $C\,\varepsilon\,\mathsf{cz}(A)$ [T]19, (10), (15)

(17) $\sim(C\,\varepsilon\,\mathsf{el}(H))$ Df. 1, (13)

(18) $\sim(C\,\varepsilon\,\mathsf{el}(\mathsf{cz}(A) \cap \mathsf{cz}(B)))$ (1), (17)

(19) $\sim(C\,\varepsilon\,\mathsf{cz}(B))$ (18), (16)

(20) $\sim(B\,\varepsilon\,\mathsf{cz}(C))$ (5), (9), (13)

(21) $[E].\ E\,\varepsilon\,\mathsf{cz}(C) \supset$

(α) $E\,\varepsilon\,\mathsf{cz}(A)$ [T]4, (16)

(β) $E = H \supset \sim E\,\varepsilon\,\mathsf{cz}(C)$ (13)

(γ) $E \neq H$ (β)

(δ) $\sim(E\,\varepsilon\,\mathsf{cz}(H))$ (13)

(ϵ) $\sim(E\,\varepsilon\,\mathsf{el}(H))$ Df. 1, (δ), (γ)

(ζ) $\sim(E\,\varepsilon\,\mathsf{cz}(A) \cap \mathsf{cz}(B))$ (2), (ϵ)

$\sim(E\,\varepsilon\,\mathsf{cz}(B))$ (ζ), (α)

(22) $\mathsf{cz}(C) \supset \mathrm{N}(\mathsf{cz}(B))$ (21)

(23) $\mathsf{el}(C) \subset \mathrm{N}(\mathsf{cz}(B))$ (12), (19), (20), (22)

(24) $[D].\ D\,\varepsilon\,\mathtt{el}(A)\,.\,\mathtt{el}(D) \subset \mathtt{N}(\mathtt{el}(B)) \supset$
(α) $D\,\varepsilon\,\mathtt{el}(A)$
(β) $[F].\ F\,\varepsilon\,\mathtt{el}(D) \supset$
(γ) $\sim(F\,\varepsilon\,\mathtt{el}(B))$
(δ) $\sim(F\,\varepsilon\,\mathtt{cz}(B))$ — [T]17, (β)
(ϵ) $\sim(F\,\varepsilon\,\mathtt{cz}(H))$ — [T]4, (9), (δ)
(ζ) $F = H \supset F\,\varepsilon\,\mathtt{cz}(B)$ — (9)
(η) $F \neq H$ — (δ),(ζ)
$\sim(F\,\varepsilon\,\mathtt{el}(H))$ — Df. 1, (ϵ), (η)
(θ) $\mathtt{el}(D) \supset \mathtt{N}(\mathtt{el}(H))$ — (β)
$D\,\varepsilon\,\mathtt{el}(C)$ — (10), (α), (θ)
Th. — (10), (23), (24)

Page 2

[T]27. $A \neq B\,.\,\sim(A\,\varepsilon\,\mathtt{cz}(B))\,.\,\sim(B\,\varepsilon\,\mathtt{cz}(A)) \supset [\exists C].\ C\,\varepsilon\,\mathtt{el}(A)\,.\,\mathtt{el}(C) \subset \mathtt{N}(\mathtt{el}(B))\,.$
$[D].\ D\,\varepsilon\,\mathtt{el}(A)\,.\,\mathtt{el}(D) \subset \mathtt{N}(\mathtt{el}(B)) \supset D\,\varepsilon\,\mathtt{el}(C)$ — [T]25, [T]26

[T]28. $A \neq B\,.\,\sim(A\,\varepsilon\,\mathtt{cz}(B)) \supset [\exists C].\ [C\,\varepsilon\,\mathtt{el}(A)\,.\,\mathtt{el}(C) \supset \mathtt{N}(\mathtt{el}(B))\,.$
$[D].\ D\,\varepsilon\,\mathtt{el}(A)\,.\,\mathtt{el}(D) \subset \mathtt{N}(\mathtt{el}(B)) \supset D\,\varepsilon\,\mathtt{el}(C)$ — [T]24, [T]27
[Drewnowski takes T24 and substitution: b/a, a/b.]

[T]29. $A\,\varepsilon\,\mathtt{obj}\,.\,B\,\varepsilon\,\mathtt{obj}\,.\,\sim(A\,\varepsilon\,\mathtt{el}(B)) \supset [\exists C].\ C\,\varepsilon\,\mathtt{el}(A)\,.\,\mathtt{el}(C) \subset \mathtt{N}(\mathtt{el}(B))\,.$
$[D].\ D\,\varepsilon\,\mathtt{el}(A)\,.\,\mathtt{el}(D) \subset \mathtt{N}(\mathtt{el}(B)) \supset D\,\varepsilon\,\mathtt{el}(C)$ — [T]28, Df. 1

Page 7

[Leśniewski's axiomatics for mereology from lectures in 1922/23. We omit sketches of derivations and commentaries on theses that are not related to the main proof for the equivalence of Leśniewski's and Drewnowski's systems.]

A1. $[A,B]\colon A\,\varepsilon\,\mathtt{el}(B)\,.\,B\,\varepsilon\,\mathtt{el}(A).\supset_. A = B$
A2. $[A,B,C]\colon A\,\varepsilon\,\mathtt{el}(B)\,.\,B\,\varepsilon\,\mathtt{el}(C).\supset .A\,\varepsilon\,\mathtt{el}(C)$
Df. 1. $[A,a]{::}\ A\varepsilon\mathtt{Kl}(a).\equiv\therefore A\varepsilon\mathtt{obj}\,.\,\mathtt{ex}(a)\,.\,a\subset\mathtt{el}(A) \therefore [D]: a\subset\mathtt{el}(D) \supset A\varepsilon el(D)$
Df. 2. $[A,B]\colon A\,\varepsilon\,\mathtt{zw}(B).\equiv_. A\,\varepsilon\,\mathtt{obj}\,.\,B\,\varepsilon\,\mathtt{obj}\,.\,\mathtt{el}(A) \supset \mathtt{N}(\mathtt{el}(B))$
A3. $[a]\colon \mathtt{ex}(a).\supset_. \mathtt{ex}(\mathtt{Kl}(a))$
A4. $[A,B]{::}\ A\,\varepsilon\,\mathtt{obj}\,.\,B\,\varepsilon\,\mathtt{obj}\,.\,[D].\ D\,\varepsilon\,\mathtt{el}(A)\,.\,D\,\varepsilon\,\mathtt{zw}(B) \supset$
$[\exists E].\ E\,\varepsilon\,\mathtt{el}(A)\,.\,E\,\varepsilon\,\mathtt{zw}(B)\ .\ E\,\varepsilon\,\mathtt{N}(\mathtt{el}(D)) \therefore\supset A\,\varepsilon\,\mathtt{el}(B)$
A5. $[A,B]\colon A\,\varepsilon\,\mathtt{el}(B).\supset .B\,\varepsilon\,\mathtt{obj}$

[Theses:]

1. $[A,B]\colon A\,\varepsilon\,\mathtt{zw}(B).\equiv_. B\,\varepsilon\,\mathtt{zw}(A)$
2. $[D]\colon D\,\varepsilon\,\mathtt{el}(A)\,.\,D\,\varepsilon\,\mathtt{zw}(A) \supset [\exists E].\ E\,\varepsilon\,\mathtt{el}(A)\,.\,E\,\varepsilon\,\mathtt{zw}(A)\,.\,E\,\varepsilon\,\mathtt{N}(\mathtt{el}(B))$
3. $[A]\colon A\,\varepsilon\,\mathtt{obj}.\supset_. A\,\varepsilon\,\mathtt{el}(A)$
4. $[A,B]{::}\ A\,\varepsilon\,\mathtt{N}(\mathtt{el}(B))\,.\,B\,\varepsilon\,\mathtt{obj} \supset{::} [\exists D]{::}\ D\,\varepsilon\,\mathtt{el}(A)\,.\,D\,\varepsilon\,\mathtt{zw}(B) \therefore$
$[E]\colon\ E\,\varepsilon\,\mathtt{el}(A)\,.\,E\,\varepsilon\,\mathtt{zw}(B) \supset E\,\varepsilon\,\mathtt{el}(D)$
5. $[A,B]\colon A\,\varepsilon\,\mathtt{N}(\mathtt{el}(B))\,.\,B\,\varepsilon\,\mathtt{obj} \supset [\exists D].\ D\,\varepsilon\,\mathtt{el}(A)\,.\,D\,\varepsilon\,\mathtt{zw}(B)$
6. $[A,B]{::}\ A\varepsilon\mathtt{obj} \supset [D]\colon D\varepsilon\mathtt{el}(A) \supset \mathtt{el}(D)\,\Delta\,\mathtt{el}(B)) \therefore\supset A\varepsilon\mathtt{el}(B) \vee \sim(B\varepsilon\mathtt{obj})$
7. $[A,B,a]\colon A\,\varepsilon\,\mathtt{Kl}(a)\,.\,B\,\varepsilon\,\mathtt{Kl}(a) \supset A = B$

Page 8

8. [unreadable formula]
9. $[A, B].\ A \,\varepsilon\, \mathtt{el}(B) \supset \mathtt{el}(A) \vartriangle \mathtt{el}(B)$
10. [[T]1 repeated]
11. $[A, B, C].\ A \,\varepsilon\, \mathtt{zw}(B) \,.\, C \,\varepsilon\, \mathtt{el}(B) \supset A \,\varepsilon\, \mathtt{zw}(C)$
12. $[A, B].\ B \,\varepsilon\, \mathtt{zw}(C) \,.\, \mathtt{el}(A) \vartriangle \mathtt{el}(B) \supset A \,\varepsilon\, \mathtt{N}(\mathtt{el}(C))$
13. $[A, B, D, a].\ A \,\varepsilon\, \mathtt{Kl}(a) \,.\, D \,\varepsilon\, \mathtt{el}(B) \,.\, a \subset \mathtt{zw}(B) \supset D \,\varepsilon\, \mathtt{zw}(A)$
14. [unreadable formula]
15. $[A, B].\ A \,\varepsilon\, \mathtt{Kl}(a) \supset [D].\ D \,\varepsilon\, \mathtt{el}(A) \supset [\exists C].\ C \,\varepsilon\, a \,.\, (\mathtt{el}(C) \vartriangle \mathtt{el}(D)$

3. The supplement of the transcription and a few systematic remarks

We now present two versions of mereology considered by Drewnowski using modern notation. We shall give explicit details of the background to his enquiries. We add the primitive two-argument predicate constant 'ε' to the first-order language. We take classical first-order logic extended by the following specific axiom of Leśniewski's ontology:

$$\forall a, b(a \,\varepsilon\, b \leftrightarrow \exists c\; c \,\varepsilon\, a \wedge \forall c, d(c \,\varepsilon\, a \wedge d \,\varepsilon\, a \to c \,\varepsilon\, d) \wedge \forall c(c \,\varepsilon\, a \to c \,\varepsilon\, b)). \quad \text{(AO)}$$

It is well-know that from (AO) we obtain the following theses [see, e.g. 6, p. 236]:

$$\forall a(a \,\varepsilon\, a \leftrightarrow \exists c\; c \,\varepsilon\, a \wedge \forall c, d(c \,\varepsilon\, a \wedge d \,\varepsilon\, a \to c \,\varepsilon\, d)),$$
$$\forall a, b(a \,\varepsilon\, b \to a \,\varepsilon\, a), \quad (\mathrm{qr}_\varepsilon)$$
$$\forall a, b, c(a \,\varepsilon\, b \wedge b \,\varepsilon\, c \to a \,\varepsilon\, c),$$
$$\forall a, b, c(a \,\varepsilon\, b \wedge b \,\varepsilon\, b \to b \,\varepsilon\, a),$$
$$\forall a, b, c(a \,\varepsilon\, b \wedge b \,\varepsilon\, c \to b \,\varepsilon\, a). \quad (\mathrm{t}_\varepsilon)$$

There are assumed ontological definitions of the following constants: the name constant '$\mathtt{V}$' (in Leśniewski's ontology and mereology, for the general name 'object'), the function constants '$-$' and '$\cap$' (for the one-argument and two-argument name-forming functors 'not' and 'and'),[4] the one-argument predicate '$\mathtt{ex}$' (for the one-argument sentence-forming functor 'exists'), and the two-argument predicates '$=$', '$\neq$', '$\subset$' and '$\vartriangle$'

[4] Actually, formulas introducing the notions of *term negation* and *object* are cases of meta-ontological definitional schemata: $\forall u(u \,\varepsilon\, \mathsf{f}(x_1, \ldots, x_n) \leftrightarrow u \,\varepsilon\, x_i \wedge \varphi_{\mathsf{f}})$, for $x_i \in \{u, x_1, \ldots, x_n\}$, and $\forall u(u \,\varepsilon\, \mathsf{n} \leftrightarrow u \,\varepsilon\, u \wedge \varphi_{\mathsf{n}})$, where $\varphi_{\mathsf{f}}, \varphi_{\mathsf{n}}$ has the same free variables as it has on the left sides of the equivalences [see, eg. 6, p. 237, or 8].

of *identity*, of *non-identity*, of *inclusion* and of *intersection*, respectively (for the two-argument sentence-forming functors 'is identical to', 'is not identical to', 'any ... is ... ' and 'some ... is ... ', respectively):

$$\forall a,b(a \mathrel{\varepsilon} \mathtt{V} \leftrightarrow a \mathrel{\varepsilon} a), \tag{df V}$$

$$\forall a,b(a \mathrel{\varepsilon} -b \leftrightarrow a \mathrel{\varepsilon} a \wedge \neg\, a \mathrel{\varepsilon} b), \tag{df −}$$

$$\forall a,b,c(a \mathrel{\varepsilon} b \cap c \leftrightarrow a \mathrel{\varepsilon} b \wedge a \mathrel{\varepsilon} c), \tag{df ∩}$$

$$\forall a(\mathtt{ex}(a) \leftrightarrow \exists b\; b \mathrel{\varepsilon} a), \tag{df ex}$$

$$\forall a,b(a = b \leftrightarrow a \mathrel{\varepsilon} b \wedge b \mathrel{\varepsilon} a), \tag{df =}$$

$$\forall a,b(a \neq b \leftrightarrow a \mathrel{\varepsilon} \mathtt{V} \wedge b \mathrel{\varepsilon} \mathtt{V} \wedge \neg\, a = b), \tag{df ≠}$$

$$\forall a,b(a \subset b \leftrightarrow \forall c(c \mathrel{\varepsilon} a \rightarrow c \mathrel{\varepsilon} b)), \tag{df ⊂}$$

$$\forall a,b(a \bigtriangleup b \leftrightarrow \exists c(c \mathrel{\varepsilon} a \wedge c \mathrel{\varepsilon} b)). \tag{df △}$$

3.1. Drewnowski's approach to mereology

Following Drewnowski's approach, we extend the vocabulary by the primitive function constant '$\mathtt{prt}$' (for the one-argument name-forming functor 'part of'). We rewrite his two axioms (cf. Aks 1 and Aks 2, Pg 1):

$$\begin{aligned}\forall a,b(a \mathrel{\varepsilon} \mathtt{prt}(b) \;\rightarrow\; & b \mathrel{\varepsilon} \mathtt{V} \wedge \mathtt{prt}(a) \subset \mathtt{prt}(b) \wedge \exists c((c = b \vee c \mathrel{\varepsilon} \mathtt{prt}(b)) \wedge \\ & c \neq a \wedge \neg\, a \mathrel{\varepsilon} \mathtt{prt}(c) \wedge \neg\, c \mathrel{\varepsilon} \mathtt{prt}(a) \wedge \mathtt{prt}(c) \subset -\mathtt{prt}(a) \wedge \\ & \forall d((d = b \vee d \mathrel{\varepsilon} \mathtt{prt}(b)) \wedge d \neq a \wedge \neg\, d \mathrel{\varepsilon} \mathtt{prt}(a) \wedge \neg\, a \mathrel{\varepsilon} \mathtt{prt}(d) \wedge \\ & \mathtt{prt}(d) \subset -\mathtt{prt}(a)) \rightarrow (d = c \vee d \mathrel{\varepsilon} \mathtt{prt}(c)))),\end{aligned} \tag{AD1}$$

$$\begin{aligned}\forall a(\mathtt{ex}(a) \rightarrow \exists b(& \forall c(c \mathrel{\varepsilon} a \rightarrow (c = b \vee c \mathrel{\varepsilon} \mathtt{prt}(b))) \wedge \\ & \forall d(\forall e(e \mathrel{\varepsilon} a \rightarrow (e = d \vee e \mathrel{\varepsilon} \mathtt{prt}(d))) \rightarrow (b = d \vee b \mathrel{\varepsilon} \mathtt{prt}(d))))).\end{aligned} \tag{AD2}$$

Like Drewnowski, we take an ontological definition of the function constants '$\mathtt{ing}$', '$\mathtt{extr}$' and '$\mathtt{Kl}$' (for the one-argument name-forming functors 'ingrediens of', 'exterior of' and 'collective class of', respectively; cf. Df. 1, Pg 9):

$$\forall a,b(a \mathrel{\varepsilon} \mathtt{ing}(b) \leftrightarrow a = b \vee a \mathrel{\varepsilon} \mathtt{prt}(b)), \tag{df ing}$$

$$\forall a,b(a \mathrel{\varepsilon} \mathtt{extr}(b) \leftrightarrow a \mathrel{\varepsilon} \mathtt{V} \wedge b \mathrel{\varepsilon} \mathtt{V} \wedge \mathtt{ing}(a) \subset -\mathtt{ing}(b)), \tag{df extr}$$

$$\begin{aligned}\forall a,b(a \mathrel{\varepsilon} \mathtt{Kl}(b) \leftrightarrow\; & a \mathrel{\varepsilon} \mathtt{V} \wedge \mathtt{ex}(b) \wedge b \subset \mathtt{ing}(a) \wedge \\ & \forall c(b \subset \mathtt{ing}(c) \rightarrow a \mathrel{\varepsilon} \mathtt{ing}(c))).\end{aligned} \tag{df Kl}$$

The above extension of the ontology we call **MD**.

Notice that directly from (df =) and (df ing) we obtain (cf. T15, Pg 9):

$$\forall a\big(a \mathrel{\varepsilon} \mathtt{V} \to a \mathrel{\varepsilon} \mathtt{ing}(a)\big). \tag{r$_{\text{ing}}$}$$

From (qr$_\varepsilon$), (df V) and (AD1) we get (cf. T1, Pg 1):

$$\forall a\big(a \mathrel{\varepsilon} \mathtt{prt}(b) \to a \mathrel{\varepsilon} \mathtt{V} \wedge b \mathrel{\varepsilon} \mathtt{V}\big). \tag{ob$_{\text{prt}}$}$$

Now we take (ob$_{\text{prt}}$) and (df =) to obtain (cf. T9, Pg 9):

$$\forall a\big(a \mathrel{\varepsilon} \mathtt{ing}(b) \to a \mathrel{\varepsilon} \mathtt{V} \wedge b \mathrel{\varepsilon} \mathtt{V}\big). \tag{ob$_{\text{ing}}$}$$

Moreover, from (AD1) and (df ⊂) we get (cf. T2 and T4 on Pg 1):

$$\forall a, b, c\big(a \mathrel{\varepsilon} \mathtt{prt}(b) \wedge b \mathrel{\varepsilon} \mathtt{prt}(c) \to a \mathrel{\varepsilon} \mathtt{prt}(c)\big). \tag{t$_{\text{prt}}$}$$

So, by (t$_{\text{prt}}$), (df ing), (df =) and (t$_\varepsilon$) we obtain (cf. T13, Pg 9):

$$\forall a, b, c\big(a \mathrel{\varepsilon} \mathtt{ing}(b) \wedge b \mathrel{\varepsilon} \mathtt{ing}(c) \to a \mathrel{\varepsilon} \mathtt{ing}(c)\big). \tag{t$_{\text{ing}}$}$$

Using the definitions and ontological theses we will give simpler but equivalent forms of axioms (AD1) and (AD2).

For (AD1) firstly notice that, by (qr$_\varepsilon$), (df V), (t$_\varepsilon$), (df =), (df ≠), (df ⊂) and (df −) we obtain:

$$\forall a, c\big(c \neq a \wedge \neg c \mathrel{\varepsilon} \mathtt{prt}(a) \wedge \mathtt{prt}(c) \subset -\mathtt{prt}(a) \;\to\; \mathtt{ing}(c) \subset -\mathtt{ing}(a)\big).$$

Secondly, from (r$_{\text{ing}}$) we obtain:

$$\forall a, c\big(\mathtt{ing}(c) \subset -\mathtt{ing}(a) \;\to\; c \neq a \wedge \neg a \mathrel{\varepsilon} \mathtt{prt}(c) \wedge \neg c \mathrel{\varepsilon} \mathtt{prt}(a) \wedge \mathtt{prt}(c) \subset -\mathtt{prt}(a)\big).$$

Hence, using (df ing), (qr$_\varepsilon$), (df V), (df extr), axiom (AD1) we can write in the following equivalent form:

$$\forall a, b(a \mathrel{\varepsilon} \mathtt{prt}(b) \;\to\; b \mathrel{\varepsilon} \mathtt{V} \wedge \mathtt{prt}(a) \subset \mathtt{prt}(b) \wedge \exists c(c \mathrel{\varepsilon} \mathtt{ing}(b) \wedge c \mathrel{\varepsilon} \mathtt{extr}(a) \wedge \forall d(d \mathrel{\varepsilon} \mathtt{ing}(b) \wedge d \mathrel{\varepsilon} \mathtt{extr}(a) \to d \mathrel{\varepsilon} \mathtt{ing}(c)))). \tag{AD1$'$}$$

From (AD1′) and (r$_{\text{ing}}$) we obtain (cf. T5, Pg 11):

$$\forall a \; \neg a \mathrel{\varepsilon} \mathtt{prt}(a). \tag{irr$_{\text{prt}}$}$$

Suppose for a contradiction that $a \mathrel{\varepsilon} \mathtt{prt}(a)$. Then for some c we have $c \mathrel{\varepsilon} \mathtt{ing}(a)$ and $c \mathrel{\varepsilon} \mathtt{extr}(a)$. But $c \mathrel{\varepsilon} \mathtt{ing}(c)$. Hence $\neg c \mathrel{\varepsilon} \mathtt{extr}(a)$, by (df extr).

Of course, by (t$_{\text{prt}}$) and (irr$_{\text{prt}}$) we have:

$$\forall a, b\big(a \mathrel{\varepsilon} \mathtt{prt}(b) \;\to\; \neg b \mathrel{\varepsilon} \mathtt{prt}(a)\big). \tag{as$_{\text{prt}}$}$$

So from (df ing), (df =), (t$_\varepsilon$), (as$_{\text{prt}}$) and (irr$_{\text{prt}}$) we have (cf. T10, Pg 9):

$$\forall a, b\big(a \mathrel{\varepsilon} \mathtt{ing}(b) \wedge b \mathrel{\varepsilon} \mathtt{ing}(a) \;\to\; a = b\big). \tag{antis$_{\text{ing}}$}$$

For (AD2), using (df ing) and (df ⊂), axiom (AD2) we can write as:

$$\forall a(\mathtt{ex}(a) \rightarrow \exists b(a \subset \mathtt{ing}(b) \wedge \forall d(a \subset \mathtt{ing}(d) \rightarrow b\, \varepsilon\, \mathtt{ing}(d)))). \qquad (\mathrm{AD2}')$$

Hence, by (df Kl), (df ex), ($\mathrm{ob}_{\mathtt{ing}}$), we have the following equivalent form of axiom (AD2) (cf. T16, Pg 9):

$$\forall a(\mathtt{ex}(a) \ \rightarrow\ \mathtt{ex}(\mathtt{Kl}(a))). \qquad (\mathrm{AD2}'')$$

3.2. Leśniewski's approach to mereology from 1922

Let us now turn to the axiomatics for mereology that Drewnowski knew from Leśniewski's lectures. To obtain this system we add to the language of ontology a primitive function constant 'ing' and the symbols defined by definitions (df ex), (df =), (df ≠), (df ⊂), (df △), (df Kl) and (df extr). Moreover, we use axiom (AO) and the following five: (AL1) := ($\mathrm{antis}_{\mathtt{ing}}$), (AL2) := ($\mathrm{t}_{\mathtt{ing}}$), (AL3) := (AD2″) and

$$\forall a, b(a\, \varepsilon\, \mathtt{V} \wedge b\, \varepsilon\, \mathtt{V} \wedge \forall d(d\, \varepsilon\, \mathtt{ing}(a) \wedge d\, \varepsilon\, \mathtt{extr}(b) \rightarrow \exists e(e\, \varepsilon\, \mathtt{ing}(a) \wedge e\, \varepsilon\, \mathtt{extr}(b) \wedge e\, \varepsilon\, \text{-}\,\mathtt{ing}(d))) \rightarrow a\, \varepsilon\, \mathtt{ing}(b)), \qquad (\mathrm{AL4})$$

$$\forall a, b(a\, \varepsilon\, \mathtt{ing}(b) \rightarrow b\, \varepsilon\, \mathtt{V}). \qquad (\mathrm{AL5})$$

Moreover, we accept the following definition of the function symbol 'prt':

$$\forall a, b(a\, \varepsilon\, \mathtt{prt}(b) \leftrightarrow a\, \varepsilon\, \mathtt{ing}(b) \wedge \neg\, a = b). \qquad (\mathrm{df}\ \mathtt{prt})$$

From (df prt), (AL5), ($\mathrm{qr}_{\varepsilon}$), (df V) and (df ≠) we also have:

$$\forall a, b(a\, \varepsilon\, \mathtt{prt}(b) \leftrightarrow a\, \varepsilon\, \mathtt{ing}(b) \wedge a \neq b).$$

The thus characterized mereology we call **ML22**.

Notice that in **ML22** the formula ($\mathrm{r}_{\mathtt{ing}}$) is provable. Indeed, at first directly from (AL4) we get:

$$\forall a(a\, \varepsilon\, \mathtt{V} \wedge \forall d(d\, \varepsilon\, \mathtt{ing}(a) \wedge d\, \varepsilon\, \mathtt{extr}(a) \rightarrow \exists e(e\, \varepsilon\, \mathtt{ing}(a) \wedge e\, \varepsilon\, \mathtt{extr}(a) \wedge e\, \varepsilon\, \text{-}\,\mathtt{ing}(d))) \rightarrow a\, \varepsilon\, \mathtt{ing}(a)).$$

So it is enough to show that the following formula is also a thesis:

$$\forall d(d\, \varepsilon\, \mathtt{ing}(a) \wedge d\, \varepsilon\, \mathtt{extr}(a) \rightarrow \exists e(e\, \varepsilon\, \mathtt{ing}(a) \wedge e\, \varepsilon\, \mathtt{extr}(a) \wedge e\, \varepsilon\, \text{-}\,\mathtt{ing}(d)).$$

For this purpose, we assume that:

$$\exists d(d\, \varepsilon\, \mathtt{ing}(a) \wedge d\, \varepsilon\, \mathtt{extr}(a) \wedge \forall e(e\, \varepsilon\, \mathtt{ing}(a) \wedge e\, \varepsilon\, \mathtt{extr}(a) \rightarrow \neg\, e\, \varepsilon\, \text{-}\,\mathtt{ing}(d))),$$

so we use an auxiliary name constant $\mathtt{d}$ such that (i) $\mathtt{d}\, \varepsilon\, \mathtt{ing}(a)$, (ii) $\mathtt{d}\, \varepsilon\, \mathtt{extr}(a)$ and (iii): $\forall e(e\, \varepsilon\, \mathtt{ing}(a) \wedge e\, \varepsilon\, \mathtt{extr}(a) \rightarrow \neg\, e\, \varepsilon\, \text{-}\,\mathtt{ing}(\mathtt{d}))$. Notice that from (i), ($\mathrm{qr}_{\varepsilon}$) and (df $\mathtt{V}$) we get $\mathtt{d}\,\varepsilon\,\mathtt{V}$. Moreover, from from (i) and (AL5) we have $a\,\varepsilon\,\mathtt{V}$. Now, by (iii), we have (iv): $\mathtt{d}\,\varepsilon\,\mathtt{ing}(a) \wedge \mathtt{d}\,\varepsilon\,\mathtt{extr}(a) \rightarrow \neg\,\mathtt{d}\,\varepsilon\,\mathtt{-ing}(\mathtt{d})$. Now from (i), (ii) and (iv) we have: $\neg\,\mathtt{d}\,\varepsilon\,\mathtt{-ing}(\mathtt{d})$. Hence $\mathtt{d}\,\varepsilon\,\mathtt{ing}(\mathtt{d})$, by (df $\mathtt{-}$). From this, (i), (df $\mathtt{extr}$), (df $\mathtt{C}$) and (df $\mathtt{-}$) we get $\neg\,\mathtt{d}\,\varepsilon\,\mathtt{extr}(a)$; which contradicts (ii).

3.3. Equivalence of both approaches to mereology

The subject of Drewnowski's analysis was to show the equivalence of **ML22** and **MD**.

In point 3.1 we showed that axioms (AL1), (AL2), (AL3) and (AL5) are derivable in **MD**. Axiom (AL4) is directly derived from T29 and the latter may be obtained from T28 and T8 (or (df $\mathtt{ing}$)). The commentary to the derivation of T24 (Pg 11) contains an error or a fragment of old notes: T24 is to be derived from T3 and (df $\mathtt{ing}$) and not from T23 (which is actually not needed in the main proof). T23 has a complex proof — it employs theses: T2, T3, T4, T5, T6, T7, T14, T15, T16, T17, T21, T22, T25 and T26. Theses T18 and T19 are used in the derivation of T20 which is a definition (df $\mathtt{prt}$) in **ML22**. Theses T11, T12, T23, and T24b are inferentially redundant for Drewnowski's approach. The whole argumentation is formally correct. Finally, (df $\mathtt{prt}$) is derivable from (df $\mathtt{ing}$), ($\mathrm{irr}_{\mathtt{prt}}$), (df $\mathtt{=}$) and (t_{ε}).

The derivability of the axioms of the system **MD** in the system **ML22** is not shown in the manuscript. To this end, we note that in **ML22** we have the following theses:

- ($\mathrm{ob}_{\mathtt{prt}}$), which we obtain from (AL5), (df $\mathtt{prt}$), ($\mathrm{qr}_{\varepsilon}$) and (df $\mathtt{V}$);
- ($\mathrm{as}_{\mathtt{prt}}$), which we obtain from ($\mathrm{antis}_{\mathtt{ing}}$) and (df $\mathtt{prt}$);
- ($\mathrm{t}_{\mathtt{prt}}$), which we obtain from ($\mathrm{t}_{\mathtt{ing}}$), ($\mathrm{antis}_{\mathtt{ing}}$), (df $\mathtt{prt}$), (df $\mathtt{=}$) and (t_{ε});
- (AD2′), which we obtain from (AD2″) and (df $\mathtt{Kl}$).

Furthermore, notice that (AL4) can be transformed by transposition to:

$$\forall a,b\big(a\,\varepsilon\,\mathtt{V} \wedge b\,\varepsilon\,\mathtt{V} \wedge \neg\, a\,\varepsilon\,\mathtt{ing}(b) \rightarrow \exists d(d\,\varepsilon\,\mathtt{ing}(a) \wedge d\,\varepsilon\,\mathtt{extr}(b) \wedge \forall e(e\,\varepsilon\,\mathtt{ing}(a) \wedge e\,\varepsilon\,\mathtt{extr}(b) \rightarrow \neg\, e\,\varepsilon\,\mathtt{-ing}(d)))\big).$$

So, by ($\mathrm{ob}_{\mathtt{ing}}$) and (df $\mathtt{-}$), we come to the analogue of the so-called *strong super-supplementation principle*:

$$\forall a,b\big(a\,\varepsilon\,\mathtt{V} \wedge b\,\varepsilon\,\mathtt{V} \wedge \neg\, a\,\varepsilon\,\mathtt{ing}(b) \rightarrow \exists d(d\,\varepsilon\,\mathtt{ing}(a) \wedge d\,\varepsilon\,\mathtt{extr}(b) \wedge \forall e(e\,\varepsilon\,\mathtt{ing}(a) \wedge e\,\varepsilon\,\mathtt{extr}(b) \rightarrow e\,\varepsilon\,\mathtt{ing}(d)))\big). \quad (\mathrm{SSP}^{\varepsilon}+)$$

Moreover, from ($\mathrm{antis}_{\mathtt{ing}}$), (df prt) and ($\mathrm{ob}_{\mathtt{prt}}$) we obtain:

$$\forall a, b\big(a\,\varepsilon\,\mathtt{prt}(b) \;\rightarrow\; a\,\varepsilon\,\mathtt{V} \wedge b\,\varepsilon\,\mathtt{V} \wedge \neg\, b\,\varepsilon\,\mathtt{ing}(a)\big).$$

Hence and from ($\mathrm{SSP}^{\varepsilon}+$) we have the analogue of the so-called *weak super-supplementation principle*:

$$\forall a, b(a\,\varepsilon\,\mathtt{prt}(b) \rightarrow \exists c(c\,\varepsilon\,\mathtt{ing}(b) \wedge c\,\varepsilon\,\mathtt{extr}(a) \wedge \forall d(d\,\varepsilon\,\mathtt{ing}(b) \wedge d\,\varepsilon\,\mathtt{extr}(a) \rightarrow d\,\varepsilon\,\mathtt{ing}(c)))). \quad (\mathrm{WSP}^{\varepsilon}+)$$

Thus, from ($\mathrm{ob}_{\mathtt{prt}}$), ($\mathrm{t}_{\mathtt{prt}}$) and ($\mathrm{WSP}^{\varepsilon}+$) we obtain (AD1′), which is equivalent to (AD1). Axiom (df ing) is derivable from (df prt), ($\mathrm{r}_{\mathtt{ing}}$), (df =) and ($\mathrm{t}_{\varepsilon}$). Finally, axiom (AD2) follows from (AD2′) and (df ing).

Why was Drewnowski interested in formalizing mereology using the primitive concept of *part*? In fact, the first version of mereology given by Leśniewski in 1916, used *part* as the primitive concept. Leśniewski reformulated that axiomatics in 1918, and in 1920 he created another one, this time using the primitive concept of *ingrediens*.[5] We think that Drewnowski's motives become clearer through two observations that we will make about **ML22** which we will formulate in the next two points. We formulate them in the next two points.

4. Collective classes as mereological sums

Our first note is addressed to definition (df Kl). Actually, (df Kl) expresses a different idea to Leśniewski's original definition of *collective class* used in approaches from 1916, 1918, and 1920. Let us recall symbol '`Cl`' and the following formulation of its ontological definition:

$$\forall a, b(a\,\varepsilon\,\mathtt{Cl}(b) \leftrightarrow a\,\varepsilon\,a \wedge b \subset \mathtt{ing}(a) \wedge \forall c(c\,\varepsilon\,\mathtt{ing}(a) \rightarrow \exists d(d\,\varepsilon\,b \wedge \mathtt{ing}(c) \,\triangle\, \mathtt{ing}(d)))). \quad (\mathrm{df\,Cl})$$

Formulas (df Kl) and (df Cl) are not logically equivalent. Anticipating our further analysis, we would say that '`Kl`' can be understood as the least upper bound of the range of a given non-empty concept (or a given non-empty general name). '`Cl`' represents the original idea of *collective class* (in the modern terminology: *mereological sum*) of elements that

[5] The above-mentioned axiomatics are discussed and compared in [11, pp. 118–122].

belong to the range of a given non-empty concept (general name).[6] The symbols '$\mathtt{Kl}$' and '$\mathtt{Cl}$' have extensionally different meanings.

Nevertheless, in Leśniewski's mereology from 1920 the following formula is derivable:[7]

$$\forall a, b(a \mathrel{\varepsilon} \mathtt{Cl}(b) \leftrightarrow a \mathrel{\varepsilon} \mathtt{V} \wedge \mathtt{ex}(b) \wedge b \subset \mathtt{ing}(a) \wedge \forall c(b \subset \mathtt{ing}(c) \rightarrow a \mathrel{\varepsilon} \mathtt{ing}(c))).$$

This remarkable fact opens up the possibility of using (df $\mathtt{Kl}$) or (df Cl) equivalently in the version of Leśniewski's system under consideration, i.e., we have:

$$\forall a, b(a \mathrel{\varepsilon} \mathtt{Cl}(b) \leftrightarrow a \mathrel{\varepsilon} \mathtt{Kl}(b)).$$

In fact, the same situation happens in the case of **ML22**. Let us note that in **ML22** (as well as in **MD**, of course) we have:

$$\forall a, b(a \mathrel{\varepsilon} \mathtt{Kl}(b) \leftrightarrow a \mathrel{\varepsilon} \mathtt{V} \wedge b \subset \mathtt{ing}(a) \wedge \\ \forall c(c \mathrel{\varepsilon} \mathtt{ing}(a) \rightarrow \exists d(d \mathrel{\varepsilon} b \wedge \mathtt{ing}(c) \mathbin{\Delta} \mathtt{ing}(d)))).$$

For the proof of the "$\rightarrow$"-part we need (df $\mathtt{Kl}$), (AL5), ($\mathrm{t}_{\mathrm{ing}}$), and '$a \mathrel{\varepsilon} \mathtt{V} \wedge b \mathrel{\varepsilon} \mathtt{V} \wedge \neg\, a \mathrel{\varepsilon} \mathtt{ing}(b) \rightarrow \exists c(c \mathrel{\varepsilon} \mathtt{ing}(a) \wedge c \mathrel{\varepsilon} \mathtt{extr}(b))$'. The latter follows from (AL4) on the basis of classical logic. For the proof of the "$\leftarrow$"-part we need (AL1), (AL4) and ($\mathrm{r}_{\mathrm{ing}}$).

5. Super-supplementation principles

The second interesting observation, concerns axiom (AL4) itself. It is worth noting that Leśniewski did not use (AL4) or any of its equivalents in his published axiomatics for mereology. As we have shown, axiom (AL4) imples the analogue of the strong super-supplementation principle ($\mathrm{SSP}^{\varepsilon}+$), which corresponds to the idea that for any two objects a and b which cross each other or which are exterior to each other, there exists "the biggest of the remaining parts" of a, which is exterior to b. Just this principle Drewnowski called in [4] "the postulate of the existence of subtractions" and treated it as a *specific assumption* of "the most perfect extension of the set theory" [4, p. 116], which in his opinion was

[6] Formulas (df $\mathtt{Kl}$) and (df Cl) correspond to the abbreviations $\mathtt{Lub}_{\varphi} z$ and $\mathtt{Sum}_{\varphi} z$ from Section 6, respectively. We refer the reader to an efficient analysis of the original descriptions of the concept of collective class presented in [7, 9].

[7] This formula has been proved on the basis of Leśniewski's mereology by Tarski already in 1921 [cf. 5, pp. 327–328].

just mereology. As he claimed, the so-called specific assumptions "do not express anything absolute [...] — it is more appropriate to formulate them as respective conditions and mention them in a shortened version in the antecedents of the claims of the theory [...]" [4, p. 67]. He wrote about "the postulate of the existence of subtractions" in the following way:

> An example of such a shortcut is the condition saying that if in a given domain a_3 any object y_2 is not included in object x_2, then a given object z_2 is a non-identity, the remaining part when object x_2 has been subtracted from object y_2. [...] That condition can be called the postulate of the existence of subtractions. [4, p. 67]

but he explained it, making reference to another principle:

> That postulate is fulfilled explicitly for instance in the domain of physical bodies because whatever remains after a part of a body has been removed always, remains a physical body. [4, p. 67]

The latter one could be expressed in our notation as (WSP$^{\varepsilon}$+), which says that if a is a part of b, then there exists object that is "the biggest of the remaining parts" of b and exterior to a. Obviously, (WSP$^{\varepsilon}$+) follows from (AD1′), which ie equivalent to (AD1).

If Drewnowski was interested in the question of finding a theory in which (WSP$^{\varepsilon}$+) is raised to the rank of a specific postulate (and thus made it possible to formulate a proof of the principle (SSP$^{\varepsilon}$+)), then the analyzed notes just express this kind of interest. This is how we understand the information added as the introduction to the whole manuscript.

As we said before, Leśniewski's own axiomatics from 1916, as well as the versions from 1918 and 1920, did not include any formulation of (AL4)/(SSP$^{\varepsilon}$+). The same applies to (WSP$^{\varepsilon}$+) and its equivalents. What Drewnowski achieved was an axiomatics with the primitive concept of *part* and the principle (WSP$^{\varepsilon}$+). Let us note that thesis T3 from Drewnowski's system is a component of axiom (AD1), and with the use of (df ing) and (df extr) it is equivalent to (WSP$^{\varepsilon}$+) which is (WSP$^{\varepsilon}$+).

As far as we know, Drewnowski's result is original and was not repeated in any version of mereology built on ontology, created up to 1954.[8] We also did not find contemporary formalizations that would take any formulation analogous to (WSP$^{\varepsilon}$+) as an axiom.

[8] This can be seen thanks to Sobociński's synthetic study [10].

6. A modern look at two old systems

We want now to reconsider Drewnowski's approach using modern mereological tools which extract mereological notions from the ontological context. Following this path, we use the first-order language $\mathcal{L}$ with the identity '$=$' and the two-argument predicates '$\sqsubset$' ("is a proper part of"), '$\sqsubseteq$' ("is an ingrediens"), '$\wr$' ("is exterior to") and '$\mathtt{O}$' ("overlaps"). The formulas '$x \sqsubset y$', '$x \sqsubseteq y$' and '$x \wr y$' correspond to '$x \,\varepsilon\, \mathtt{cz}(y)$', '$x \,\varepsilon\, \mathtt{ing}(y)$' and '$x \,\varepsilon\, \mathtt{extr}(y)$', respectively.

Furthermore, in $\mathcal{L}$ for any its formula φ which contains the free variable 'x' and does not contain the variable 'z' as free we put the following abbreviation:

$$\mathtt{Lub}_{\varphi} z \quad \text{stands for:} \quad \forall x(\varphi \to x \sqsubseteq z) \wedge \forall y(\forall x(\varphi \to x \sqsubseteq y) \to z \sqsubseteq y).$$

Informally, $\ulcorner \mathtt{Lub}_{\varphi} z \urcorner$ says that z is a least upper bound of the φs.[9] Moreover, the formula $\ulcorner \mathtt{Lub}_{\varphi} z \wedge \exists x\, \varphi \urcorner$ is a modern counterpart of $\ulcorner z \,\varepsilon\, \mathtt{Kl}(b) \urcorner$, where b is a general name of the members of the set $\{x : \varphi(x)\}$. Thus, if $\exists x\, \varphi(x)$, then $\ulcorner \mathtt{Lub}_{\varphi} \urcorner$ is a counterpart of $\ulcorner \mathtt{Kl}(b) \urcorner$.

6.1. A reconstruction of Drewnowski's system MD

We take as primitive the predicate '$\sqsubset$'. The other three predicates are defined as follows:

$$\forall x, y(x \sqsubseteq y \leftrightarrow x \sqsubset y \vee x = y), \tag{df$\sqsubseteq$}$$

$$\forall x, y\big(x \wr y \leftrightarrow \neg \exists z(z \sqsubseteq x \wedge z \sqsubseteq y)\big), \tag{df$\wr$}$$

$$\forall x, y\big(x \mathbin{\mathtt{O}} y \leftrightarrow \exists z(z \sqsubseteq x \wedge z \sqsubseteq y)\big), \tag{df$\mathtt{O}$}$$

Directly from the definitions it follows that the predicates '$\wr$' and '$\mathtt{O}$' are symmetrical and '$\sqsubseteq$' is reflexive:

$$\forall x\, x \sqsubseteq x. \tag{r$_{\sqsubseteq}$}$$

To axioms and rules of the first-order logic with identity in $\mathcal{L}$ we add the following axioms:

$$\forall x, y, z(x \sqsubset y \wedge y \sqsubset z \to x \sqsubset z), \tag{t$_{\sqsubset}$}$$

$$\forall x, y\big(x \sqsubset y \to \exists z(z \sqsubseteq y \wedge z \wr x \wedge \forall u(u \sqsubseteq y \wedge u \wr x \to u \sqsubseteq z))\big), \tag{WSP+}$$

and the following axiom schema:

$$\exists x\, \varphi \;\to\; \exists z\, \mathtt{Lub}_{\varphi} z, \tag{exLub}$$

[9] In [1, 12] the term 'minimal upper bound of the ϕs' is used and the abbreviations $F_{\varphi} z$ and $Mub(z, \phi_x)$, respectively.

where φ contains the free variable 'x' and does not contain the variable 'z' as free. Axioms ($\mathrm{t}_{\sqsubset}$) and (WSP+) correspond to ($\mathrm{t}_{\mathrm{ing}}$) and ($\mathrm{WSP}^{\varepsilon}+$), respectively. Let us remind that from ($\mathrm{ob}_{\mathrm{prt}}$), ($\mathrm{t}_{\mathrm{prt}}$) and ($\mathrm{WSP}^{\varepsilon}+$) we obtain (AD1′), which is equivalent to (AD1). Thus, the conjunction ($\mathrm{t}_{\sqsubset}$) and (WSP+) is equivalent to the counterpart of (AD1) in $\mathcal{L}$. Finally, the schema (exLub) corresponds to (AD2″), which is equivalent to (AD2).

Just as ($\mathrm{WSP}^{\varepsilon}+$) can be transformed to (WSP+), also ($\mathrm{SSP}^{\varepsilon}+$) can be reformulated as follows:

$$\forall x, y(\neg\, x \sqsubseteq y \rightarrow \exists z(z \sqsubseteq x \wedge z \wr y \wedge \forall u(u \sqsubseteq x \wedge u \wr y \rightarrow u \sqsubseteq z))). \quad \text{(SSP+)}$$

Following the terminology proposed by Pietruszczak [9, pp. 93–94], we call (WSP+) and (SSP+), respectively: *weak super-supplementation* and *strong super-supplementation principles* (in [7, p. 106] the latter is called *super strong supplementation principle*). From the above principles we obtain *weak supplementation* and *strong supplementation principles*:

$$\forall x, y\big(x \sqsubset y \rightarrow \exists z(z \sqsubseteq y \wedge z \wr x)\big), \qquad \text{(WSP)}$$

$$\forall x, y\big(\neg\, x \sqsubseteq y \rightarrow \exists z(z \sqsubseteq x \wedge z \wr y)\big). \qquad \text{(SSP)}$$

As noted in [9, remarks II.3.1 and III.6.1], in the light of ($\mathrm{t}_{\sqsubset}$), conditions (WSP) and (WSP+) may be given the following equivalent form, respectively:

$$\forall x, y\big(x \sqsubset y \rightarrow \exists z(z \sqsubset y \wedge z \wr x)\big),$$

$$\forall x, y\big(x \sqsubset y \rightarrow \exists z(z \sqsubset y \wedge z \wr x \wedge \forall u(u \sqsubset y \wedge u \wr x \rightarrow u \sqsubseteq z))\big).$$

Furthermore, as noted in [7, p. 83], from (WSP) we obtain (cf. the proof of conditions ($\mathrm{irr}_{\mathrm{prt}}$)):

$$\forall x \ \neg\, x \sqsubset x. \qquad (\mathrm{irr}_{\sqsubset})$$

Hence and from ($\mathrm{t}_{\sqsubset}$) we have:

$$\forall x, y(x \sqsubset y \ \rightarrow \ \neg\, y \sqsubset x). \qquad (\mathrm{as}_{\sqsubset})$$

Of course, by ($\mathrm{t}_{\sqsubset}$), ($\mathrm{as}_{\sqsubset}$) and ($\mathrm{irr}_{\sqsubset}$) we have:

$$\forall x, y, z(x \sqsubseteq y \wedge y \sqsubseteq z \rightarrow x \sqsubseteq z), \qquad (\mathrm{t}_{\sqsubseteq})$$

$$\forall x, y, z(x \sqsubseteq y \wedge y \sqsubseteq x \rightarrow x = y) \qquad (\mathrm{antis}_{\sqsubseteq})$$

Furthermore, ($\mathrm{antis}_{\sqsubseteq}$) entails the uniqueness of $\mathtt{Lub}_{\varphi}$, i.e., for any formula φ which contains the free variable 'x' and does not contain the variables 'z' and 'u' as free we have:

$$\forall z, u(\mathtt{Lub}_{\varphi} z \wedge \mathtt{Lub}_{\varphi} u \ \rightarrow \ z = u).$$

Now let us take the formula $\varphi_0 :=$ '$x \sqsubseteq u \wedge x \sqsubseteq v$'. Then for φ_0 from axiom schema (exLub) we obtain:

$$\exists x(x \sqsubseteq u \wedge x \sqsubseteq v) \rightarrow$$
$$\exists z\big(\forall x(x \sqsubseteq u \wedge x \sqsubseteq v \rightarrow x \sqsubseteq z) \wedge \forall y(\forall x(x \sqsubseteq u \wedge x \sqsubseteq v \rightarrow x \sqsubseteq y) \rightarrow z \sqsubseteq y)\big).$$

Hence and from ($\mathrm{t}_{\sqsubseteq}$) we have the following condition:

$$\forall u, v\big(\neg\, u \wr\, v \;\rightarrow\; \exists z\big(\forall y(y \sqsubseteq z \leftrightarrow y \sqsubseteq u \wedge y \sqsubseteq v)\big)\big), \tag{c$\exists\sqcap$}$$

which states the conditional existence of an object, this being the "product" of two overlapping objects.

In [9, Theorem III.6.2; in Eng. trans. Theorem III.6.11] it is proved that ($\mathrm{t}_{\sqsubseteq}$), (WSP+) and (c$\exists\sqcap$) entail (SSP+).

6.2. A reconstruction of Leśniewski's system ML22

In the language $\mathcal{L}$ we present the system of classical mereology which is a modern analogue of Leśniewski's system **ML22**. So we take as primitive the predicate '$\sqsubseteq$'. Furthermore, we accept (df $\wr$), (df $\circ$) and the following definition for '$\sqsubset$':

$$\forall x, y(x \sqsubset y \leftrightarrow x \sqsubseteq y \wedge x \neq y). \tag{df$\sqsubset$}$$

Directly from (df $\sqsubset$) we obtain ($\mathrm{irr}_{\sqsubset}$).

The system is characterized by the addition to axioms and rules of first-order logic with identity in $\mathcal{L}$ three axioms ($\mathrm{t}_{\sqsubseteq}$), ($\mathrm{antis}_{\sqsubseteq}$), (SSP+) and the axiom schema (exLub).

Let us recall that from (SSP+) we obtain (SSP). In [7, pp. 91 and 157] it is proved that ($\mathrm{r}_{\sqsubseteq}$) follows from ($\mathrm{t}_{\sqsubseteq}$) and (SSP). Thus, we see that our system is a modern analogue of Leśniewski's system **ML22**.

Finally, notice that from ($\mathrm{antis}_{\sqsubseteq}$) we have ($\mathrm{as}_{\sqsubset}$); and ($\mathrm{antis}_{\sqsubseteq}$) and ($\mathrm{t}_{\sqsubseteq}$) entail ($\mathrm{t}_{\sqsubset}$).

Remark. In [1] Cotnoir and Varzi proposed the system for classical mereology, CM, by adding to axioms and rules of first-order logic with identity in $\mathcal{L}$ the axiom schema (exLub) and axioms ($\mathrm{r}_{\sqsubseteq}$), ($\mathrm{t}_{\sqsubseteq}$), ($\mathrm{antis}_{\sqsubseteq}$), (df $\wr$), (df $\sqsubset$) and the following remainder principle:

$$\forall x, y\big(\neg\, x \sqsubseteq y \rightarrow \exists z \forall u(u \sqsubseteq z \leftrightarrow u \sqsubseteq x \wedge u \wr\, y)\big). \tag{RP}$$

In [9, Eng. trans.; Theorem III.6.3] it is proved that (RP) and ($\mathrm{r}_{\sqsubseteq}$) entail (SSP+); and (SSP+) and ($\mathrm{t}_{\sqsubseteq}$) entail (RP). Thus, we see that Cotnoir–Varzi's system is equivalent to our system. ⊣

6.3. Definitional equivalence of both modern systems

We will show that both modern systems presented above are definitionally equivalent.

Firstly, in point 6.1 we showed that all axiom ($t_{\sqsubseteq}$), ($antis_{\sqsubseteq}$) and (SSP+) of the modern counterpart of **LM22** are theses of the modern counterpart of **MD**. Furthermore, (df $\sqsubset$) is derivable from (df $\sqsubseteq$) and ($irr_{\sqsubset}$).[10]

Secondly, we showed that ($t_{\sqsubset}$) is a thesis of the modern counterpart of **ML22**. In [9, Lemma III.6.1] it is proved that (SSP+), ($as_{\sqsubset}$), and ($r_{\sqsubseteq}$) entail (WSP+). Finally, (df $\sqsubseteq$) is derivable from (df $\sqsubset$) and ($r_{\sqsubseteq}$).

6.4. Mereological sums

Let us recall that (df $\mathtt{Kl}$) expresses different idea than Leśniewski's original definition of *collective class* used in approaches from 1916, 1918, 1920 and after 1922. The original idea of Leśniwski's collective classes is presented by the definition (df Cl). Its modern counterpart can be expressed as follows:

$$\mathtt{Sum}_\varphi z \quad \text{stands for:} \quad \forall x(\varphi \to x \sqsubseteq z) \wedge \forall y(y \sqsubseteq z \to \exists x(\varphi \wedge y \circ x)),$$

where φ contains the free variable 'x' and does not contain the variable 'z' as free. Informally, $\ulcorner \mathtt{Sum}_\varphi z \urcorner$ says that z is a mereological sum of the φs and it is a modern counterpart of $\ulcorner z \,\varepsilon\, \mathtt{Cl}(b) \urcorner$, where b is a general name of members of $\{x : \varphi(x)\}$.

From ($r_{\sqsubseteq}$) we obtain:

$$\exists z\, \mathtt{Sum}_\varphi z \;\to\; \exists x\, \varphi.$$

Moreover, by Lemma II.8.2 from [7], from ($t_{\sqsubseteq}$) and (SSP) we obtain:

$$\forall z(\mathtt{Sum}_\varphi z \;\to\; \mathtt{Lub}_\varphi z).$$

Hence and from ($antis_{\sqsubseteq}$) we obtain the uniqueness of $\mathtt{Sum}_\varphi$, i.e., for any formula φ which contains the free variable 'x' and does not contain the variables 'z' and 'u' as free we have [see also 7, pp. 91 and 156]:

$$\forall z, u(\mathtt{Sum}_\varphi z \wedge \mathtt{Sum}_\varphi u \;\to\; z = u).$$

[10] The proof may be formulated following the path from Drewnowski's manuscript completed by us in the previous sections.

Furthermore, by Lemma V.3.2 from [7], conditions ($\mathrm{as}_{\sqsubset}$), ($\mathrm{t}_{\sqsubset}$) and (WSP) we obtain the following:

$$\forall z(\mathtt{Sum}_{\varphi} z \wedge \mathtt{Lub}_{\varphi} u \;\rightarrow\; z = u).$$

In [7, Proposition V.6.3] it is proved that conditions ($\mathrm{r}_{\sqsubseteq}$), ($\mathrm{t}_{\sqsubseteq}$), ($\mathrm{antis}_{\sqsubseteq}$) and (SSP+) entail the following:

$$\forall z(\mathtt{Lub}_{\varphi} z \wedge \exists x\, \varphi \;\rightarrow\; \mathtt{Sum}_{\varphi} z).$$

Thus, in both modern reconstructions of **MD** and **ML22** we obtain:

$$\forall z(\mathtt{Sum}_{\varphi} z \;\leftrightarrow\; \mathtt{Lub}_{\varphi} z \wedge \exists x\, \varphi),$$

which corresponds to the condition '$\forall a, b(a \mathrel{\varepsilon} \mathtt{Cl}(b) \;\leftrightarrow\; a \mathrel{\varepsilon} \mathtt{Kl}(b))$' on p. 204. Therefore, in both modern reconstructions of **MD** and **ML22** the following axiom schema also holds:

$$\exists x\, \varphi \;\rightarrow\; \exists z\, \mathtt{Sum}_{\varphi} z, \qquad (\mathrm{ex}\mathtt{Sum})$$

where φ contains the free variable 'x' and does not contain the variable 'z' as free.

In [7, 9] it is proven that by taking axiom schema (ex`Lub`), axioms (SSP+) and (WSP+) cannot be replaced by (SSP) and (WSP), respectively. In addition, in [7, 9] it is proven that such replacements are possible if we take (ex`Sum`) instead of (ex`Lub`).

Acknowledgments. We would like to express our gratitude to Mr Jacek Drewnowski for making the analyzed manuscript available. We are also grateful to Andrzej Pietruszczak for important and informative discussions and especially for his significant help in reformulating the last paragraph.

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