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TOWARDS CONTINGENT WORLD DESCRIPTIONS IN DESCRIPTION LOGICS

Abstract. The philosophical, logical, and terminological junctions between Description Logics (DLs) and Modal Logic (ML) are important because they can support the formal analysis of modal notions of 'possibility' and 'necessity' through the lens of DLs. This paper introduces functional contingents in order to (i) structurally and terminologically analyse 'functional possibility' and 'functional necessity' in DL world descriptions and (ii) logically and terminologically annotate DL world descriptions based on functional contingents. The most significant contributions of this research are the logical characterisation and terminological analysis of functional contingents in DL world descriptions. The ultimate goal is to investigate how modal operators can — logically and terminologically — be expressed within DL world descriptions.

Keywords: conditional information; contingent interpretation; contingent world description; description logics; functional contingents; knowledge representation; modality; three-valued semantics

1. Introduction

In the recent decades, knowledge representation in information and computer sciences has experienced significant improvements [see 14, 33, 63]. Underlying description logics (DLs) are now among the most widely used knowledge representation formalisms in semantics-based systems. DLs have emerged from semantic networks [47] and frame-based systems [40]. Most DLs are decidable fragments of predicate logic (PL). More specifically, DLs are PL-based terminological systems developed out of the attempt to represent knowledge, with a formal semantics, in order

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to establish a common ground for human and machine interactions [see 2, 3, 18, 48, 56].

This research deals with the notations of 'possibility' and 'necessity' within DL world descriptions, so the concepts of 'possibility' and 'necessity' have to be taken into account. The idea of thinking about possibilities as well as necessities within possible worlds complements the developments of logics with modalities, qualities, conditions, and their philosophical reflections on the world. My most central assumption based on possibilistic approaches is that the explicit statements are at least possible in the sense that they are logically and conceptually consistent. More specifically, the possibilities express what has had the potential of being valid and, subsequently, being meaningful. Accordingly, the necessities expose the certain available beliefs and, consequently, express what has been valid and meaningful in all possible worlds.

Modal logic (ML) is the most well-known logic of possibilities, necessities, and other conceptions related to them [see 10, 25]. ML provides a formal basis for expressing possibilities and necessities as well as for defining a semantics in terms of possible worlds [see 51]. Regarding the strong (i) logical dependencies between DLs and PL, and (ii) syntactical relationships between ML and DLs,[1] the foremost objective of this research is the logical and terminological analysis of knowledge representation based on conditional information. The overarching goal is investigating how modal notations can — terminologically and logically — be analysed within DL world descriptions. This research defines DL-based functional contingents in order to analyse contingent world descriptions. The most significant contribution will be the logical-terminological analysis of the interconnections between DL world descriptions and ML notations of 'possibility' and 'necessity'.[2] By defining a three-valued semantics, this article will reveal how to provide semantics for logical/terminological structures based on conditional information.

[1] The following sections will demonstrate that most DLs syntactically correspond to modal logic.

[2] DLs and ML are logical formalisms to capture, represent, and express the structure of variant forms of inferential and reasoning procedures.

2. Literature Review

Several logical approaches focus on checking validity, soundness, and completeness within knowledge analysis through possible and probable worlds. As mentioned, modal logic is the most well-known logic of possibilities, necessities, and other concepts related to them. Fuzzy logic approaches in the theory of possibility [16, 19, 36, 49, 50, 66], belief functions [9, 17, 38, 54, 55, 65], and possibility measures [1, 15, 20, 23, 34, 35, 59] are other salient approaches to the analysis of knowledge through possible worlds. Also, Doxastic Logic has been concerned with reasoning about beliefs. It has perhaps 'belief' as a modal operator [see 39, 53].

2.1. Probabilistic Approaches in Description Logics

Focusing specifically on DLs and terminological systems, there has been a strong interest in expressive probabilistic description logics [see 11, 30, 37]. The works just cited have also focused on the connections between expressive probabilistic description logics and the first-order logic of probability. Note that this research does not deal with the notation of probability, but only with possibility.

2.1.1. Possibility and probability.

The term 'probability' expresses the state or the fact of being probable of either happening or being true. The probability of event E can be seen to be equivalent to the quantificational measure of the likelihood that E will occur. Therefore, the probability of fact F is equivalent to the quantificational measure of the likelihood that F will be true and acceptable.

In contrast, the term 'possibility' is concerned with the state of being possible. Being possible has two characteristics: (i) having the potential of happening or being done and (ii) having the potential of being satisfactory and acceptable. More precisely, the possibility of event E is concerned with E's qualificational state of having the potential of happening and being done. Also, the possibility of fact F corresponds with F's qualificational state of being true (as well as satisfactory and acceptable).

2.2. Possibilistic Description Logics

There has been significant research in possibilistic approaches. [43] developed an epistemic operator for description logics. [28] focused on proof methods in possibilistic logic and on possibilistic logic applications to terminological logics, [32] modelled imprecise arguments in DLs, [21] dealt with uncertainty, possibility, and fuzzy features in DLs, [57] and [58] handled fuzziness[3] in DLs within the Semantic Web context, and [12] offered a reasoning framework based on fuzzy DL $\mathcal{SROIQ}$ [see 31].[4] [44] sees DLs through the lens of possibilistic logic and focuses on developing a possibilistic extension to DLs. It has associated a DL-based formula with a number in $[0, 1]$. Furthermore, [44] offered an adequate syntax and semantics for a possibilistic extension of DLs. In addition, [45, 46] have extended DLs with uncertainty reasoning in possibilistic logic. Furthermore, [26] and [27] offered reasoning frameworks for ontologies based on inconsistent possibilistic description logics. [41] offered a possibilistic DL extension for an uncertain geographic ontology, and [7] created a possibilistic extension of the Web Ontology Language based on [41]. Moreover, [8] designed a possibilistic description logic for uncertain geographic information.

Note that there are numerous works on DLs with rules and other normative notions (e.g., RuleML & Fuzzy RuleML [62] and SWRL [64]), as well on defeasible and non-monotonic DLs. The most common feature of possibilistic approaches in knowledge representation systems is representing the degree(s) of compatibility of an interpretation with available beliefs, where the beliefs are produced based on incomplete knowledge.

3. Description Logics

Description Logics (DLs) are the most well-known (terminological) knowledge representation formalisms in semantics-based systems [see 2, 3, 4, 13, 52, 56]. DLs represent knowledge in terms of (i) *individuals*

[3] Mentioning fuzzy DLs does not mean mixing up possibilistic and fuzzy formalisms, but taking into account that possibilistic logics fall under uncertainty theory. More specifically, the statements in a possibilistic logic are either true or false (to some possibility), whereas in fuzzy logics statements are true (to a certain degree). In this context, fuzzy DLs can show how we can formally represent the floating degrees of truth (between 0 (stands for absolute falsity) and 1 (stands for absolute truth)) within terminological systems. Undoubtedly, fuzzy-based approaches have, both formally and mathematically, supported the analysis of possibility and necessity measures [e.g. 22, 29].

[4] The $\mathcal{SROIQ}$ is the underlying description logic of the Web Ontology Language (OWL).

(that are equivalent to constant symbols in predicate logic), (ii) *concepts* (that are equivalent to unary predicates in predicate logic), and (iii) *roles* (that are equivalent to n-ary predicates in predicate logic and can be either relations or properties). More specifically, a role expresses a relationship between individuals or it assigns a property to an individual. A role is a relation defined with some valence greater than or equal to 0.

In description logics, a concept corresponds to a distinct (conceptual) entity. Also, it can be regarded as a class of other entities (e.g., objects, subjects). It shall be taken into consideration that concepts and their interrelationships are, in the form of hierarchical structures, used to create a terminology. Subsequently, the individuals are regarded as instances of concepts. For example, the individual `john` can be an instance of the concept *Student*. The predicates (either unary or n-ary) are the most important building blocks in predicate logic. The most significant fact about predicate logic (which is, terminologically, the parent of DLs) is that the unary predicate P in a formula is capable of covering something (e.g., the variable x[5]) and, in fact, P can describe x. Thus, we can have the logical term '$P(x)$'. Subsequently, the world description '$P(x)$' expresses that the variable x (that can be any possible individual) is an instance of predicate P; thus x comes under the label of P. Therefore, P can describe x. After the transformation of a predicate-based formula (in predicate logic) into a concept-based formula (in description logic), the predicate P manifests itself in the form of a (possibly specified) concept (like C).

In DLs, there are three kinds of atomic symbols: (i) individuals, e.g., `bob`, `pizza`, (ii) atomic concepts, e.g., *Person*, *Bird*, and *Food*, and (iii) atomic roles, e.g., *hasMother*, *isEating*, and *isMoving*. Atomic symbols are elementary descriptions from which we inductively build complex (more-specified) descriptions based on concept (and role) constructors. More specifically, the individual `bob` is related to itself by means of the relation of valence 0. The term 'Fred is a student' (formally: *Student*(`fred`)) is structured based on the relation of valence 1. Also, the terms 'Sebastian is married to Juliana' (formally: *marriedTo*(`sebastian`, `juliana`)), '10 is greater than 3' (formally: *greaterThan*(10, 3)), and 'Bob is the father of Alice' (formally: *hasFather*(`alice`, `bob`) are structured based on the relations of valence 2. Obviously, there are relations of greater valences as well.

[5] The variable x is a relation of valence 0. It can express any possible individual.

3.1. The Syntactic Relationship between DLs and ML

The set of the main connectors in $\mathcal{ALC}$ (Prototypical DL Attributive Language with Complements) is: {conjunction ($\sqcap$), disjunction ($\sqcup$), negation[6]($\neg$), existential restriction ($\exists$), universal restriction ($\forall$)}. In addition, we have tautology ($\top$), contradiction ($\bot$), and, as mentioned, atomic concepts and atomic roles.

As mentioned earlier, most DLs are syntactically modal logics. More specifically, $\mathcal{ALC}$ is developed using modal logic **K**[7] as its foundation. In other words, $\mathcal{ALC}$ is a syntactic variant of modal logic **K**. More precisely, I assume that the DL symbols '$\neg$' (for 'not') and '$\Rightarrow$' (for 'if ..., then ...') are the translations of the ML symbols '$\sim$' (for 'not'), '$\rightarrow$' (for 'if ..., then ...'). The DL symbols '$\sqcap$' (for 'and'), '$\sqcup$' (for 'or'), and '$\Leftrightarrow$' (for 'if and only if ..., then ...') are definable from DL '$\neg$' and '$\Rightarrow$' the same way as in propositional logic and, in fact, as in the modal logic **K**. More specifically, modal logic **K** defines the logical symbols '&' (for 'and'), '$\vee$' (for 'or'), and '$\leftrightarrow$' (for 'if ..., then ...') from '$\sim$' (for 'not') and '$\rightarrow$' (for 'if ..., then ...').

Such a syntactic similarity can be found between ML and other members of the family of DLs as well. For example, the description logic $\mathcal{SR}$ (that denotes $\mathcal{ALC}$ extended with all kinds of rule-based axioms and self-constructs) is a syntactic variant of Propositional Dynamic Logic [61]. Dynamic logics are modal logics for representing the states and the events of dynamic systems. The language of dynamic logics is both an assertion language able to express properties of computation states, and a programming language able to express properties of system transitions between these states. They are logics of programs, and permit us to describe and reason about states of affairs, processes, changes, and results [see 60]. Also, there are some DLs that are syntactic variants of the Deterministic Propositional Dynamic Logic [see 5, 6].

3.2. Knowledge Bases in DLs

A DL knowledge base usually consists of the terminological axioms (that describe the underlying terminologies and vocabularies), and assertional

[6] In some DLs, such as in $\mathcal{AL}$, negation is permitted on atomic concepts only, while in more expressive DLs it is permitted on complex concepts as well.

[7] **K** was named after Saul Aaron Kripke, who is an American logician and philosopher. Kripke is well-known for his valuable works on the semantics of modal logic.

axioms (that describe the world). The *concept inclusion* and *role inclusion* axioms (in the form of $C \sqsubseteq D$ and $R \sqsubseteq S$, where C and D stand for two concepts, and R and S stand for two roles) are the most fundamental terminologies. In addition, (ii) the *concept equality* and *role equality* axioms (in the form of $C \equiv D$ and $R \equiv S$) are other terminological axioms and are generally defined from concept inclusions and role inclusions. Furthermore, (iii) the *concept assertions* and *role assertions* (in the form of $C(a)$ and $R(a_1, a_2, \ldots, a_n)$, where C stands for an atomic concept, R stands for an atomic role, and a_i (for $i \in [1, n]$) stands for an individual symbol) are the most fundamental descriptions of the world. Note that any specified description of the world is expressible based on fundamental descriptions of that world [see 5, 6, 42].

3.3. Terminological Interpretations in DLs

Note that the formal semantics of a term in DLs is interpretable based on individuals, concepts, and roles. In fact, they are the non-logical symbols in logical descriptions; hence, they do not independently have any logical consequence in a world description. Therefore, we need to utilise terminological interpretation in order to become involved with providing a semantics. A terminological interpretation (like $\mathcal{I}$) consists of (i) a non-empty set Δ (that is the interpretation domain and consists of any variable that occurs in any of the concept descriptions), and (ii) an interpretation function (like '$^{\mathcal{I}}$'). The function '$^{\mathcal{I}}$' assigns every individual symbol to an element $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$. Also, it assigns to every atomic concept A (or every atomic unary predicate) a set $A^{\mathcal{I}} \in \Delta^{\mathcal{I}}$, and to every atomic role P (or every atomic binary predicate) a binary relation $P^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$. Table 1 represents the syntax and semantics of concept constructors in $\mathcal{ALC}$ [see 5, 6, 42].

Table 2 reports the terminological and assertional axioms in DLs [see 5, 6, 42]. Note that an interpretation is called a *model* of an axiom if it can satisfy the terminological axioms and fundamental world descriptions.

4. Functional Roles in Description Logics

In Description Logics, the *functional roles* (or *features*) are special kinds of roles (relations). Thus, $N_F \subseteq N_R$, where N_F and N_R stand for the 'set of functional roles' and 'set of roles', respectively. Functional roles can

Syntax	**Semantics**
A	$A^I \subseteq \Delta^I$
r	$r^I \subseteq \Delta^I \times \Delta^I$
$\top$	Δ^I
$\bot$	$\emptyset$
$C \sqcap D$	$(C \sqcap D)^I = C^I \cap D^I$
$C \sqcup D$	$(C \sqcup D)^I = C^I \cup D^I$
$\neg C$	$(\neg C)^I = \Delta^I \setminus C^I$
$\exists r.C$	$\{a \mid \exists b.(a,b) \in r^I \wedge b \in C^I\}$
$\forall r.C$	$\{a \mid \forall b.(a,b) \in r^I \supset b \in C^I\}$

Table 1. $\mathcal{ALC}$ syntax and semantics

Name	**Syntax**	**Semantics**
concept inclusion axiom	$C \sqsubseteq D$	$C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
role inclusion axiom	$R \sqsubseteq S$	$R^{\mathcal{I}} \subseteq S^{\mathcal{I}}$
concept equality axiom	$C \equiv D$	$C^{\mathcal{I}} = D^{\mathcal{I}}$
role equality axiom	$R \equiv S$	$R^{\mathcal{I}} = S^{\mathcal{I}}$
concept assertion	$C(a)$	$a^{\mathcal{I}} \in C^{\mathcal{I}}$
role assertion	$R(a,b)$	$(a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$

Table 2. Terminological axioms and world descriptions in DLs

be seen as roles that are structurally (and existentially) functions and, hence, they can express functional actions, movements, and procedures. According to the functional behaviour of functional roles, a functional role associates a single value (that can be regarded as the member of a singleton[8]) to its input parameter (that is also the only member of a singleton). According to $F_R(a,b)$, we can interpret that $F_R\colon \{a\} \to \{b\}$ and, in fact, $F_R(a) \equiv b$.

The roles *motherOf*, *fatherOf*, *scoreOf*, and *lastNameOf* are functional roles, whereas *parentOf* and *childOf* are not. The term 'the father of Alice' can produce the functional role *fatherOf* (for the individual `alice`). As demonstrated above, a functional role can be applied to an individual symbol in order to make a functional relation between it and another individual symbol. Consequently, the word 'if' can be regarded as the most significant functional word in natural languages.

[8] A singleton is a set which contains exactly one element.

According to the statement 'Bob is the father of Alice', 'father of' is a functional role that relates Bob to Alice. In fact, the functional role *fatherOf* maps `bob` onto `alice`. Formally, *fatherOf*(`alice`) $\approx$ `bob`. The functional role *fatherOf* is produced based on the role *hasFather*. In fact, *fatherOf* is terminologically supported by *hasFather*. Also, *fatherOf*$^{-}$ expresses the inverse role of *fatherOf*. Formally speaking:

$$\mathit{hasFather} \approx \mathit{fatherOf}^{-}$$

PROPOSITION. Relying on the relationship *hasFather* (between the individuals `alice` and `bob`) that supports the describability of them by the concepts *Daughter* and *Father*, the functional role *fatherOf* is modelled. Subsequently, *fatherOf* maps *Father*(`bob`) onto *Daughter*(`alice`).

5. Functional Contingents in Description Logics

Assume that $A(a)$, $B(b)$, and $R(a,b)$ are three world descriptions in the knowledge base $\mathcal{K}$, where the individuals a and b are known as the instances of the concepts A and B. Also, the role R represents a binary relation between a and b. Formally speaking: $\mathcal{K} \models \{A(a),\ B(b),\ R(a,b)\}$. In fact, the fundamental world descriptions $A(a)$, $B(b)$, and $R(a,b)$ are semantically satisfied by our terminological knowledge $\mathcal{K}$. As proposed above, we can interpret that the world description $R(a,b)$ terminologically supports the production of the functional role F. F maps the concept assertion $A(a)$ onto the concept assertion $B(b)$. Therefore, F is satisfiable by $\mathcal{K}$. Note that the central focus of world description analysis will be on this functional role.

5.1. Handling the Possibility and Necessity of World Descriptions

The introduction of the *Possibility Functional Contingent* $\mathcal{P}$ and the *Necessity Functional Contingent* $\mathcal{N}$ are now due, these being structurally functional roles. They represent the possibility and necessity of a world description. The central assumption is that any functional contingent can cover its inner concept/role assertions. Actually the term 'contingent' is a property and contingency is a relationship that supports a functional role, so the following rules apply:

1. $\mathcal{P}(A(a))$ expresses that 'it is possible that $A(a)$'.
2. $\neg\mathcal{P}(A(a))$ expresses that 'it is impossible that $A(a)$'.

3. $\mathcal{N}(A(a))$ expresses that 'it is necessary that $A(a)$'.

4. $\neg\mathcal{N}(A(a))$ expresses that 'it is unnecessary that $A(a)$'.

5. $\mathcal{P}(R(A(a), B(b)))$ expresses that 'it is possible that $R(A(a), B(b))$'. *Specific Analysis:* $\mathcal{P}(R(A(a), B(b)))$ expresses the existence of, at least, one functional role (like F_i) of the concept assertion $A(a)$. Formally, the *possibility functional contingent* is — based on role assertion $R(A(a), B(b))$ — interpretable as follows:

$$\exists i[(\mathcal{K} \models \{A(a), B(b), R_i(a, b)\}) \Rightarrow F_i(A(a), B(b))].$$

This means that there exists — at least — one role assertion that is satisfied by $\mathcal{K}$ and can support the construction of a functional role of $A(a)$. Consequently, that functional role maps $A(a)$ onto $B(b)$.[9]

6. $\neg\mathcal{P}(R(A(a), B(b)))$ expresses that it is impossible that $R(A(a), B(b))$. *Specific Analysis:* according to $\neg\mathcal{P}(R(A(a), B(b)))$, there is no functional role of the concept assertion $A(a)$ that can be represented in the form of $F_i(A(a), B(b))$ and can map $A(a)$ onto $B(b)$.

7. $\mathcal{N}(R(A(a), B(b)))$ expresses that 'it is necessary that $R(A(a), B(b))$'. *Specific Analysis:* $\mathcal{N}(R(A(a), B(b)))$ expresses the existence of all possible functional roles of the concept assertion $A(a)$ in the form of $F_i(A(a), B(b)))$. Formally, the *necessity functional contingent* is, based on role assertion $R(A(a), B(b))$, interpretable as follows:

$$\forall i[(\mathcal{K} \models \{A(a), B(b), R_i(a, b)\}) \Rightarrow F_i(A(a), B(b))].$$

This means that for all possible role assertions that are satisfied by $\mathcal{K}$, there are functional roles that can map $A(a)$ onto $B(b)$.

8. $\neg\mathcal{N}(R(A(a), B(b)))$ expresses that $R(A(a), B(b))$ does not necessarily hold. *Specific Analysis:* $\neg\mathcal{N}(R(A(a), B(b)))$ expresses that the existence of the functional role '$F_i(A(a), B(b))$' that can map $A(a)$ onto $B(b)$ is not necessary.

5.2. Definability Analysis

The world description *hasFather*(`alice`, `bob`) expresses the facts that Alice has a father and Bob is Alice's father. According to the contingent world description $\mathcal{P}$(*hasFather*(`alice`, `bob`)), we can conclude that it is possible that Alice has a father, and it is possible that Bob is Alice's father. Actually the focus of possibility has been on (i) having a father (by

[9] The functional role $F_i(A(a), B(b))$ expresses that F_i maps $A(a)$ onto $B(b)$, formally: $F_i(A(a)) \equiv B(b)$.

Alice), (ii) being a father (by Bob) for Alice, and (iii) the interrelationships between 'having a father (by Alice)' and 'being a father (by Bob)'. We need to interpret $\mathcal{P}$ as a *functional role* of the functional role '*fatherOf*' in order to, terminologically, analyse $\mathcal{P}(\mathit{hasFather}(\mathtt{alice}, \mathtt{bob}))$. In fact, $\mathcal{P}$ is definable at the states at which the roles 'having a father (by Alice)' and 'being a father (by Bob)' have been defined and are meaningful. Note the following consequences:

1. According to $\mathcal{P}(\mathit{hasFather}(..., ...))$, the functional contingent $\mathcal{P}$ can logically be valid and meaningful at the states at which 'having a father' is defined and meaningful. Therefore, regarding $\mathcal{P}(\mathit{hasFather}(\mathtt{alice}, \mathtt{bob}))$, the definability of the relation 'having a father' between the individual `alice` and the individual `bob` is the logical premise of the definability of the functional contingent $\mathcal{P}$.

2. According to $\mathcal{P}(\mathit{hasFather}(\ldots, \ldots)) \Rightarrow \mathcal{P}(\mathit{hasFather}(\mathtt{alice}, \ldots))$, it is possible that someone has a father and, therefore, it is possible that Alice has a father. Informally, the possibility of the proposition 'Alice has a father' is valid and meaningful if and only if the possibility of 'having a father' is valid and meaningful.

3. According to $\mathcal{P}(\mathit{hasFather}(\ldots, \ldots)) \Rightarrow \mathcal{P}(\mathit{hasFather}(\ldots, \mathtt{bob}))$, it is possible that someone has a father and, therefore, it is possible that Bob is the father of that person. Informally, the possibility of the proposition 'Bob is the father of someone who has a father' is valid and meaningful if and only if the possibility of 'having a father' is valid and meaningful.

4. The possibility of the proposition 'Alice has a father, and Bob is Alice's father' is valid and meaningful if and only if (i) the possibility of having a father (by Alice) and (ii) the possibility of being Alice's father (by Bob) are valid and meaningful. Then, the possibility of the proposition 'Bob is the father of Alice' is valid and meaningful if and only if (i) the possibility of having a father (by Alice), (ii) the possibility of being a father (by Bob), and (iii) the possibility of having Bob as father (by Alice) are valid and meaningful.

6. Semantic Analysis

Let $\mathcal{T} = \{\mathit{Daughter} \sqsubseteq \mathit{Person}, \mathit{Father} \sqsubseteq \mathit{Person}\}$ be the set of terminological axioms in knowledge base $\mathcal{K}$. Also, $\mathcal{W} = \{\mathit{Daughter}(\mathtt{alice}), \mathit{Father}(\mathtt{bob}), \mathit{hasFather}(\mathtt{alice}, \mathtt{bob})\}$ is the set of fundamental world descriptions (i.e., assertional axioms) in knowledge base $\mathcal{K}$. Therefore, $\mathcal{K} = (\mathcal{T}, \mathcal{W})$ and, thus, $\mathcal{T}$ and $\mathcal{W}$ can semantically be satisfied by $\mathcal{K}$. Formally, $\mathcal{K} \models \{\mathcal{T}, \mathcal{W}\}$.

A *Contingent Assertional Axiom* (fundamental contingent world description) is representable in the form of (*world description*, *world description's value*). The values can be either 'F' (stands for Falsity) or 'T' (stands for Truth) or 'V' (stands for Vagueness). Note that the interval $[0, 1]$ has been suggested in the possibilistic logic semantics, e.g., [24, 44]. It is worth mentioning that every possibilistic logic is a weighted logic, where each classical logic formula is associated with a number in the interval $[0, 1]$. Accordingly, the semantics of possibilistic DL has usually been defined by a possibility distribution (like π) over the set I of all classical description logic interpretations, formally: $\pi\colon I \to [0, 1]$. Thus, $\pi(I)$ represents the degree of compatibility of I with available information. However, my offered semantics is (a) three-valued (based on Truth, Falsity, and Vagueness) and (b) only offered for contingent world descriptions in DLs.

In order to be more specific on this three-valued logic, we need to define *Contingent Interpretations* $\mathcal{I}_c$. As you will see, any contingent interpretation can provide a semantic basis for satisfying the functional contingents $\mathcal{P}$, $\mathcal{N}$ and, subsequently, for satisfying the fundamental contingent world descriptions. The contingent interpretation '$\mathcal{I}_c$' is provided in order to satisfy the concepts of:

- functional necessity (in the form of $\mathcal{I}_n$),
- functional possibility (in the form of $\mathcal{I}_p$), and
- functional impossibility (in the form of $\mathcal{I}_{imp}$).

In other words, the contingent interpretation $\mathcal{I}_c$ is utilised to provide semantic models of functional contingents within DL world descriptions. More specifically, if $\mathcal{I}_c$ can satisfy any of the members of $\mathcal{W}$ and, respectively, can satisfy the relevant contingent functions of that world description, then it will provide a *Contingent Model*.

6.1. Assertional Axioms for Functional Contingents

1. $\mathcal{P}(A(a))$ is a possible concept assertion. Semantically,
   (a) based on the possibility interpretation '$\mathcal{I}_p$', $|\mathcal{P}(A(a))|^{\mathcal{I}_p} = T$.
   (b) based on the impossibility interpretation '$\mathcal{I}_{imp}$', $|\mathcal{P}(A(a))|^{\mathcal{I}_{imp}} = F$.

   (c) based on the necessity interpretation '$\mathcal{I}_n$', $|\mathcal{P}(A(a))|^{\mathcal{I}_n} = V$. Actually, we only know that $A(a)$ is possible. So, there is no evidence that $A(a)$ is necessary.
2. $\mathcal{N}(A(a))$ is a necessary concept assertion. Then, semantically,
   (a) $|\mathcal{N}(A(a))|^{\mathcal{I}_n} = T$. Informally, $A(a)$ is necessary. Therefore, $A(a)$ is necessarily necessary.
   (b) $|\mathcal{N}(A(a))|^{\mathcal{I}_p} = T$. Informally, $A(a)$ is necessary. Therefore, $A(a)$ is necessarily possible.
   (c) $|\mathcal{N}(A(a))|^{\mathcal{I}_{imp}} = F$. Informally, $A(a)$ is necessary. Therefore, $A(a)$ is necessarily not impossible.
3. $\mathcal{P}(R(a, b))$ is a possible role assertion. Semantically,
   (a) $|\mathcal{P}(R(a, b)|^{\mathcal{I}_p} = T$.
   (b) $|\mathcal{P}(R(a, b)|^{\mathcal{I}_{imp}} = F$.
   (c) $|\mathcal{P}(R(a, b)|^{\mathcal{I}_n} = V$. Here, we only know that a and b are possibly related to each other (by means of R). In fact, there is no evidence that $R(a, b)$ is necessary.
4. $\mathcal{N}(R(c, d))$ is a necessary role assertion. Semantically,
   (a) $|\mathcal{N}(R(a, b))|^{\mathcal{I}_n} = T$.
   (b) $|\mathcal{N}(R(a, b))|^{\mathcal{I}_p} = T$.
   (c) $|\mathcal{N}(R(a, b))|^{\mathcal{I}_{imp}} = F$. Here, we know that a and b are necessarily related to each other (by means of R). Therefore, $R(a, b)$ is necessarily not impossible.

PROPOSITION. According to the aforementioned items, the functional contingents $\mathcal{P}$ and $\mathcal{N}$ are mappings from their central world descriptions into the values $\{T, F, V\}$.

Considering $\mathcal{K} = (\mathcal{T}, \mathcal{W})$, where $\mathcal{T} = \{\mathit{Daughter} \sqsubseteq \mathit{Person}, \mathit{Father} \sqsubseteq \mathit{Person}\}$ and $\mathcal{W} = \{\mathit{Daughter}(\texttt{alice}), \mathit{Father}(\texttt{bob}), \mathit{hasFather}(\texttt{alice}, \texttt{bob})\}$, I shall draw your attention to the following examples:

Example 1. According to the contingent role assertion '$\mathcal{P}(\mathit{hasFather}(\texttt{alice}, \texttt{bob}))$', 'it is possible that Alice has a father, and it is possible that Bob is Alice's father'. In this example, $(\mathcal{P}(\mathit{hasFather}(\texttt{alice}, \texttt{bob}))^{\mathcal{I}_p} = T$, $(\mathcal{P}(\mathit{hasFather}(\texttt{alice}, \texttt{bob}))^{\mathcal{I}_{imp}} = F$, and $(\mathcal{P}(\mathit{hasFather}(\texttt{alice}, \texttt{bob}))^{\mathcal{I}_n} = T$. Therefore, $\mathcal{I}_p$ is the possibility model of $\mathit{hasFather}(\texttt{alice}, \texttt{bob})$ and $\mathcal{I}_n$ is the necessity model of $\mathcal{P}(\mathit{hasFather}(\texttt{alice}, \texttt{bob}))$.

Example 2. According to the contingent concept assertion '$\mathcal{N}(\mathit{Father}(\texttt{bob}))$', it is necessary that Bob is a father. In this example, $(\mathit{Father}(\texttt{bob}))^{\mathcal{I}_n} = T$, $(\mathit{Father}(\texttt{bob}))^{\mathcal{I}_p} = T$, and $(\mathit{Father}(\texttt{bob}))^{\mathcal{I}_{imp}} = F$. Therefore, $\mathcal{I}_n$ is the necessity model of $\mathcal{N}(\mathit{Father}(\texttt{bob}))$. Obviously,

$\mathcal{N}(Father(\texttt{bob})) \Rightarrow \mathcal{P}(Father(\texttt{bob}))$, then $\mathcal{I}_n$ is the necessity model of the possibility world description '$\mathcal{P}(Father(\texttt{bob}))$' as well.

Example 3. According to the contingent concept assertion '$\mathcal{P}(Daughter(\texttt{maria}))$', it is possible that Maria is a daughter. In this example, $(Daughter(\texttt{maria}))^{\mathcal{I}_p} = T$, $(Daughter(\texttt{maria}))^{\mathcal{I}_{imp}} = V$, and $(Daughter(\texttt{maria}))^{\mathcal{I}_{imp}} = V$. Therefore, $\mathcal{I}_p$ is the possibility model of $\mathcal{P}(Daughter(\texttt{maria}))$.

Example 4. According to the contingent role assertion '$\mathcal{P}(hasFather(\texttt{alice}, \texttt{john}))$', it is possible that Alice has a father and it is possible that John is Alice's father. In this example, $(\mathcal{P}(hasFather(\texttt{alice}, \texttt{john})))^{\mathcal{I}_p} = T$, $(\mathcal{P}(hasFather(\texttt{alice}, \texttt{john})))^{\mathcal{I}_n} = V$, and $(\mathcal{P}(hasFather(\texttt{alice}, \texttt{john})))^{\mathcal{I}_{imp}} = V$.

6.2. Axiomatisation

Axiom 1. $\mathcal{N}(A(a)) \Rightarrow A(a)$.
This axiom expresses that necessary concept assertions are definitely valid. It is axiomatised based on the concept of 'reflexivity'.

Specific Analysis: We are certain that the individual a is necessarily an instance of the concept A. Therefore, $A(a)$ is necessarily valid. In fact, $A(a)$ is valid.

Axiom 2. $\mathcal{N}(R(a,b)) \Rightarrow R(a,b)$.
This axiom expresses that necessary role assertions are definitely valid. It is axiomatised based on the concept of 'reflexivity'.

Specific Analysis: We are certain that the individuals a and b are necessarily connectable to each other (by means of R). Therefore, $R(a,b)$ is necessarily valid. In fact, $R(a,b)$ is valid.

Axiom 3. $\mathcal{N}(A(a)) \Rightarrow \mathcal{P}(A(a))$.
This axiom expresses that necessary concept assertions are possible. It is axiomatised based on the concept of 'seriality'.

Specific Analysis: We are certain that 'it is necessary that $A(a)$' is valid. Then, the individual a is necessarily describable under the label of the concept A. Therefore, it is definitely possible to describe a as an instance of the concept A. Then, it is possible to describe a by A. So, $A(a)$ is possible.

Axiom 4. $\mathcal{N}(R(a,b)) \Rightarrow \mathcal{P}(R(a,b))$.
This axiom expresses that necessary role assertions are possible. It is axiomatised based on the concept of 'seriality'.

Specific Analysis: We are certain that 'it is necessary that $R(a, b)$' is valid. Then, the individuals a and b are necessarily relatable to each other (by means of R). Therefore, it is definitely possible to relate a and b by means of R. Then, $R(a, b)$ is possible.

Axiom 5. $A(a) \Rightarrow \mathcal{N}(\mathcal{P}(A(a)))$.
This axiom expresses that any valid concept assertion is necessarily possible. It is axiomatised based on the concept of 'symmetry'.

Specific Analysis: We know (and are certain) that $A(a)$ is valid. Therefore, the concept A can describe the individual a. Thus, it is definitely possible to describe a by A. Then, a is necessarily possibly described by A. Hence, $A(a)$ is necessarily possible.

Axiom 6. $R(a, b) \Rightarrow \mathcal{N}(\mathcal{P}(R(a, b)))$.
This axiom expresses that any valid role assertion is necessarily possible. It is axiomatised based on the concept of 'symmetry'.

Specific Analysis: We know (and are certain) that $R(a, b)$ is valid. Therefore, it is definitely possible to relate a and b by means of R. Then, a and b are necessarily possibly related by R. Hence, $R(a, b)$ is necessarily possible.

Axiom 7. $\mathcal{N}(A(a)) \Rightarrow \mathcal{N}(\mathcal{N}(A(a)))$.
This axiom represents the iteration (and transitivity) of necessary concept assertions. It expresses that necessary concept assertions are necessary.

Specific Analysis: We are certain that the individual a is necessarily an instance of the concept A. So, a can necessarily be described by A. Thus, 'a can necessarily be described by A' is definitely valid. Therefore, it is necessary that a can necessarily be described by A.

Axiom 8. $\mathcal{N}(R(a, b)) \Rightarrow \mathcal{N}(\mathcal{N}(R(a, b)))$.
This axiom represents the iteration (and transitivity) of necessary role assertions. It expresses that necessary role assertions are necessary.

Specific Analysis: We are certain that the individuals a and b are necessarily related to each other by means of R. So, a and b can necessarily be related to each other (based on R). In fact, 'a and b can necessarily be related by means of R' is definitely valid. Therefore, it is necessary that a and b can necessarily be related to each other (by means of R).

Axiom 9. $\mathcal{N}(\mathcal{N}(A(a))) \Rightarrow \mathcal{N}(A(a))$.
This axiom expresses that the necessity of a necessary concept assertion implies the necessity of that concept assertion. It is axiomatised based on the concept of 'density'.

Specific Analysis: This axiom is analysable based on axiom 1. We are certain that it is necessary that the individual a can necessarily be described by the concept A. Therefore, a can necessarily be described by A. Hence, it is necessary that $A(a)$.

Axiom 10. $\mathcal{N}(\mathcal{N}(R(a,b))) \Rightarrow \mathcal{N}(R(a,b))$.
This axiom expresses that the necessity of a necessary role assertion implies the necessity of that role assertion. It is axiomatised based on the concept of 'density'.

Specific Analysis: This axiom is analysable based on axiom 2. We are certain that it is necessary that the individuals a and b can necessarily be related to each other (by means of R). Therefore, a and b can necessarily be related to each other (by means of R). Hence, it is necessary that $R(a,b)$.

Axiom 11. $\mathcal{P}(A(a)) \Rightarrow \mathcal{N}(\mathcal{P}(A(a)))$.
This axiom expresses that possible concept assertions are necessarily possible.

Specific Analysis: We know that the individual a might be an instance of the concept A. Thus, it is possible to represent a under the label of A. Hence, it is definitely possible to represent a under the label of A. So, it is necessarily possible to describe a by A. In fact, $A(a)$ is necessarily possible.

Axiom 12. $\mathcal{P}(R(a,b)) \Rightarrow \mathcal{N}(\mathcal{P}(R(a,b)))$.
This axiom expresses that possible role assertions are necessarily possible.

Specific Analysis: We know that the individuals a and b might be connected to each other by means of R. Thus, it is possible to represent $R(a,b)$. Hence, it is definitely possible to relate a and b by means of R. So, it is necessarily possible to relate a and b by means of R. In fact, $R(a,b)$ is necessarily possible.

Axiom 13. $\mathcal{N}[\mathcal{N}(A(a)) \Rightarrow A(a)]$.
This axiom expresses that the necessity of a concept assertion necessarily implies the validity of that concept assertion. It is axiomatised based on the concept of 'shift-reflexivity'.

Specific Analysis: This axiom is analysable based on axiom 1. We know that the individual a is necessarily describable by the concept A. Therefore, 'a is necessarily describable by A' necessarily implies that 'a can be described by A'. This means that it is necessary that $A(a)$ necessarily implies $A(a)$. Hence, $A(a)$ is valid.

Axiom 14. $\mathcal{N}[\mathcal{N}(R(a,b)) \Rightarrow R(a,b)]$.
This axiom expresses that the necessity of a role assertion necessarily implies the validity of that role assertion. It is axiomatised based on the concept of 'shift-reflexivity'.

Specific Analysis: This axiom is analysable based on axiom 2. We know that the individuals a and b are necessarily connectable (by means of R). Therefore, 'a and b are necessarily connectable (by means of R)' necessarily implies that 'a and b are connectable (by means of R)'. This means that it is necessary that $R(a,b)$ necessarily implies $R(a,b)$. Hence, $R(a,b)$ is valid.

Axiom 15. $\mathcal{P}(\mathcal{N}(A(a))) \Rightarrow \mathcal{N}(\mathcal{P}(A(a)))$.
This axiom expresses that the possibility of the necessity of a concept assertion implies the necessity of its possibility. It is axiomatised based on the concept of 'convergency'.

Specific Analysis: Suppose that it is possible that $A(a)$ is necessary. Then, it is not necessary that $A(a)$ is necessary. Therefore, $A(a)$ is necessarily possible (and not necessary). In fact, $\mathcal{N}(\mathcal{P}(A(a)))$. Note that this axiom has a strong correlation with axiom 3.

Axiom 16. $\mathcal{P}(\mathcal{N}(R(a,b))) \Rightarrow \mathcal{N}(\mathcal{P}(R(a,b)))$.
This axiom expresses that the possibility of the necessity of a role assertion implies the necessity of its possibility. It is axiomatised based on the concept of 'convergency'.

Specific Analysis: Suppose that it is possible that $R(a,b)$ is necessary. Then, $R(a,b)$ is not necessarily necessary. Therefore, $R(a,b)$ is necessarily possible (and not necessary). In fact, $\mathcal{N}(\mathcal{P}(R(a,b)))$. Note that this axiom has a strong correlation with axiom 4.

Axiom 17. $[\mathcal{N}(A(a) \Rightarrow B(b))] \Rightarrow [\mathcal{N}(A(a)) \Rightarrow \mathcal{N}(B(b))]$.
This axiom represents the distribution of necessity over concept assertions.

Specific Analysis: We know that $A(a)$ and $B(b)$ are valid. We also know that $A(a)$ necessarily implies $B(b)$. Therefore, the necessity of $B(b)$ is deduced from the necessity of $A(a)$. For example, we know that *Father*(`bob`) and *Daughter*(`alice`). We also know that *Father*(`bob`) necessarily implies *Daughter*(`alice`). Therefore, the necessity of *Daughter* (`alice`) is deduced from the necessity of *Father*(`bob`). Informally, we know that Bob is a father and Alice is a daughter. We also know that 'being a father by Bob' necessarily implies 'being a daughter by Alice'. In fact, we know that Bob has become a father and, subsequently, Alice has become a daughter. Therefore, the necessity of 'being a daughter by Alice' is deduced from the necessity of 'being a father by Bob'.

Axiom 18. $[\mathcal{N}(R(a,b) \Rightarrow S(c,d))] \Rightarrow [\mathcal{N}(R(a,b)) \Rightarrow \mathcal{N}(S(c,d))]$. This axiom represents the distribution of necessity over role assertions.

Specific Analysis: We know that $R(a,b)$ and $S(c,d)$ are valid. We also know that $R(a,b)$ necessarily implies $S(c,d)$. Therefore, the necessity of $S(c,d)$ is deduced from the necessity of $R(a,b)$. For example, we know that *hasMother*(`bob`, `mary`) and we know that *hasFather*(`alice`, `bob`). We also know that *hasMother*(`bob`, `mary`) necessarily implies *hasFather* (`alice`, `bob`). Therefore, the necessity of *hasFather*(`alice`, `bob`) is deduced from the necessity of *hasMother*(`bob`, `mary`). Informally, we know that Bob has a mother and Mary is his mother and we know that Alice has a father, and Bob is Alice's father. We also know that 'being the mother of Bob (by Mary)' necessarily implies 'being the father of Alice (by Bob)'. Therefore, the necessity of 'being the father of Alice (by Bob)' is deduced from the necessity of 'being the mother of Bob (by Mary)'.

6.3. Semantics of Negative Contingent World Descriptions

Regarding the following items, the notions of 'satisfiability', 'logical consequence', and 'tight logical consequence for possibilistic knowledge bases' are definable as similar to [24, 28].

1. $\mathcal{P}(A(a)) \equiv \neg\mathcal{N}(\neg A(a))$.

Specific Analysis. If we know that A can possibly describe a, then it will not be necessary that A cannot describe a. This means that it will not be necessary that A cannot describe a. Similarly, if we know that it is not necessary that A cannot describe a, then it will not be necessary that A cannot describe a. This means that A will possibly describe a. In fact, $[\mathcal{P}(A(a)) \Rightarrow \neg\mathcal{N}(\neg A(a))] \sqcap [\neg\mathcal{N}(\neg A(a)) \Rightarrow \mathcal{P}(A(a))]$. This means that: $\mathcal{P}(A(a)) \equiv \neg\mathcal{N}(\neg A(a))$.

2. $\mathcal{P}(R(a,b)) \equiv \neg\mathcal{N}(\neg R(a,b))$.

Specific Analysis. If we know that R can possibly relate a and b, then it will not be necessary that R does not relate a and b with each other. This means that it will not be necessary that R does not relate a and b with each other. Similarly, if we know that it is not necessary that R does not relate a and b with each other, then it will not be necessary that R does not relate a and b with each other. Thus, R will possibly relate a and b with each other. In fact, $[\mathcal{P}(R(a,b)) \Rightarrow \neg\mathcal{N}(\neg R(a,b))] \sqcap$

$[\neg\mathcal{N}(\neg R(a,b)) \Rightarrow \mathcal{P}(R(a,b))]$. This means that: $\mathcal{P}(R(a,b)) \equiv \neg\mathcal{N}(\neg R(a,b))$.

3. $\mathcal{N}(A(a)) \equiv \neg\mathcal{P}(\neg A(a))$.

Specific Analysis. If we know that A necessarily describes a, then there will be no possibility that A cannot describe a. Hence, it will be impossible that A cannot describe a. Similarly, if we know that it is impossible that A cannot describe a, then there will be no possibility that A cannot describe a. Therefore, A will necessarily be able to describe a. In fact, $[\mathcal{N}(A(a)) \Rightarrow \neg\mathcal{P}(\neg A(a))] \sqcap [\neg\mathcal{P}(\neg A(a)) \Rightarrow \mathcal{N}(A(a))]$. This means that: $\mathcal{N}(A(a)) \equiv \neg\mathcal{P}(\neg A(a))$.

4. $\mathcal{N}(R(a,b)) \equiv \neg\mathcal{P}(\neg R(a,b))$.

Specific Analysis. If we know that it is necessary that $R(a,b)$, then there will be no possibility that $R(a,b)$ will not be valid. In other words, it will be impossible that $R(a,b)$ will not be valid. Similarly, if we know that it is impossible that $R(a,b)$ is not valid, then there will be no possibility that $R(a,b)$ will not be valid. Therefore, $R(a,b)$ will necessarily be valid. In fact, $[\mathcal{N}(R(a,b)) \Rightarrow \neg\mathcal{P}(\neg R(a,b))] \sqcap [\neg\mathcal{P}(\neg R(a,b)) \Rightarrow \mathcal{N}(R(a,b))]$. This means that: $\mathcal{N}(R(a,b)) \equiv \neg\mathcal{P}(\neg R(a,b))$.

6.3.1. Semantic representation of negative contingent world descriptions

1. $\neg(\mathcal{P}(A(a)), T) \equiv (\neg\mathcal{P}(A(a)), F)$.
2. $\neg(\mathcal{P}(A(a)), F) \equiv (\neg\mathcal{P}(A(a)), T)$.
3. $\neg(\mathcal{P}(A(a)), V) \equiv (\neg\mathcal{P}(A(a)), V)$.
4. $\neg(\mathcal{P}(R(a,b)), T) \equiv (\neg\mathcal{P}(R(a,b)), F)$.
5. $\neg(\mathcal{P}(R(a,b)), F) \equiv (\neg\mathcal{P}(R(a,b)), T)$.
6. $\neg(\mathcal{P}(R(a,b)), V) \equiv (\neg\mathcal{P}(R(a,b)), V)$.
7. $\neg(\mathcal{N}(A(a)), T) \equiv (\neg\mathcal{N}(A(a)), F)$.
8. $\neg(\mathcal{N}(A(a)), F) \equiv (\neg\mathcal{N}(A(a)), T)$.
9. $\neg(\mathcal{N}(A(a)), V) \equiv (\neg\mathcal{N}(A(a)), V)$.
10. $\neg(\mathcal{N}(R(a,b)), T) \equiv (\neg\mathcal{N}(R(a,b)), F)$.
11. $\neg(\mathcal{N}(R(a,b)), F) \equiv (\neg\mathcal{N}(R(a,b)), T)$.
12. $\neg(\mathcal{N}(R(a,b)), V) \equiv (\neg\mathcal{N}(R(a,b)), V)$.

Consider the following examples:

Example 5. We know that Martin is not a father. Therefore, $|\mathcal{P}(\mathit{Father}(\mathtt{martin}))|^{\mathcal{I}_p} = (\mathcal{P}(\mathit{Father}(\mathtt{martin})), F)$. Hence, it is impossible to describe Martin by the concept *Father*. In fact, the term 'Martin is a father' is impossible. Consequently, it is deducible that $\neg\mathit{Father}(\mathtt{martin})$.

Example 6. Bob is the father of Alice. So $|\mathcal{P}(\mathit{hasFather}(\mathtt{alice}, \mathtt{john}))|^{\mathcal{I}_p} = (\mathcal{P}(\mathit{hasFather}(\mathtt{alice}, \mathtt{john})), F)$. Thus, it is impossible to relate Alice and John by the relation *hasFather*. In fact, the term 'Alice has a father and John is Alice's father' is impossible. Consequently, it is deducible that $\neg \mathit{hasFather}(\mathtt{alice}, \mathtt{john})$.

Example 7. We are not certain that James is a father (or not). So, the term 'James is a father' is semantically vague. In fact, based on our conditional information, it is unnecessary that 'James is a father'. However, it is possible that 'James is a father'. Consequently, we can represent our knowledge by either:

1. $|\neg\mathcal{N}(\mathit{Father}(\mathtt{james}))|^{\mathcal{I}_n} = (\mathcal{N}(\mathit{Father}(\mathtt{james})), V)$, or
2. $|\neg\mathcal{N}(\mathit{Father}(\mathtt{james}))|^{\mathcal{I}_p} = (\mathcal{P}(\mathit{Father}(\mathtt{james})), T)$.

Example 8. Suppose that we are not certain that $\mathit{hasMother}(\mathtt{alice}, \mathtt{ann})$ (i.e., we are not certain that 'Alice has a mother and Ann is Alice's mother'). Thus, we can conclude that the term 'Alice has a mother and Ann is Alice's mother' is possible. Formally, $|\mathcal{P}(\mathit{hasMother}(\mathtt{alice}, \mathtt{ann}))|^{\mathcal{I}_p} = (\mathcal{P}(\mathit{hasMother}(\mathtt{alice}, \mathtt{ann})), T)$. In addition, based on our conditional information, it is unnecessary that 'Alice has a mother and Ann is Alice's mother'. Formally: $|\neg\mathcal{N}(\mathit{hasMother}(\mathtt{alice}, \mathtt{ann}))|^{\mathcal{I}_n} = (\mathcal{N}(\mathit{hasMother}(\mathtt{alice}, \mathtt{ann})), V)$. Note that the term 'Alice has a mother and Ann is Alice's mother' is based on the conjunction of the terms 'Alice has a mother' and 'Ann is Alice's mother'. Alice (as a human being) certainly has (or has had) a mother. So, 'Alice has a mother' is necessarily valid. In fact, we can conclude that the vagueness of the term 'Alice has a mother and Ann is Alice's mother' is because of the vagueness of the term 'Ann is Alice's mother'. In fact, $(\mathcal{N}(\mathit{hasMother}(\mathtt{alice}, \mathtt{ann})), V) \Rightarrow [(\mathcal{N}(\mathit{Daughter}(\mathtt{alice}), T) \wedge \mathcal{N}((\mathit{Mother}(\mathtt{ann})), V)]$. Obviously, a contingent role assertion has been expressed which is based on the conjunction of two contingent concept assertions in order to be semantically interpreted.

7. Conclusions

The main focus of this research has been on possibility and necessity of DL world descriptions. This paper has initially focused on DL fundamental world descriptions (that are in the form of either concept assertions or role assertions) and has introduced functional contingents $\mathcal{P}$ (for possibility) and $\mathcal{N}$ (for necessity) over fundamental world descriptions.

$\mathcal{P}$ and $\mathcal{N}$ are interpreted as the functional roles of the most central concept assertions and role assertions in every world description. Functional contingents support structural analyses of the concepts of 'being-functionally-possible' and 'being-functionally-necessary'. Based on the role assertion $R(a,b)$, the functional role F (that is supported by the role R) associates the singleton $\{b\}$ (that consists of the individual b) with the singleton $\{a\}$ (that consists of the individual a).

According to $W = \{A(a), B(b), R(a,b)\}$, the individuals a and b can be described by the concepts A and B. This research has proved that the validity of the possibility of $R(a,b)$ is equivalent to the existence of — at least — one functional role of $A(a)$, like $F_i(A(a))$, that can be mapped onto $B(b)$. Therefore, there is — at least — one possible R that can functionally relate a and b with each other. In addition, it has been proved that the validity of the necessity of $R(a,b)$ is equivalent to the existence of all possible functional roles of $A(a)$ (that can be mapped onto $B(b)$). The concept of functional necessity means that there is always a functional relation that can functionally relate a and b with each other.

This research has introduced contingent interpretations (or $\mathcal{I}_c$) in order to handle the semantics of functional contingency. The basic assumption is that any $\mathcal{I}_c$ can provide a semantic basis for satisfying functional contingents $\mathcal{P}$ and $\mathcal{N}$ and, subsequently, for satisfying fundamental contingent world descriptions. Contingent interpretations are utilised to provide semantic models for functional possibilities and functional necessities within DL world descriptions. Relying on contingent interpretations, a three-valued semantics (based on Truth, Falsity, and Vagueness) has been analysed. It is concluded that the functional contingents $\mathcal{P}$ and $\mathcal{N}$ are two kinds of mappings from their central world descriptions into the values $\{T, F, V\}$.

This paper has finally offered several axioms and annotated DL world descriptions based on functional contingents. Furthermore, a semantic analysis of negative contingent world descriptions has been offered.

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