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## CONTENT IMPLICATION AND YABLO'S SEQUENCE OF SENTENCES

**Abstract.** This paper is a continuation of [Łukowski, 2019], where it is shown that just like sets, sentences can also be understood in two ways: distributively or collectively. A distributive understanding of sets leads to the Russell antinomy, and a distributive understanding of sentences to liar antinomy. A collective understanding of sets frees up the set theory from Russell's antinomy. Taking a similar approach to sentences no liar like paradoxes appear. The aim of the paper is to examine Yablo's problem from this collective perspective. Given its nature, by using the content implication connective it becomes possible to assign logical values to all Yablo's sentences. Moreover, it seems that Yablo's problem is not a case of circularity.

**Keywords:** Yablo's paradox; paradox; self-reference; circularity; content implication

### 1. Introduction

Ever since Russell, the popular view has been that paradoxical sentences are paradoxical because of circularity, especially through self-reference. Obviously, self-reference does not necessarily lead to contradiction. Among many instances proving that self-reference is not necessary for paradox, the truth teller sentence seems to be the most spectacular. However, a counterexample proving that self-reference is necessary for paradox was unknown until Stephen Yablo proposed his famous infinite sequence of sentences in which every sentence states the falsehood of all sentences with bigger indices than it [Yablo, 1985, 1993]. According to Yablo's intension the sequence should be an example of the liar-like

paradox free from circularity and so it should disprove Russell's opinion that every liar-like paradox is essentially associated with circularity. Summing up his construction in the last sentence of his paper from 1993 Yablo writes: "I conclude that self-reference is neither necessary nor sufficient for liar-like paradox" [Yablo, 1993, p. 252]. Let us recall the Yablo problem in its weaker form<sup>1</sup> using the following sequence of sentences:

$$\begin{aligned} S_0 &= \text{"For } k > 0, S_k \text{ is false"} \\ S_1 &= \text{"For } k > 1, S_k \text{ is false"} \\ S_2 &= \text{"For } k > 2, S_k \text{ is false"} \\ &\vdots \\ S_n &= \text{"For } k > n, S_k \text{ is false"} \\ &\vdots \end{aligned}$$

Let us suppose that  $S_0$  is a true sentence, then all  $S_n$  for  $n > 0$  are false. Let  $k > 0$  be any natural number. Of course,  $S_k$  is false. However, since for any  $n > k$  all sentences  $S_n$  are false,  $S_k$  is true — a contradiction. Thus, it is impossible to assign a logical value for any  $S_n$  with  $n > 0$ . It means that also  $S_0$  cannot be consistently valuated.

In publishing his famous logical problem Yablo opened a long-running and still unresolved discussion on the real nature of the paradox. Among others, the main question is whether Yablo's paradox involves circularity. Although our paper does not contribute to these discussions let us mention that the most important opinions about possible answers on this question are presented in: [Beall, 1999, 2001; Bolander, 2008; Bringsjord and van Heuveln, 2003; Bueno and Colyvan, 2003a,b; Colyvan, 2009; Cook, 2006; Hardy, 1995; Hsiung, 2013; Ketland, 2004; Leitgeb, 2002; Luna, 2009a; Priest, 1997; Schlenker, 2007a,b; Sorensen, 1998; Yablo, 2004]. Yablo's problem has inspired others to search for Yablo-like (so called, *yabloesque*) versions of other logical problems: [Beall, 1999; Cieřlinski and Urbaniak, 2013; Goldstein, 1994, 2013; Luna, 2009a,b; Sorensen, 1998; Uzquiano, 2004].

This list of publications shows how important the problem discovered by Yablo is. All these papers have made significant contributions to our

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<sup>1</sup> At first, in [Yablo, 1985, 1993], the paradox is presented in stronger form in term of untruth. This weaker version comes from [Yablo, 2004]. See also [Colyvan, 2009].

knowledge about circularity, and especially self-reference. However, this long-running debate is based on the unspoken assumption that Yablo's paradox cannot be solved in the same way as the liar antinomy. Since the content implication connective can be successfully used for the solution of the liar problem [see [Łukowski, 1997, 2006, 2011](#)], this paper proposes a similar attempt at solving Yablo's paradox.

## 2. Propositional logic with content implication

An aim of the paper [[Łukowski, 2019](#)] was to present the idea of two quite different approaches to sentences, inspired by, respectively, distributive and collective set theories. A distributive interpretation of the liar sentence leads to the liar antinomy whereas a collective interpretation does not. The first approach is typical for the truth-functional sense of a sentence, reducing the sentence to its logical value. Since every logical value is an indivisible whole, sentences are treated also as indivisible wholes that can be only substituted for each other. The collective approach is quite different. A sentence is identified with its content. It means that every sentence is a whole composed of parts which are other sentences, as the content is composed of parts being some other contents — exactly as in the case of sets in collective set theory. This contentual or collective perspective frees the liar sentence from its antinomial consequence.

The structure of the sentence, represented by its main connective, decides which approach is appropriate for the correct and successful understanding of the sentence. Some sentences should be understood in a distributive way, while others, in a collective manner. Both alternative ways of thinking about sentences are analogous to the set-theoretical case. Some sets, as abstract objects defined only by their elements, are understood as distributive. Other sets, understood as fragments of a three-dimensional space are called "collective". In a similar way, it is possible to distinguish two alternative understandings of sentences, one truth-functional, and the second contentual. The main difference between the distributive and collective approaches to sentences is the treatment of a sentence's sense, either as an indivisible whole or a whole consisting of parts being senses of other sentences. In the distributive approach a sense/content of a sentence  $p \rightarrow q$  is an indivisible whole as are the senses/contents of  $p$  and  $q$ . Indeed, the content of the sentence  $p \rightarrow q$  expresses the fact that what is said by  $p$  entails what is said

by  $q$ . That is why neither the content of  $p$ , nor the content of  $q$  can be understood as parts of the content of  $p \rightarrow q$ . Neither what is said by  $p$ , nor by  $q$  can be parts of what is said by  $p \rightarrow q$ .

Another approach is proposed in the calculus with the content implication connective represented by the colon symbol “:”. A collective sentence  $p : q$  says that  $p$  says what is said by  $q$ . In other words, the content of  $q$  is a part of the content of  $p$ .

A complete presentation of a language extended by the connective of content implication and propositional logic is included in the paper [Łukowski, 2019]. Here, we recall only most basic facts [see also Łukowski, 1997, 2006, 2011].

$\mathcal{L}_C = \langle \text{For}_{\text{CCL}}, \neg, \wedge, \vee, \rightarrow, \leftrightarrow, : \rangle$  is the *Contentual Propositional Language* (CCL-language), i.e., a language for classical propositional logic extended by the new connective “:” of content implication. Formulas:

$$((\alpha : \beta) \wedge (\beta : \delta)) \rightarrow (\alpha : \delta), \quad (\text{A1})$$

$$(\alpha \wedge \beta) : \alpha, \quad (\text{A2})$$

$$(\alpha \wedge \beta) : (\beta \wedge \alpha), \quad (\text{A3})$$

$$\alpha : (\alpha \wedge \alpha), \quad (\text{A4})$$

$$((\alpha : \beta) \wedge (\beta : \alpha)) \rightarrow ((\neg\alpha : \neg\beta) \wedge (\neg\beta : \neg\alpha)), \quad (\text{A5})$$

$$((\alpha : \beta) \wedge (\beta : \alpha) \wedge (\delta : \gamma) \wedge (\gamma : \delta)) \rightarrow \\ ((\alpha \S \delta) : (\beta \S \gamma)) \wedge ((\beta \S \gamma) : (\alpha \S \delta)), \quad (\text{A6})$$

$$((\alpha : \beta) \wedge (\delta : \gamma)) \rightarrow ((\alpha \circ \delta) : (\beta \circ \gamma)), \quad (\text{A7})$$

$$(\alpha : \beta) \rightarrow (\alpha \rightarrow \beta), \quad (\text{A8})$$

for  $\S \in \{\rightarrow, \leftrightarrow, :\}$  and  $\circ \in \{\wedge, \vee\}$ . Moreover, *Modus Ponens* (MP):

$$\text{if } \alpha \rightarrow \beta \text{ and } \alpha \text{ are theses, then } \beta \text{ is a thesis} \quad (\text{MP})$$

is the only inference rule of CCL. One of the most important CCL-theses is  $\alpha : \alpha$ , a trivial formula easily inferred by (A1), (A2) and (A4).

The CCL-model is a matrix  $\mathfrak{M} = \langle \mathfrak{A}, D \rangle$  such that  $\mathfrak{A} = \langle A, -, \cap, \cup, \Rightarrow, \Leftrightarrow, \supset \rangle$  is an algebra similar to  $\mathcal{L}_{\text{CCL}}$ ,  $D$  is a nonempty subset of  $A$  and for all  $a, b \in A$ :

1.  $a = a \cap a$ ,
2.  $a \cap b = b \cap a$ ,
3.  $a \cap (b \cap c) = (a \cap b) \cap c$ ,
4.  $\neg a \in D$  iff  $a \notin D$ ,
5.  $a \cap b \in D$  iff  $a \in D$  and  $b \in D$ ,

6.  $a \cup b \in D$  iff  $a \in D$  or  $b \in D$ ,
7.  $a \Rightarrow b \in D$  iff  $a \notin D$  or  $b \in D$ ,
8.  $a \supset b \in D$  iff  $a = b \cap c$ , for some  $c \in A$ .

Semantic inference is defined in a standard way:

$$\begin{aligned} \Pi \models_{\text{CCL}} \alpha \text{ iff for arbitrary CCL-model } \mathfrak{M} = \langle A, D \rangle \text{ and} \\ v \in \text{Hom}(\mathcal{L}_{\text{CCL}}, \mathfrak{A}), v(\alpha) \in D, \text{ if for any } \beta \in \Pi, v(\beta) \in D. \end{aligned}$$

A class of all CCL-matrices gives a semantics adequate for CCL.

The eighth condition shows that a relation between sentences combined by the connective of content implication is not truth-functional. The logical value of the sentence  $p : q$  is independent of the logical values of  $p$  and  $q$  with one exception only —  $p : q$  cannot be true, if  $p$  is true and  $q$  is false. The logical value of the sentence  $p : q$  depends on the relation between the forms of  $p$  and  $q$ . It is easy to see that the only tautologies of CCL with the content implication connective as the main functor are the formulas  $\alpha : \alpha$  and  $(\alpha_1 \wedge \dots \wedge \alpha_n) : \alpha_i$ , for  $i \in \{1, \dots, n\}$ . Thus, if every sentence is an *ingredient*, in Leśniewski's sense, of itself, then it is a whole, or some other sentence, as its part.<sup>2</sup> The content of a true sentence can be composed by the senses of true sentences only. The content of a false sentence can contain some parts which are the contents of true sentences. For example, the false sentence “January 2018 was so warm in Poland that some crocuses bloomed” says something true, namely that this January was really warm in Poland.

### 3. The solution of Yablo's paradox

In a language with the content implication connective it becomes possible to express the contents of all sentences composing Yablo's well-known sequence:

$$\begin{aligned} S_0 : \text{For } k > 0, S_k \text{ is false} \\ S_1 : \text{For } k > 1, S_k \text{ is false} \\ S_2 : \text{For } k > 2, S_k \text{ is false} \\ \vdots \end{aligned}$$

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<sup>2</sup> “Definition I. The expression ‘ingredient of object  $A$ ’ is used to denote  $A$ , and every part of  $A$ . That is: Object  $B$  is an ingredient of object  $A$  if and only if either  $B$  is  $A$  or  $B$  is part of  $A$ . Nowadays the term ‘ingredient’ is often simply rendered as ‘part’ and Leśniewski's ‘part’ is called ‘proper part’” [Simons, 2015].

$S_n$  : For  $k > n$ ,  $S_k$  is false

⋮

Consequently, by using the content implication connective, every sentence says what is said by the appropriate infinite conjunction of sentences:

$$S_0 : (\neg S_1 \wedge \neg S_2 \wedge \neg S_3 \wedge \cdots \wedge \neg S_k \wedge \cdots)$$

$$S_1 : (\neg S_2 \wedge \neg S_3 \wedge \neg S_4 \wedge \cdots \wedge \neg S_k \wedge \cdots)$$

$$S_2 : (\neg S_3 \wedge \neg S_4 \wedge \neg S_5 \wedge \cdots \wedge \neg S_k \wedge \cdots)$$

⋮

$$S_n : (\neg S_{n+1} \wedge \neg S_{n+2} \wedge \neg S_{n+3} \wedge \cdots \wedge \neg S_{n+k} \wedge \cdots)$$

⋮

Let us assume that besides the obvious fact that every sentence says what it says ( $S_n : S_n$ , for  $n \in \mathbb{N}$ ), every Yablo sentence does not say anything more than what is expressed by the above infinite formulas. It means that, also:

$$(\neg S_1 \wedge \neg S_2 \wedge \neg S_3 \wedge \cdots \wedge \neg S_k \wedge \cdots) : S_0$$

$$(\neg S_2 \wedge \neg S_3 \wedge \neg S_4 \wedge \cdots \wedge \neg S_k \wedge \cdots) : S_1$$

$$(\neg S_3 \wedge \neg S_4 \wedge \neg S_5 \wedge \cdots \wedge \neg S_k \wedge \cdots) : S_2$$

⋮

$$(\neg S_{n+1} \wedge \neg S_{n+2} \wedge \neg S_{n+3} \wedge \cdots \wedge \neg S_{n+k} \wedge \cdots) : S_n$$

⋮

In the paper [Łukowski, 2019] is presented a relation between content implication and Suszko's non-Fregean identity [Bloom and Suszko, 1972; Suszko, 1975]. Here, let us only recall that

$$((\alpha : \beta) \wedge (\beta : \alpha)) \leftrightarrow (\alpha \equiv_c \beta)$$

where “ $\equiv_c$ ” is Suszko's identity strengthened by the axioms:

$$(\alpha \wedge \alpha) \equiv_c \alpha,$$

$$(\alpha \wedge \beta) \equiv_c (\beta \wedge \alpha),$$

$$(\alpha \wedge \beta) \wedge \gamma \equiv_c \alpha \wedge (\beta \wedge \gamma).$$

Using the connective of identity, one can present Yablo sentences as:

$$\begin{aligned}
 S_0 &\equiv_c \neg S_1 \wedge \neg S_2 \wedge \neg S_3 \wedge \dots \wedge \neg S_k \wedge \dots \\
 S_1 &\equiv_c \neg S_2 \wedge \neg S_3 \wedge \neg S_4 \wedge \dots \wedge \neg S_k \wedge \dots \\
 S_2 &\equiv_c \neg S_3 \wedge \neg S_4 \wedge \neg S_5 \wedge \dots \wedge \neg S_k \wedge \dots \\
 &\vdots \\
 S_n &\equiv_c \neg S_{n+1} \wedge \neg S_{n+2} \wedge \neg S_{n+3} \wedge \dots \wedge \neg S_{n+k} \wedge \dots \\
 &\vdots
 \end{aligned}$$

It leads to a simple understanding of all the sentences:

$$\begin{aligned}
 S_0 &\equiv_c \neg S_1 \wedge S_1 \\
 S_1 &\equiv_c \neg S_2 \wedge S_2 \\
 S_2 &\equiv_c \neg S_3 \wedge S_3 \\
 &\vdots \\
 S_n &\equiv_c \neg S_{n+1} \wedge S_{n+1} \\
 &\vdots
 \end{aligned}$$

and also a simple solution of the Yablo problem: all Yablo sentences are logically false.

Unfortunately, there are no infinite formulas either in the CCL-language or Suszko's SCI-language. It means that every formula with an infinite conjunction must be replaced by an appropriate sequence formulas:

$$\begin{aligned}
 S_0 &: \neg S_1, S_0 : \neg S_2, S_0 : \neg S_3, \dots, S_0 : \neg S_k, \dots \\
 S_1 &: \neg S_2, S_1 : \neg S_3, S_1 : \neg S_4, \dots, S_1 : \neg S_k, \dots \\
 S_2 &: \neg S_3, S_2 : \neg S_4, S_2 : \neg S_5, \dots, S_2 : \neg S_k, \dots \\
 &\vdots \\
 S_n &: \neg S_{n+1}, S_n : \neg S_{n+2}, S_n : \neg S_{n+3}, \dots, S_n : \neg S_{n+k}, \dots \\
 &\vdots
 \end{aligned}$$

Since  $S_0 : \neg S_1$  and  $S_1 : S_1$ , then  $S_0 \wedge S_1 : S_1 \wedge \neg S_1$ . Similarly,  $S_0 \wedge S_k : S_k \wedge \neg S_k$ , for any  $k \in \mathbb{N}$ . Thus, the conjunction of  $S_0$  and any other Yablo

sentence expresses a contradiction, so it is false. Obviously, this fact holds not only for  $S_0$ . Indeed, for any  $n, k \in \mathbb{N}$ , and  $k > 0$ ,  $S_n : \neg S_{n+k}$  and  $S_n : S_n$ . Thus,

$$S_n \wedge S_{n+k} : S_{n+k} \wedge \neg S_{n+k}.$$

In summary, it should be observed that every conjunction of any two different Yablo sentences is false. It means that among Yablo sentences at most one sentence is true. Otherwise, some conjunctions would be true, which is excluded. Thus, there are possible only two cases: either only one Yablo sentence is true or all Yablo sentences are false.

### 3.1. Only one Yablo sentence is true

As was already mentioned, for any  $n, m, k \in \mathbb{N}$ , such that  $m > 1$ ,

$$S_n : \neg S_{n+m} \wedge \cdots \wedge \neg S_{n+m+k}$$

as well as

$$S_n : S_n$$

It means that, for any  $n, m, k \in \mathbb{N}$ , such that  $m > 1$ ,

$$S_n : (\neg S_{n+m} \wedge \cdots \wedge \neg S_{n+m+k}) \wedge S_n.$$

Now assume that,  $S_{n_0}$  is the only true sentence. It means that, for any  $n \neq n_0$ ,  $S_n$  is false, and so, for any  $m, k \in \mathbb{N}$ , where  $m > 1$ , the conjunction  $\neg S_{n_0+m} \wedge \cdots \wedge \neg S_{n_0+m+k}$  is true. Of course,  $(\neg S_{n_0+m} \wedge \cdots \wedge \neg S_{n_0+m+k}) \wedge S_{n_0}$  is also true. It is different for all sentences  $S_n$  with  $n \neq n_0$ . Indeed, since  $S_n$  is false, the conjunction  $(\neg S_{n+m} \wedge \cdots \wedge \neg S_{n+m+k}) \wedge S_n$  also is false.

Now, let us assume that  $\mathfrak{M} = \langle \mathfrak{A}, D \rangle$  is a CCL-model and  $v \in \text{Hom}(\mathcal{L}_{\text{CCL}}, \mathfrak{A})$  a valuation such that for any  $i \in \mathbb{N}$ ,  $v(S_i) = C_i$ . Of course every  $C_i \in A$ , and moreover

$$C_n = (-C_{n+m} \cap \cdots \cap -C_{n+m+k}) \cap C_n \cap a_{n,m,k}$$

for some  $a_{n,m,k} \in A$ . (1) If  $n \neq n_0$ , then  $a_{n,m,k}$  can be any element from  $A$ . Of course,  $C_n$  does not belong to  $D$  as an intersection of  $(-C_{n+m} \cap \cdots \cap -C_{n+m+k})$ ,  $C_n$ , and  $a_{n,m,k}$ , where the first belongs to  $D$ , the second is not, and the third is unknown. Thus, regardless of  $a_{n,m,k}$ ,  $C_n \notin D$ . (2) When  $n = n_0$ , then  $a_{n,m,k}$  cannot be any element from  $A$ . In this case,  $(-C_{n_0+m} \cap \cdots \cap -C_{n_0+m+k}) \in D$  and  $C_{n_0} \in D$ . Thus, to avoid contradiction,  $a_{n,m,k}$  also must belong to  $D$ .

The case when  $S_0$  is the only true sentence seems to be the most elegant, especially from the point of view of an infinite characterization of all sentences. The first sentence saying that all next sentences are false is a true sentence. And all next sentences really are false. The semantic interpretation could then simply be:

$$\begin{aligned}
 C_0 &= (\neg C_1 \cap \neg C_2 \cap \neg C_3 \cap \dots \cap \neg C_k \cap \dots) \cap C_0 \\
 C_1 &= (\neg C_2 \cap \neg C_3 \cap \neg C_4 \cap \dots \cap \neg C_k \cap \dots) \cap C_1 \\
 C_2 &= (\neg C_3 \cap \neg C_4 \cap \neg C_5 \cap \dots \cap \neg C_k \cap \dots) \cap C_2 \\
 &\vdots \\
 C_n &= (\neg C_{n+1} \cap \neg C_{n+2} \cap \neg C_{n+3} \cap \dots \cap \neg C_{n+k} \cap \dots) \cap C_n \\
 &\vdots
 \end{aligned}$$

Unfortunately, this characteristic is not approved because CCL-language does not contain infinite formulas.

### 3.2. All Yablo sentences are false

In light of the considerations of the previous paragraph, it would be easiest to assume that all sentences are false. Moreover, such a solution coincides with the “infinite” case discussed at the beginning of the third section. Thus, let us assume that for any  $n \in \mathbb{N}$ ,  $S_n$  is false, and so,  $\neg S_n$  is true. Thus, every conjunction  $\neg S_{n+m} \wedge \dots \wedge \neg S_{n+m+k}$  is true but every  $(\neg S_{n+m} \wedge \dots \wedge \neg S_{n+m+k}) \wedge S_n$  is false. The semantic interpretation simple:  $a_{0,m,k} = v(S_0) = C_0 \in D$ , and for any  $n > 0$ ,  $a_{n,m,k} = v(S_n) = C_n \notin D$ .

### 3.3. Comment

Some doubts may arise over the claim that the sentence  $S_n : (\neg S_{n+m} \wedge \dots \wedge \neg S_{n+m+k}) \wedge S_n$  is false thanks to its own falsehood. The first is that it is because we consider infinitely many sentences that the conjunction of any two of them is contradictory. However, there is another reason for this fact, namely that for any  $\alpha$ ,  $\alpha : \alpha$  is the CCL-tautology. This trivial tautology has a completely non-trivial justification detailed in [Łukowski, 2019]. This explanation refers to what is known as *Buridan's thesis*, which claims that a correct understanding of the meaning of a sentence requires the temporary acceptance (as a working assumption) of

the truthfulness of that sentence. More precisely, the correct recognition of the logical value of the sentence must be preceded by the recognition of the meaning of the sentence. However, the correct recognition of the meaning of a sentence is possible only if the truth of the sentence is assumed. For example, let us consider the sentence “Warsaw is located west of Lisbon”. We understood well all the expressions composing the sentence: “Warsaw”, “is located”, “west of” and “Lisbon”. However, in order to understand the whole sentence, we need to combine the meanings of these expressions and recognize the situation which this sentence describes. Thus, at this stage, we understand that the sentence says that Warsaw is located west of Lisbon. In other words, we understand that the sentence expresses the situation that Warsaw is west of Lisbon. But this situation is expressed by the sentence only on the condition that this sentence is treated as true. If this sentence were treated as false, we would understand it as saying that Warsaw may lie north or east or south of Lisbon — but not west of it. Therefore, if we think that this sentence says that Warsaw is west of Lisbon, it is only because we understand this sentence as true. Only under this condition the sense of the sentence is not reversed by us. After recognizing the real meaning of a sentence, we can compare the situation expressed in the sentence with reality. Because Warsaw is not west but east of Lisbon, we recognize the sentence as false.

#### 4. Conclusion

In the standard approach, here called the “distributive approach”, the Yablo problem is unsolvable just as the liar problem is. The new, “collective” perspective reformulates the whole problem in such a way that the contents of sentences start to play an important role. Consequently, we can demonstrate the validity of one of two solutions. Either all Yablo sentences are false, or exactly one is true. Furthermore, it seems that the way in which we have formalised the contents of the sentences shows that Yablo’s problem is not one of circularity.

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