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## HUMBLE CONNEXIVITY

**Abstract.** In this paper, I review the motivation of connexive and strongly connexive logics, and I investigate the question why it is so hard to achieve those properties in a logic with a well motivated semantic theory. My answer is that strong connexivity, and even just weak connexivity, is too stringent a requirement. I introduce the notion of *humble connexivity*, which in essence is the idea to restrict the connexive requirements to possible antecedents. I show that this restriction can be well motivated, while it still leaves us with a set of requirements that are far from trivial. In fact, formalizing the idea of humble connexivity is not as straightforward as one might expect, and I offer three different proposals. I examine some well known logics to determine whether they are humbly connexive or not, and I end with a more wide-focused view on the logical landscape seen through the lens of humble connexivity.

**Keywords:** connexive logic; strong connexivity; unsatisfiability; paraconsistency; conditional logic; modal logic

### 1. Introduction

This paper is an attempt to answer a particular challenge to the enterprise of connexive logic. It was put to me some years ago by David Makinson.<sup>1</sup>

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<sup>1</sup> Not only is he, by giving me this challenge, responsible for the existence of this paper, he also gave a number of suggestions that were of tremendous help to me in writing this paper; section 6 in particular owes its inclusion and form to these suggestions. Two others have had an equally great impact on this paper, and they happen to be the editors of this volume. The idea of humble connexivity originates in my joint work with Hitoshi Omori. Even if what I'll have to say is probably more opinionated than he would have put it, I would not have been able to form

He said (quoted from dim memory, but corroborated by his own memory of the event):

I think that the idea of connexive logic leads up a blind alley. The connexive principles look convincing at first glance, but just a little thought will show that these are only appearances. For example, in  $(A \rightarrow \neg A)$ , take  $A$  to be an outright contradiction such as  $B \wedge \neg B$ , and the statement will look perfectly fine.

That day, I had little to answer. Now, however, I think I have the answer that I should have given back then. Sometimes you wake up realizing which witty and pithy reply you should have given the day before. Other times, it takes some years, and the reply is not pithy at all but paper length. In any case, here it is.

## 2. Background: Connexivity, Weak and Strong

Let us start at the beginning. Usually, connexivity is understood to lie in adherence to the following principles:

ARISTOTLE      $\neg(A \rightarrow \neg A)$  and  $\neg(\neg A \rightarrow A)$  are valid.

BOETHIUS      $(A \rightarrow B) \rightarrow \neg(A \rightarrow \neg B)$  and  $(A \rightarrow \neg B) \rightarrow \neg(A \rightarrow B)$  are valid.<sup>2</sup>

As it happens, few of the known logics are actually connexive. Most prominently, classical logic is not connexive. Moreover, classical logic would even become *trivial* if ARISTOTLE and BOETHIUS were added as new axioms, so in a connexive logic, certain classical validities will have to be dropped. Though non-classical logics are, for the most part, arrived at by dropping certain things from classical logic, few if any of the usual non-classical ideas lead naturally to something that is connexive.

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these opinions without him. Heinrich Wansing has long been my guide to all things connexive (and not just mine), and he has given me a detailed list of comments that much improved this piece. Two referees have also given very helpful comments, and some improvements are due to observations by Hannes Leitgeb and Lavinia Picollo and the audience at the Third Connexive Logic Workshop in Kyoto in 2017. I thank them all, and remain responsible for all mistakes and opinions expressed.

<sup>2</sup> Nothing I have to say hinges on the difference between  $\neg(A \rightarrow \neg A)$  and  $\neg(\neg A \rightarrow A)$ , nor on the difference between  $(A \rightarrow B) \rightarrow \neg(A \rightarrow \neg B)$  and  $(A \rightarrow \neg B) \rightarrow \neg(A \rightarrow B)$ , and I will often just discuss one of these variations and leave the others to be understood implicitly. Sometimes, when I take a thought to apply in obviously similar ways to both ARISTOTLE and BOETHIUS, I will even just mention  $\neg(A \rightarrow \neg A)$  and ask the reader to think of all four principles.

Before I try to give a diagnosis of why this is so, a quick note on the names of the connexive theses is in order: The intent of my paper is conceptual, not historical. That is, I do not consider the question what Aristotle and Boethius thought, and whether it is captured by the principles named after them. However, the interested reader should consider this piece side by side with Wolfgang Lenzen's contribution to this collection, as he argues that the charitable way of reading these authors is to take them to mean something very close to what I will develop here.<sup>3</sup>

For the purposes of this paper, let us start with the observation that these principles certainly seem plausible at first blush. Read out with the usual natural language correspondences to the formal vocabulary, it is hard not to feel a strong pre-theoretic intuition that these principles express logical truths. "It is not the case that if  $A$  is the case, not- $A$  is the case", for example, is as plausibly true as it is clumsy phrased. The same is true if we transpose the conditional to the subjunctive mood: "It is not the case that if  $A$  were the case, not- $A$  would be the case".

Though I have come to think that the connexive theses are most interesting when thought about as natural language conditionals, it remains true that they also sound intuitively right when  $\rightarrow$  is read as entailment or implication: "It is not the case that a statement should entail its own negation". Plausible, indeed, at least at first sight.

In earlier work, I have pointed out that ARISTOTLE and BOETHIUS by themselves might not be doing full justice to these intuitions. In [3], I suggested that in order to do so, a logic should additionally satisfy the following conditions:<sup>4</sup>

UNSAT1 In no model,  $A \rightarrow \neg A$  is satisfiable (for any  $A$ ).

UNSAT2 In no model  $(A \rightarrow B)$  and  $(A \rightarrow \neg B)$  are simultaneously satisfiable (for any  $A$  and  $B$ ).

It seemed, and still seems, to me that whatever reason you might have to require "It is not the case that if  $A$  is the case, not- $A$  is the

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<sup>3</sup> Our papers were developed in complete isolation, and we only found out about their surprising convergence when we were both invited to give talks at the Third Connexive Logic Workshop in Kyoto in 2017. The paper submitted to this issue by Wansing and Unterhuber, "Connexive Conditional Logic. Part 1", also has some overlap with my topic, especially with the material in section 7. Furthermore, the editors have pointed out to me that [15] also expresses ideas that go in a similar direction as the notion of humble connexivity does.

<sup>4</sup> Again, remember that variations like  $\neg A \rightarrow A$  are omitted in all clauses that follow and are to be understood implicitly.

case” to be logically true should also rule out any satisfiable instance of “If  $A$  is the case, not- $A$  is the case”. Note that this was not just an idle exercise in pedantry, as there were connexive logics discussed in the literature that *had* satisfiable instances of  $A \rightarrow \neg A$ , etc. I coined the term *strong connexivity* for the property that is made up of all four conditions above (ARISTOTLE, BOETHIUS, UNSAT1 and UNSAT2). Correspondingly, I called logics that only satisfy the earlier two conditions *weakly connexive*.<sup>5</sup>

More recently, [2], Luis Estrada-González and Elisángela Ramírez-Cámara found it useful to single out the last two conditions, UNSAT1 and UNSAT2, and called logics satisfying those conditions (but not necessarily ARISTOTLE and BOETHIUS) *Kapsner-strongly connexive*. I felt both flattered and bemused by this development, as I had certainly not intended to make any case for those conditions by themselves. I have warmed up to the idea that they might have some merit in certain settings, though, and I will write a bit more about this below.

While I am still not completely certain that UNSAT1 and UNSAT2 are worth investigating *on their own* in this way, I have surely not come to doubt my argument for adding them to the connexive theses ARISTOTLE and BOETHIUS. Nonetheless, it must be acknowledged that strong connexivity seems to be a very demanding requirement. Even though connexive logic, as a research field, is living through a small renaissance these days,<sup>6</sup> there have been no proposals for a truly satisfying strongly connexive logic since the idea was introduced, at least none that I am aware of. What I mean by “truly satisfying” in this context is mainly that the logic should have an intelligible and well-motivated semantics. There *are* strongly connexive logics, but they tend to be many-valued logics in which an intuitive reading of the values is missing.<sup>7</sup> In my view, this amounts to considerable (even if clearly defeasible) evidence

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<sup>5</sup> Note that any weakly connexive system will have to be paraconsistent, else it will collapse into triviality. It might be thought that certain applications of paraconsistent logics will also make it doubtful whether the UNSAT-clauses are really warranted. I will come back to this and other topics related to paraconsistency towards the end of the paper.

<sup>6</sup> See, e.g., references in [17, 18], as well as the contribution to the present volume.

<sup>7</sup> With Hitoshi Omori, I myself have been working on ideas that have gotten us closer to strong connexivity than any other attempt we know of in a logic that is a close relative to the one we introduced in [5]. In the end, we must admit to still fall slightly short of the pure idea of strong connexivity, but our efforts are interesting in

that strong connexivity was too much to ask for. And indeed, I have come to believe that this is the case.

It therefore looks to me that, starting out from strong connexivity, we might have to weaken our requirements again. There are (at least) three options here:

(A) We go back to weak connexivity and look for ways in which the original motivation of strong connexivity was mistaken. Maybe the kinds of instances of  $A \rightarrow \neg A$  etc. that are satisfiable all have some interesting property that makes their satisfiability plausible (while it does not undermine the plausibility of the logical truth of  $\neg(A \rightarrow \neg A)$  etc.) Someone generally sympathetic to paraconsistent logics might have a story to tell along these lines.<sup>8</sup> I am suspicious of the viability of this route, but I am ready to be convinced otherwise.

(B) We could go the other way and consider the validity of ARISTOTLE and BOETHIUS as relatively unimportant compared to UNSAT1 and UNSAT2. That is, we could instead go on to look for Kapsner-strong connexive logics as the true solution to the intuitions driving the connexive enterprise. Until recently, this would have seemed a rather absurd option to me, but as I mentioned above, I am in the process of changing my mind. At this point I believe that there might be a place for Kapsner-strongly connexive logics in a full picture, but I will not pursue the line in this piece.<sup>9</sup>

(C) The last option is the one I want to investigate in this paper: I want to weaken the requirements of strong connexivity in a quite dif-

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themselves, and they also spawned the ideas in this piece. That work will be published elsewhere, but I will allow myself to allude to it again below.

<sup>8</sup> Again, a logic that is only weakly connexive will have to be paraconsistent in order to avoid triviality.

<sup>9</sup> As there will be a small point of contact with the material below, let me give just a very small glimpse of what I have in mind. I am at the moment exploring the idea that connexivity is fully at home in the realm of counterfactual conditionals, while in indicative conditionals the plausibility of ARISTOTLE and BOETHIUS is a matter of pragmatics rather than semantics. In particular, these principles strike us as plausible because a presupposition fails when things like “If  $A$  is the case, then not- $A$  is the case” are asserted, namely the presupposition that the antecedent of an indicative conditional is an epistemic possibility for the speaker (see [6]). If one takes a Strawsonian view on presupposition failures, according to which they lead to statements that lack a truth value and whose negations also lack a truth value, one will feel encouraged to look for something like Kapsner-strongly connexive systems (possibly slightly altered by the ideas which I develop in this piece below), at least in order to deal with indicatives.

ferent dimension from the line between strong and weak connexivity. I will introduce this new line of demarcation in the next section.

### 3. Humble Connexivity

As I already mentioned in footnote 7, the idea to this new line of demarcation came to me as I was working with Hitoshi Omori on a new family of constructive logics that we augmented with a subjunctive conditional. Omori and I realized that often, the problematic part of getting strong connexivity seems to be wrestling with impossible antecedents. This suggested the following: Impossible antecedents are generating too much trouble, so what we should go for is a weaker notion of connexivity that only applies to *possible* antecedents, or at least *non-contradictory* ones. We should stick to requiring ARISTOTLE, BOETHIUS and the two UNSAT-clauses to hold, but only *if the antecedents are possible*. I will call this more restrained idea of connexivity *humble connexivity*, and I will get to how to make this idea formally more precise in the next sections. In this section, I want to make the notion informally plausible, first.

Before I get to my arguments to that end, just a quick note on the terminology I adopted: Hypothetically, someone persuaded by these arguments, but not by my earlier arguments for strong connexivity, might want to disregard the unsatisfiability clauses, even in their humbled form. For the sake of completeness, then, we might want to call this impoverished set of requirements *weak humble connexivity* and the full set *strong humble connexivity*. For ease of communication, though, I will here refer to the latter simply as humble connexivity, just because I believe it to express the right requirements.

Now, what speaks for humble connexivity is not just that it might simply be easier to meet that lowered bar, as opposed to the earlier requirements of strong connexivity. I believe that it is also philosophically a most natural move, all considerations of technical feasibility aside.

There are two ways to argue for humble connexivity, and, as far as I can see, only one to argue against it, all of them starting with different answers to the question what we should do with conditionals with impossible antecedents:

First, we might say that conditionals with impossible antecedents are pretty much opaque to our intuitions. If something impossible were the case, then who knows what else would be the case? Everything?

Nothing? Something? All of these answers have been given, and it might be doubted that this dispute can be resolved at all. In view of this, it would seem prudent to only ask for humble connexivity, just to be sure that we aren't overplaying our hand by asking for too much. Taking this agnostic stance will not mean that we will be disappointed by a non-humbly connexive logic, only that we will be satisfied with a humble one.

The second, more full-blooded answer would be the one I was given by David Makinson: We indeed have a good idea about statements of the form  $(A \rightarrow \neg A)$  when  $A$  is impossible: They are true!

Certainly, if the impossibility of the antecedent stems from it being contradictory, there is a very strong case for this answer to be made. One might, for example, think it is right because one believes in EXPLOSION, which will get us there immediately. EXPLOSION, of course, is a contested principle, so it is worthwhile to note that much less questionable principles are sufficient as well. This has been clear at least since Storrs McCall made his contribution to Anderson and Belnap's *Entailment*; here is his derivation ([9, p. 463] notation adjusted):

1.  $(A \wedge \neg A) \rightarrow A$
2.  $A \rightarrow (\neg A \vee A)$
3.  $(A \wedge \neg A) \rightarrow (\neg A \vee A)$
4.  $((A \wedge \neg A) \rightarrow \neg(A \wedge \neg A))$

The last step packs together a DeMorgan law and double negation elimination. The latter of which might have intuitionists grumble a bit, so it might be worthwhile to point out that it isn't strictly needed, as the following adjustment of the derivation shows:

1.  $(A \wedge \neg A) \rightarrow \neg A$
2.  $\neg A \rightarrow (\neg A \vee \neg \neg A)$
3.  $(A \wedge \neg A) \rightarrow (\neg A \vee \neg \neg A)$
4.  $((A \wedge \neg A) \rightarrow \neg(A \wedge \neg A))$

This uses only principles endorsed by adherents of almost all major non-classical logics (note that the DeMorgan law is one of the three that intuitionists accept).<sup>10</sup>

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<sup>10</sup> As I was looking this part of McCall's contribution to *Entailment* up, I was startled to find a line of thought that seems very close to the one in this paper. McCall is thinking of a calculus for "events or states of affairs that occur at the same time" and wants to restrict substitutivity in such a way that you cannot get from  $(A \wedge B) \rightarrow A$  to  $(A \wedge \neg A) \rightarrow A$ , because  $A$  and  $\neg A$  just cannot be occurring at the same time.

In any case, whether because of these derivations or some other reason, if you believe that  $(A \rightarrow \neg A)$  is true when  $A$  is contradictory or otherwise impossible, you should certainly welcome the restriction to possible antecedents in the humble requirements.

Both of the views explored so far, then, point us towards humble connexivity. Those who will insist on taking the *third* remaining option at this point, namely to argue that we can be sure that  $(A \rightarrow \neg A)$  is false<sup>11</sup> even if  $A$  is contradictory, will have to carry on in their search for a non-humble strongly connexive logic. If they strike gold, we will be satisfied as well, as any strongly connexive logic will trivially satisfy the demands of humble connexivity. But even if such a success is to be had (something I am by now mildly doubtful of), there is value in discussing our restriction, because it will make connexivity an interesting topic for all those who hold one of the two other views about such statements.

As I said above, I think that giving one of the first two answers seems plausible for a range of notions of (im)possibility. Certainly, this is true for logical possibility, in the sense that the antecedent should not be outright contradictory; for the purposes of this paper, it might well be enough to stop right there. But intuitively, probably also slightly less blatant forms of impossibility should be filtered out. Mathematical impossibility looks like it might well have to go, and maybe the same goes for metaphysical impossibility.<sup>12</sup>

#### 4. Is Humble Connexivity Boring?

For the reasons above, I believe that philosophically, humble connexivity is more attractive than “traditional” unrestricted connexivity, both in its weak and its strong form. It is also more attractive than having no kind

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Given such a restriction, calling for connexivity will come to the same thing as calling for humble connexivity (modulo the addition of the idea of strong connexivity, which I don't think is in conflict with the ideas McCall was trying to spell out). However, I want to apply my idea to all logics, whether or not substitutivity holds in them.

<sup>11</sup> Or a least *not true*, while  $\neg(A \rightarrow \neg A)$  is true.

<sup>12</sup> Maybe even epistemic impossibility might be something to consider here: An alternative way to account for the unacceptability of ARISTOTLE and BOETHIUS in the indicative mood which I mentioned in footnote 9 would be *not* to view it as a question of pragmatics and to make it into a semantic requirement that would look just as the above, with the diamond expressing that the statement is possibly true for all the speaker knows.



of connexivity at all, at least for those who are moved by the intuitions alluded to in the beginning of this piece. Also, I have conjectured that given this restriction, it will be possible to finally find some more satisfying logics that meet our demands, and I will give some evidence for that conjecture right below. All that is good. Is there anything that might possibly be *bad* about the requirements of humble connexivity?

One of the few ways I can think of to be unsatisfied with humble connexivity is to think that it is utterly obvious and boring.<sup>13</sup> The beef of connexivity, that argument might run, lies precisely in those things that I want to filter out, namely the contradictory premises. Of course, whether those instances of ARISTOTLE and BOETHIUS with inconsistent premises should be valid or not is a contentious question, but that is the reason why connexive logic is bold and exciting. When we dial back to humble connexivity, then what we are left with is something on which a consensus might indeed quickly be reached, but that just goes to show how the really interesting questions have been skirted.

That line of thought, however, runs into a big problem: The proposal can't be quite *that* boring and obvious, given that so many of our garden-variety logics fail to live up to it! First and foremost, classical logic is not humbly connexive. In classical logic,  $A \rightarrow \neg A$  is true when  $A$  is false, no matter  $A$ 's logical or modal status. Similarly, in intuitionistic logic  $A \rightarrow \neg A$  is provable when  $\neg A$  is provable. The same goes for Nelson logics  $N3$  and  $N4$ , as well as all the variations I introduced in [4].

Indeed, I have only found two related areas in non-classical logic in which humble connexivity seems to arise naturally. These are modal logics with strict implications and the so-called conditional logics, i.e. logics designed to account for counterfactual conditionals. However, before I can make the argument that these logics fulfill my requirements, I will need to express them in a formally more adequate way.

## 5. Expressing Humble Connexivity: Modal and Plain

Expressing humility can be difficult. "I am so humble!", for example, rarely works.

It turns out that in our case, the task is likewise not quite as straightforward as one might hope, and that some attention needs to be paid

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<sup>13</sup> I have actually heard that complaint when I first aired the idea.

to the particular circumstances. I will propose two ways of formalizing humble connexivity in this section and leave a third one, applicable in paraconsistent settings, for a later section.

First, in those cases in which we have suitable modal vocabulary available it seems only natural to employ it to express the restriction to possible statements as antecedents. In the case of BOETHIUS, this concerns not just  $A$ , but also  $(A \rightarrow B)$ , because we are driven by the recognition that unsatisfiable antecedents are generating too much trouble, and  $(A \rightarrow B)$  happens to be the antecedent of the main connective in BOETHIUS. This leads to the following proposal for what I call *modal humble connexivity*:

MODAL HUMBLE ARISTOTLE:  $\Diamond A \models \neg(A \rightarrow \neg A)$  is valid.

MODAL HUMBLE BOETHIUS:

$\Diamond A \wedge \Diamond(A \rightarrow B) \models (A \rightarrow B) \rightarrow \neg(A \rightarrow \neg B)$  is valid

MODAL HUMBLE UNSAT1: In no model,  $\Diamond A \wedge (A \rightarrow \neg A)$  is satisfiable.

MODAL HUMBLE UNSAT2: In no model,  $\Diamond A \wedge (A \rightarrow B) \wedge (A \rightarrow \neg B)$  is satisfiable.

I think that this way of phrasing the requirements is relatively straightforward, even if there might be alternatives that could also be considered (such as  $\models \Diamond A \rightarrow \neg(A \rightarrow \neg A)$ , etc.). What seems more in need of discussion is the talk of the availability of “suitable modal vocabulary” that I used in the lead-up to the conditions. However, I am afraid that seeing whether that proviso is met will involve some reader discretion. As we are absolutely general about other features of the logics in question, it seems hard to give a precise set of syntactic requirements for this modality; it will have to be seen in each individual case whether the diamond does what we want it to do. Also, as I think many levels of impossibility might be affected by the arguments for humble connexivity in the preceding section, I intend to remain somewhat uncommitted as to which kind of possibility the diamond should express. I will come back to this issue below.

In any case, I want the idea of humble connexivity to be general enough to be also applicable to logics which don’t have modal vocabulary, so I would like to suggest a second set of conditions in which I will use *unsatisfiability* as a rough proxy for impossibility.<sup>14</sup>

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<sup>14</sup> I am, after all, already in the slightly unusual business of talking about unsatis-

What this will mean in detail is that we will ask for ARISTOTLE only to hold for those  $A$  that are satisfiable. Likewise, UNSAT1 is only required to hold when  $A$  is satisfiable.

When we consider BOETHIUS, we will again have to ask for a satisfiable  $A$  and to make sure that  $(A \rightarrow B)$  is satisfiable.<sup>15</sup>

Thus, what we get is the following set of conditions, which characterizes what I shall call *plain humble connexivity*.

PLAIN HUMBLE ARISTOTLE: For any *satisfiable*  $A$ ,  $\neg(A \rightarrow \neg A)$  is valid.

PLAIN HUMBLE BOETHIUS: For any *satisfiable*  $A$  and *satisfiable*  $(A \rightarrow B)$ ,  $(A \rightarrow B) \rightarrow \neg(A \rightarrow \neg B)$  is valid.

PLAIN HUMBLE UNSAT1: In no model,  $A \rightarrow \neg A$  is satisfiable (for any *satisfiable*  $A$ ).

PLAIN HUMBLE UNSAT2: In no model,  $(A \rightarrow B)$  and  $(A \rightarrow \neg B)$  are simultaneously satisfiable (for any *satisfiable*  $A$ ).

Unfortunately, the two ways of phrasing the requirements of humility are only approximations of each other. In certain circumstances, plain and modal humble connexivity might diverge, while they will go together in other settings. We will see both of these patterns when we look at the two examples of humbly connexive logics I mentioned earlier, modal logics with strict implications and conditional logics. These families of intensional logics are close relatives of each other, and it is interesting to study how they respond to the two different versions of humble connexivity I introduced in this section. After this, I will get back to the question which version of the clauses should be used in questionable cases.

## 6. Humble Connexivity in Modal Logics with Strict Implication

First, consider normal modal logics, such as K or stronger ones like T, B, S4, S5 and others. When we define a strict implication,  $\rightarrow$ , as

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fiability in phrasing my requirements, as I want to take the ideas of strong connexivity on board.

<sup>15</sup> It might look like it would be enough to ask for the satisfiability of  $B$  to achieve this in any decent logic, but that is not quite true. Just take  $B$  to be  $\neg A$ , and then any logic that satisfies Unsats1 (for satisfiable  $A$ ) will yield an unsatisfiable antecedent here. To ask that  $A$  and  $B$  should be *simultaneously* satisfiable should do the trick in most settings and be somewhat more elegant. However, just to be explicit about what I want and to guard against unforeseen complications, I go for the slightly more unwieldy but more straightforward condition that  $(A \rightarrow B)$  is satisfiable.

$\Box(A \supset B)$  in these logics, where  $\supset$  is the material conditional, then we get systems that exhibit Modal Humble Connexivity. For example, MODAL HUMBLE ARISTOTLE,  $\Diamond A \models \neg(A \supset \neg A)$ , unfolds in this setting to be a definitional abbreviation of  $\Diamond A \models \neg\Box(A \supset \neg A)$ .

In the Kripke semantics for  $K$  (the logic characterized by the class of all frames) and all stronger systems,  $\Diamond A$  being true at a world  $w$  means that there is a world accessible from  $w$  in which  $A$  is true. In classical logic,  $(A \supset \neg A)$  is equivalent to  $\neg A$ . Thus,  $\Box(A \supset \neg A)$  says nothing more than that in every world accessible from  $w$ ,  $\neg A$  is true. Given the underlying classical logic governing the worlds, this cannot be true in  $w$  (delivering MODAL HUMBLE UNSAT1), so  $\neg\Box(A \supset \neg A)$  must be true in  $w$ , which gives us MODAL HUMBLE ARISTOTLE. It is not harder than this to see that MODAL HUMBLE BOETHIUS and MODAL HUMBLE UNSAT2 hold, as well.

However, the normal modal logics do not answer to the non-modalized version of the clauses. To see this, just consider a model, call it  $M$ , in which a given propositional parameter, call it  $p$ , is false at every world. This is a perfectly normal model. As  $p$  is arbitrarily chosen, it is surely satisfiable; a different model in which it appears true at some worlds is just as fine as the one we are considering. Nonetheless, we find in  $M$  that  $\Box(p \supset \neg p)$  holds. Thus,  $p \supset \neg p$  is satisfiable, and the non-modalized version of the clauses is seen to fail to hold. ( $M$  is not a countermodel for the modalized version of humility because  $\Diamond p$  does not hold in it).

We could try to get these modal logics to also answer to the unmodalized version of the requirements by restricting the class of models in such a way that such troublesome models as  $M$  are ruled out. That is, we might, instead of arbitrary models, only consider “intended” models that seek to capture logical possibility. One requirement for being among the intended models would be that for each world and each propositional parameter, there is an accessible world in which that parameter is true, and another world in which it is false.

This would not be a wholly unnatural move. It is relatively close to what Carnap originally proposed in *Meaning and Necessity* ([1], see also [8]), and we will see a variant of it in the next section. However, in the case of modal logic, Timothy Williamson has argued that it would be going against the spirit of Kripke’s project to make this kind of restriction to intended models. He concludes that therefore, possible worlds semantics are not very well suited for the study of logical necessity and more apt

to give us insights about metaphysical and other kinds of necessity (see [20, p. 83]). This is not the place to delve into that argument. However, whether or not we agree with Williamson, we must acknowledge the fact that in practice, the strategy of singling out intended models is usually not pursued in the study of modal logics and strict implications.

So, unless we want to go against the grain of current theorizing about modal logic, we must come to terms with the fact that the two versions of the humble requirements part ways at this point. Maybe the best simple heuristic I can offer is to use the modal version whenever a diamond expressing a notion of possibility that strikes you as affected by the arguments above is available, and to revert to the plain version only when such expressibility is not available. In any case, we will see in the next section that (luckily) there are also cases in which no such call needs to be made, as the two versions coincide.

## 7. Humble Connexivity in Conditional Logics

The second area in which humble connexivity seems to be easily attainable are the logics for counterfactual conditionals (such as “If  $A$  had been the case, then  $B$  would have been the case”) developed by Robert Stalnaker and David Lewis, and the many people working in their wake.<sup>16</sup> There is a difference between Lewis’s and Stalnaker’s systems that plays a certain role here; I will discuss Stalnaker’s system as introduced in [14] in this section and comment on Lewis’s in an appendix for the interested reader.

The account of counterfactuals that Stalnaker and Lewis give is sometimes called one of “variably strict” conditionals, which already shows the close proximity to the strict conditionals we saw in the last section. Accordingly, the semantics is a variation of the possible world semantics for modal logics that was discussed in the last section. The intuitive idea behind Stalnaker’s logic is that a conditional statement in the subjunctive mood is true if and only if  $B$  is true at the world in which  $A$  is true and which is otherwise most similar to ours.

To deal with impossible antecedents, Stalnaker posits an *impossible* world in which everything is true, and which is further away from ours

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<sup>16</sup> This includes the work by Omori and myself that I have mentioned in footnote 7, which in fact was the starting point for the line of thought that led me to the present paper.

than every possible world. It is important to note that the selection function will only pick out this impossible world in cases where the antecedent is impossible. In other words, for every possible statement, Stalnaker's models have at least one world in which this statement is true. In a sense, this is a way of restricting the semantics to intended models in the way I mentioned in the last section.<sup>17</sup>

As every statement is true in the impossible world, the non-humble versions of ARISTOTLE, BOETHIUS, UNSAT1 and UNSAT2 all fail in this system, counterexamples for all four being cases in which  $A$  is impossible.

However, if we restrict our attention to possible antecedents, then things look very different, indeed. Let us start by seeing whether the plain version of the conditions is fulfilled (I will leave off the "plain" modifier in the next three paragraphs).

As the worlds (except for the impossible world that is furthest removed from all other worlds) are all classical, in no world we find both  $A$  and  $\neg A$  true. That means that  $A \rightarrow \neg A$  cannot be true, as the truth condition looks for the closest world in which  $A$  is true, which is by assumption not the trivial but a classical one. So  $\neg A$  is false there, showing that HUMBLE UNSAT1 is met. As  $A \rightarrow \neg A$  is false for all possible statements  $A$ ,  $\neg(A \rightarrow \neg A)$  is true for all these statements, giving us HUMBLE ARISTOTLE.

For HUMBLE UNSAT2, consider a true conditional ( $A \rightarrow B$ ) with a possible antecedent  $A$ . Then  $B$  is true at the closest world in which  $A$  is true, which is a classical world. So  $\neg B$  cannot be true, and therefore ( $A \rightarrow \neg B$ ) can't, either.

Last, for HUMBLE BOETHIUS, consider a satisfiable conditional ( $A \rightarrow B$ ) with a satisfiable antecedent  $A$ . That means that there is a possible classical world  $y$  which is closest to ours and in which ( $A \rightarrow B$ ) is true. That in turn means that there is a possible classical world  $z$  which is closest to  $y$ , given that  $A$  holds in it, in which  $B$  is true, as well. But that means that  $\neg B$  is false in  $z$ , which means that ( $A \rightarrow \neg B$ ) is false in  $y$ , and  $\neg(A \rightarrow \neg B)$  true. That, finally, means that  $(A \rightarrow B) \rightarrow \neg(A \rightarrow \neg B)$  must be true at our world.

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<sup>17</sup> One of the referees has raised an interesting question at this point, namely whether Stalnaker's semantics should be seen as "intelligible and well-motivated", as I have put it earlier in this piece. I would say that it is, even if, of course, one might disagree with the ideas that stand behind the choice of the formalism. But at least these ideas are straightforwardly recognizable, such as the idea that everything would be true if something impossible were true.

So, Stalnaker's system is the first example of a well known logic in which the plain version of humility holds. Luckily, it is also an instance in which the modal version of our requirements is readily available, showing that the two sets of requirements do not *always* diverge.

Stalnaker himself defines possibility as  $\neg(A \rightarrow \neg A)$ , which makes MODAL HUMBLE ARISTOTLE completely trivial, at least from a formal point of view. The other clauses are slightly more interesting, but also clearly satisfied.

## 8. Humble Connexivity and Paraconsistency

We saw in the last sections that the notions of satisfiability and possibility (at least as expressed in modal logic) do not necessarily coincide. The same is true of the notions of unsatisfiability and *inconsistency*, if we are willing to consider paraconsistent logics.

Paraconsistency is often (though not universally) achieved by allowing a glutty truth value, call it **B**, that is to be understood roughly as "both true and false". This value is considered a designated value, and the negation of a statement with value **B** is fixed such that it also receives value **B**, so as to give a counter example to  $A \wedge \neg A \models B$ .<sup>18</sup>

Motivations for adding such a glutty value vary, and it is hard to speak in full generality here. However, it stands to reason that, at least in some cases, statements with value **B** are exactly the ones we are trying to filter out in our humble requirements, their satisfiability notwithstanding. Where paraconsistency is employed to underwrite dialethic theories, for example, this seems to be the case.

In other cases, such as the told-truth interpretation of the values in First Degree Entailment (FDE), on the other hand, it does not seem to me to be a plausible requirement to restrict the connexive principles to antecedents that do not take value **B**: Suppose that an otherwise unremarkable statement  $A$  has, by different sources, been told to us to be true and to be false. That in itself, should not be enough to allow it to feature in true statements such as  $A \rightarrow \neg A$ . After all, surely neither of our sources told us that "If  $A$  is the case, not  $A$  is the case" is true, and I don't see an intuitive story about how their information combines to support that conditional.

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<sup>18</sup> Again, this is surely not the only way to achieve paraconsistency, but it is the way paradigmatic examples, such as *LP* and *FDE*, work.

For those examples in which we decide value **B** in the antecedent signals an exception for our requirements, there is another thought to be explored: Consider a statement  $B$  which is true (in some sense) while its negation,  $\neg B$ , is also true. Shouldn't we then also expect that there might be some true conditionals of the form  $(A \rightarrow B)$  and  $(A \rightarrow \neg B)$ ? It seems so, and thus we need to also restrict our conditions to exclude these kinds of cases.

With exceptions for both the antecedent and the succedent, the conditions become, unavoidably, somewhat gnarly. Here is my suggestion:

GLUTTY HUMBLE ARISTOTLE: For any *satisfiable*  $A$  that does not take value **B**,  $\neg(A \rightarrow \neg A)$  takes a designated value.<sup>19</sup>

GLUTTY HUMBLE BOETHIUS: For any *satisfiable*  $A$  that does not take value **B** and any *satisfiable*  $(A \rightarrow B)$  that does not take value **B** and any  $B$  that does not take value **B**,  $(A \rightarrow B) \rightarrow \neg(A \rightarrow \neg B)$  takes a designated value.

GLUTTY HUMBLE UNSAT1: In no model,  $A \rightarrow \neg A$  is satisfiable (for any *satisfiable*  $A$  that does not take value **B**).

GLUTTY HUMBLE UNSAT2: In no model,  $(A \rightarrow B)$  and  $(A \rightarrow \neg B)$  are simultaneously satisfiable (for any *satisfiable*  $A$  that does not take value **B** and any  $B$  that does not take value **B**).

By pushing parts of the requirements to the object language, we can gain generality (covering also strategies for achieving paraconsistency that do not employ gluts), as well as some meta-language clarity (albeit on pain of more complicated object language expressions):

PARACONSISTENT HUMBLE ARISTOTLE: For any *satisfiable*  $A$ ,  $(A \wedge \neg A) \vee \neg(A \rightarrow \neg A)$  is valid.<sup>20</sup>

PARACONSISTENT HUMBLE BOETHIUS: For any *satisfiable*  $A$  and *satisfiable*  $(A \rightarrow B)$ ,  $(A \wedge \neg A) \vee ((A \rightarrow B) \wedge \neg(A \rightarrow B)) \vee (A \rightarrow B) \rightarrow \neg(A \rightarrow \neg B)$  is valid.

PARACONSISTENT HUMBLE UNSAT1:  $A \rightarrow \neg A$  is satisfiable only in valuations in which  $A \wedge \neg A$  is also satisfied.

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<sup>19</sup> Should the paraconsistent logic in question allow us to satisfy *every* formula whatsoever, this part of the clause can of course be omitted.

<sup>20</sup> To phrase these requirements analogously to the way I have done in the clauses for Modal Humble Connexivity, such as  $\neg(A \wedge \neg A) \vDash \neg(\neg A \rightarrow A)$ , does not quite work, as many paraconsistent logics will make  $\neg(A \wedge \neg A)$  true even if  $(A \wedge \neg A)$  is true, as well.



PARACONSISTENT HUMBLE UNSAT2:  $(A \rightarrow B)$  and  $(A \rightarrow \neg B)$  are simultaneously satisfiable only in valuations in which  $A \wedge \neg A$  is satisfied or  $(A \rightarrow B) \wedge (\neg A \rightarrow B)$  is satisfied.

As I said above, at least for some applications of paraconsistency, this seems to be a natural development. However, I must admit that so far, I have not found any example of a logic that meets these specific requirements (I will talk about paraconsistent humble connexivity for the rest of the section, even though all that is said also applies to gluttony humble connexivity).

This is surprising, at least it was to me. I mentioned above that every weakly connexive logic must be paraconsistent. It might thus well be expected that we could find some examples among the weakly connexive logics that have been proposed so far in which the violations of the UNSAT-clauses originate only in gluttony antecedents. Those would then pass muster under the new requirements. But, as far as I can see, this is not the usual pattern.

Consider Heinrich Wansing’s **C** ([16]), a constructive connexive logic that can be given a Kripke semantics in the vein of the Kripke semantics for intuitionistic logic. A model for **C** is a structure  $\langle W, \leq, v \rangle$ ,  $W$  being a non-empty set of partially ordered ( $\leq$ ) worlds and  $v$  a valuation relation, relating formulas to values 1 and 0. Gaps and gluts of these two values are allowed.

There are hereditary constraints for both 1 and 0:

- For all  $p$  and all worlds  $w$  and  $w'$ , if  $w \leq w'$  and  $w \Vdash_1 p$ , then  $w' \Vdash_1 p$ , and
- for all  $p$  and all worlds  $w$  and  $w'$ , if  $w \leq w'$  and  $w \Vdash_0 p$ , then  $w' \Vdash_0 p$ .

Consequence is defined as preservation of value 1:

$\Gamma \vDash A$  iff in every model and every  $w \in W$ , if  $w \Vdash_1 B$  for any  $B \in \Gamma$ , then  $w \Vdash_1 A$ .

Here are the clauses for the connectives:

- $w \Vdash_1 A \wedge B$  iff  $w \Vdash_1 A$  and  $w \Vdash_1 B$
- $w \Vdash_0 A \wedge B$  iff  $w \Vdash_0 A$  or  $w \Vdash_0 B$
- $w \Vdash_1 A \vee B$  iff  $w \Vdash_1 A$  or  $w \Vdash_1 B$
- $w \Vdash_0 A \vee B$  iff  $w \Vdash_0 A$  and  $w \Vdash_0 B$
- $w \Vdash_1 \neg A$  iff  $w \Vdash_0 A$
- $w \Vdash_0 \neg A$  iff  $w \Vdash_1 A$

$w \Vdash_1 A \supset B$  iff for all  $x \geq w$ ,  $x \not\Vdash_1 A$  or  $x \Vdash_1 B$

$w \Vdash_0 A \supset B$  iff for all  $x \geq w$ ,  $x \not\Vdash_1 A$  or  $x \Vdash_0 B$

Now, consider a model with just one world in which  $A$  takes value 0, but not 1.  $A \rightarrow \neg A$  takes value 1 in this model, showing that PARACONSISTENT HUMBLE UNSAT1 is not fulfilled here.

The same is true of Cantwell’s system, which I also discussed as another example for a weakly connexive logic in [3]. It is a three valued logic which, adjusting notation, can be seen as Graham Priest’s Logic of Paradox (LP) with an added conditional. Here is the matrix for the conditional:

$\rightarrow$	<b>T</b>	<b>B</b>	<b>F</b>
<b>T</b>	<b>T</b>	<b>B</b>	<b>F</b>
<b>B</b>	<b>T</b>	<b>B</b>	<b>F</b>
<b>F</b>	<b>B</b>	<b>B</b>	<b>B</b>

Here, if a statement  $A$  takes value **F**, PARACONSISTENT HUMBLE UNSATISFIABILITY 1 is violated.

A close relative of this logic that shows the same pattern (i.e., that is a weakly connexive logic the fate of which does not improve under the new conditions) adds the following conditional to LP:

$\rightarrow$	<b>T</b>	<b>B</b>	<b>F</b>
<b>T</b>	<b>T</b>	<b>B</b>	<b>F</b>
<b>B</b>	<b>B</b>	<b>B</b>	<b>B</b>
<b>F</b>	<b>B</b>	<b>B</b>	<b>B</b>

This is not too unnatural a conditional,<sup>21</sup> even though I don’t think

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<sup>21</sup> It might also not strike you as too natural, either. Viewing it from the angle of the general procedure outlined in [10], it would appear as a combination of the truth condition of a material conditional and the falsity condition of a conjunction, maybe not the most intriguing combination.

It looks pretty plausible, however, if we take the third truth value to be a (designated) truth value gap rather than a glut:

$\rightarrow$	<b>T</b>	<b>N</b>	<b>F</b>
<b>T</b>	<b>T</b>	<b>N</b>	<b>F</b>
<b>N</b>	<b>N</b>	<b>N</b>	<b>N</b>
<b>F</b>	<b>N</b>	<b>N</b>	<b>N</b>

I have argued that gaps should be treated as designated values in certain circumstances

I have seen it in the literature. However, it is likewise weakly connexive while it does not satisfy the clauses of paraconsistent humble connexivity.

That these sorts of systems fail to be paraconsistently humbly connexive shows something interesting about the dialectics around strong connexivity. I mentioned earlier, when I introduced the UNSAT-clauses, that a paraconsistent logician might want to dig in her heels and reject them. Maybe a statement  $A$  that serves as a counterexample to EXPLOSION should indeed be *expected* to satisfy  $A \rightarrow \neg A$  and the like. Fair enough, but in the logics that fail to be paraconsistently humbly connexive, these statements are now seen not to be the culprits, at least not the only ones. And proponents of logics that *do* satisfy paraconsistent humble connexivity will have no pressing need to dig in their heels, provided they manage to find such a system. At least to my mind, adherence to the requirements of paraconsistent humble connexivity speaks more strongly in favor of a logic than mere simple connexivity does (although I admit that that recommendation is less elegantly phrased).

As to the prospects of finding such a logic, the fact that I did not find any examples of a logic satisfying these requirements, of course, does not mean much. A perfectly satisfying system might well be achievable.

Similarly, the point of the preceding two sections was merely to give examples that show that humble connexivity is achievable at all. It was not meant as an endorsement of (variably) strict conditionals over other accounts, nor do I want to suggest that other ways of giving semantics for conditionals couldn't be humbly connexive. It is just that so far, I have not found any such examples. Let me now return from these concrete examples to a more general discussion.

## 9. What (Humble) Connexivity is About

The refocusing of the humble clauses gives us, I have argued, a more refined and philosophically better motivated set of conditions. In addition, I believe that it also give us a better idea what connexivity is about, and what not.

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in [4]. I did not consider this logic there, but it might be seen as a simple alternative to the logics I proposed.

After finishing the manuscript, I have become aware that Paul Egré, Lorenzo Rossi, and Jan Sprenger, as well as Luis Estrada-González and Elisángela Ramírez-Cámara are drafting papers discussing this logic, with the latter explicitly thinking of the middle value as a designated gap.

It has, for example, little to do with any particular account of *negation*. This thought is in mild<sup>22</sup> disagreement with Graham Priest, who took connexive logic to be precisely about negation, namely about an account of negation that he called “negation as cancellation” (see [11]). The idea is that an assertion of  $A$  is cancelled out by an assertion of  $\neg A$ , such that nothing at all is said after the two assertions have been made. The reason Priest sees a connection here is that this account of negation is the only one he can think of that might plausibly deal with the instances of ARISTOTLE and BOETHIUS that go *beyond* humble connexivity, i.e. those which have inconsistent antecedents.

Technically speaking, Priest develops an account that looks similar to the one presented here: There is a restriction to possible antecedents, or rather, as he is more focused on the entailment version of the connexive theses, to possible premisses. However, it is not the *requirements* that are restricted in this way, but rather the *entailment relation*: Inconsistent premisses do not entail anything. This leads to a very unusual notion of entailment, on top of an account of negation that is also quite unusual.<sup>23</sup>

In contrast, as we have decided not to pay too much attention to the problematic cases in which antecedents (or premisses) are inconsistent, we are free to call for humble connexivity, no matter our favorite account of negation. Indeed, I can’t see how any of the usual stories about negation could be in conflict with the plausibility of the humble connexive principles.

If I were asked which logical item humble connexivity is about, I would hesitate to answer. Rather than one isolated notion, humble connexivity seems to me to be about the *interplay* between negation and the conditional.<sup>24</sup> But if I were pressed to choose only one, I would say that humble connexivity has its most interesting things to say about conditionals.<sup>25</sup>

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<sup>22</sup> “Mild” because he might agree with me if he were to consider humble connexivity instead of unrestricted connexivity and simply claim that I changed the subject of the discussion.

<sup>23</sup> I should note that Priest’s piece is exploratory, and that he does not in fact commit to either connexive logic or negation as cancellation. The point he is trying to make is just that the two ideas belong together.

<sup>24</sup> This feeling is in agreement with the presentation in [17].

<sup>25</sup> A referee has encouraged me to comment on Richard Routley’s thought that connexivity is essentially about *conjunction*, in particular, about the failure of conjunction elimination (see [12] and [13]). To me, this seems a relatively absurd notion.

If what I have argued here is right, then not every conditional statement will be affected by the humble connexive requirements. In particular, counter-possible or counter-logical conditionals (e.g., “If I had squared the circle, you would have been surprised”) will receive the same treatment whether or not we accept humble connexivity. Furthermore, I have a hunch that it might be even more particularized than that, namely that the connexive theses should only apply to *counterfactual* conditionals,<sup>26</sup> and that the plausibility of the corresponding indicative instances can be explained by appeals to pragmatics rather than semantics. I have hinted at this thought already in footnote 9 and I am looking forward to develop it further later, but I will not try to unfold that argument here.

## 10. Conclusion

I have argued that the original definition of connexivity (i.e., satisfying ARISTOTLE and BOETHIUS) is both too undemanding and too demanding at the same time. It is too undemanding in not calling for the unsatisfiability clauses of strong connexivity, and it is too demanding in not making the restrictions to possible antecedents that humble connexivity makes.

Regarding the latter, here is what I now think I should have said to David Makinson six years ago:

Indeed,  $A \rightarrow \neg A$  might be a fine statement if  $A$  is contradictory. I give you that without a fight. But what about all the cases in which  $A$  is *consistent*? Even then, classical logic gives you a fifty per cent chance of  $A \rightarrow \neg A$  being true. And in the remaining half of the cases,  $\neg A \rightarrow A$  will be true. *That* is the scandal the connexive critique should focus on.

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Of course, as I have stated in the beginning of this piece, if we want to stretch the logical budget to afford us connexivity, we have to make spending cuts elsewhere; one of the areas we might wish to apply these cuts to is conjunction, as Routley has showed. But to say that this is essential to connexivity is like saying that reductions of spendings on public housing are somehow essential to governmental environmental protection policies. A much more detailed, but likewise critical discussion of Routley’s idea is in [19].

<sup>26</sup> That Stalnaker’s conditional logic is arguably the clearest example for humble connexivity I could present in this paper is in pleasant harmony with this thought, though of course it is not saying too much.

The way to express that indignation is to call for the requirements of humble connexivity.

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## A. Appendix

In this appendix, I want to briefly point out the above-mentioned difference between Stalnaker’s system and the one developed by David Lewis (see [7]) and its effect on the humble connexive principles. Where Stalnaker posits a selection *function*, Lewis is more liberal. For him, there might be more than one closest world, and there might be none. It is the second case which generates trouble, so I will ignore the first in what follows. If there is no closest possible world in which an antecedent statement holds, then, in Lewis’s semantics, the counterfactual conditional comes out true.

The cases in which there is no closest possible world in which the antecedent holds include, but are not exhausted by, the cases in which Stalnaker’s function would point to the impossible world. That is, if the antecedent is impossible, for Lewis there is no world which is most similar to ours in which it holds, while for Stalnaker there is one, namely the impossible one.

So far, so good; the problem lies with the other cases in which for Lewis there is no closest possible world in which the antecedent holds.

These are exemplified by statements like “If I were more than seven feet tall, I would be a great basketball player.” Plausibly, Lewis holds that there could not be *one* single world in which I was more than seven feet tall but which would otherwise be maximally similar to this world. For, whatever my exact height in this world might be, there is a height that is incrementally closer to seven feet, and thus closer to my actual height. And that means that there is a world in which I have that height that is slightly closer to seven feet, and that that world is closer to the actual world.

But of course, it is perfectly *possible* that I might have been more than seven feet tall, and the statement “I am more than seven feet tall” should certainly be *satisfiable* on any decent account. Nonetheless, the statement “If I were more than seven feet tall, then I would not be more than seven feet tall” will come out true in Lewis’s semantics. This shows that we are not dealing with a system that is humbly connexive, for the antecedent should certainly not be filtered out, and the last statement was a violation of UNSAT1. It also, of course, shows that there is something intuitively wrong with Lewis’s system, as has been noted by many others before. The intuitions that are violated by his account are the very same intuitions that stand behind the idea of humble connexivity.<sup>27</sup>

There is a variation proposed by Lewis himself that will remedy this situation (see [7, p. 25]). He suggests that we might ask of a true counterfactual that there should be *at least one* most similar antecedent-world. Now “If I were more than seven feet tall, then I would not be more than seven feet tall” would be false; indeed, this move would give us a system that is humbly connexive.<sup>28</sup>

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<sup>27</sup> That is to say, the fact that the truth of “If I were more than seven feet tall, then I would not be more than seven feet tall” is generally seen to speak against Lewis’s system can be read as evidence for the intuitive correctness of UNSAT1.

<sup>28</sup> In the larger scheme of things, however, this remedy appears to be only partially satisfying: It would mean that statements like “If I were more than seven feet tall, then I would be more than six feet tall” would be counted as false, which seems not to be much of an improvement over “If I were more than seven feet tall, then I would not be more than seven feet tall” being true.