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EXISTENTIAL IMPORT AND RELATIONS OF CATEGORICAL AND MODAL CATEGORICAL STATEMENTS

Abstract. I examine the familiar quadruple of categorical statements “Every F is/is not G ”, “Some F is/is not G ” as well as the quadruple of their modal versions “Necessarily, every F is/is not G ”, “Possibly, some F is/is not G ”. I focus on their existential import and its impact on the resulting Squares of Opposition. Though my construal of existential import follows modern approach, I add some extra details which are enabled by framing my definition of existential import within expressively rich higher-order partial type logic. As regards the modal categorical statements, I find that so-called void properties bring existential import to them, so they are the only properties which invalidate subalternation, and thus also contrariety and subcontrariety, in the corresponding Square of Opposition.

Keywords: existential import; categorical statements; Square of Opposition; properties; quantified modal logic; partiality; type theory

1. Introduction

It is a familiar fact that the four *categorical statements*

- a. “Every F is/is not G ”
- b. “Some F is/is not G ”

may be used to ‘decorate’ the vertices of the *Square of Opposition*, while each statement contradicts the statement written in the opposite vertex, so the Square’s diagonals represent the relation of contradictoriness.¹ On

¹ I will use the term “categorical statements” for their logical analyses as well.

the *modern reading* of the Square, however, its edges do not represent the relation of subalternation, contrariety and subcontrariety, as on the *traditional reading* by Aristotle [1] and his followers. Since subalternation is an inverse of immediate entailment, and contrariety and subcontrariety are certain weak forms of contradictoriness, there is a good reason for investigating as to why the relations are not preserved in the modern reading when they had been admitted for more than two millennia.²

In this paper I zoom in on the role of existential import, which is known as the source of invalidity of subalternation, contrariety and subcontrariety.

I will tune up categorical statements within an expressively rich higher-order partial type logic and confirm the construal of modern logic as regards their existential import (i.e., that universal categorical statements lack existential import, whereas particular categorical statements have it). I will precisely define the notion of existential import and restate the familiar of subalternation, contrariety and subcontrariety on modern reading of the Square.

From the categorical statements a. and b. one can easily construct four *modal (de dicto) categorical statements*

- a.' "Necessarily, every F is/is not G "
- b.' "Possibly, some F is/is not G ".

Immediately, an important question arises: Do they lack existential import?

I answer in the affirmative for a.', but in the negative for b.'. So-called void properties, which are barely conceived, since they cannot have an instance and so they appear to be 'non-properties', are the sole cause of the existential import of b.'-type modal categorical statements. Consequently, subalternation, and thus also contrariety and subcontrariety, do not generally hold in the corresponding 'modal' Square of Opposition.

The four modal categorical statements articulate the necessity or contingency of property possession. Such *properties* are traditionally called *essential* or *accidental*, in agreement with the mode they pertain to their bearers. By adding two comparable kinds of properties, one completes a quadruple of such properties, and determine the relation of such quadruple to the 'modal' Square of Opposition.

² For introduction to the recent extensive study of the Square of Opposition see e.g. [2, 3]. My present paper develops my earlier research [33].

A tighter kinship can be ascertained in the case of statements about *requisite* and *potential properties*

- a." "The property G is/is not a requisite of the property F "
- b." "The property G is/is not a potentiality of the property F "

which are logically equivalent to modal categorical statements a.' and b.', respectively. These statements can be considered as explicit renderings of statements such as "Horses are animals" when they are understood as analytic, i.e., necessarily true or necessarily false.

My investigations will utilize Kuchyňka's (yet unpublished) variant of Tichý's Transparent Intensional Logic (TIL) which is called *Transparent Hyperintensional Logic* (THL). THL, as well as TIL, is a many-sorted higher-order logic, a type theory. Rivalling Montague's well-known [20] approach, the most important applications of TIL — which are usually translatable to THL — are in the semantics of natural language (propositional attitudes, subjunctive conditionals, verb tenses, etc); cf. esp. [41, 42, 11, 34].

There are several reasons for adopting THL. Firstly, it is a rather extensive and versatile framework which is even capable of explicating hyperintensional phenomena of our conceptual scheme. The system naturally covers quantified modal logic, while it enables higher-order quantification over properties and relations, which is needed for my investigation as well. Another important reason is THL's accommodation of partiality, which is required for precise definition of existential import.³

The structure of the paper is as follows. After the exposition of THL's key notions in Section 2, I will offer THL's analysis of categorical statements and their main relations in Section 3. The definition of existential import occurs in Section 3.2. Modal categorical statements are proposed in Section 4.1, where I will also define the notions of requisite and potentiality. An examination of relations of modal categorical statements is provided separately in Section 4. In Section 5, I will summarize my results and give some hints for future work.

³ The reader only accustomed to FOL should consult e.g. [13, 15, 24] for quantified modal logic and e.g. [20, 15, 22] for intensional logic.

2. Key notions of Transparent hyperintensional logic

2.1. Constructions as hyperintensions

THL resembles the typed λ -calculus because its language \mathcal{L} uses terms such as

$$“X” \mid “x” \mid “[FX]” \mid “\lambda x[Fx]”.$$

The crucial idea of Tichý’s approach to logic is to understand the terms as linguistic devices directly representing so-called *constructions*.

Constructions are structured abstract entities. They are (not necessarily effective) *algorithmic computations* which typically yield common set-theoretical objects (cf. [40, 41] for careful description and defence of the notion). Postulating constructions as genuine entities enables us to model meanings as language-independent, fine-grained entities, as *hyperintensions* (cf., e.g., the seminal paper by Cresswell [9] for the notion of hyperintensionality). Meaning is thus an algorithm that computes an expression’s denotatum — an idea that has been recently elaborated also by Moschovakis [21] and Muskens [23].

For a straightforward reason for the adoption of hyperintensions consider e.g. the argument having the premise “ A believes that $8 = 8$ ” from which it is surely invalid to infer “ A believes that $8 = \sqrt{64}$ ”. However, since standard *possible world semantics* (PWS) (some say: intensional semantics) is incapable of discriminating between different meanings of logically equivalent expressions, the agent is modelled as believing an unstructured proposition that is true in all possible worlds; the sentences thus report the same state of affairs and the argument is therefore wrongly evaluated as valid. A move from PWS intensions to sentences (or even formulas) as objects of beliefs can be easily rejected by Church’s familiar translation argument which shows that the agent’s beliefs cannot be correctly modelled as attitudes towards pieces of particular notation. Tichý’s constructions present a reasonable balance between the two mentioned extremes: meanings are modelled as structured constructions of expressions’ denotata (such as PWS propositions), and the argument above is rightly evaluated as invalid.

Dependently on a valuation (‘assignment’) v (cf. the next paragraph for its specification), constructions v -construct objects. Objects are denotational values of \mathcal{L} ’s terms, so the semantics for \mathcal{L} has both algorithmic and denotational levels: \mathcal{L} ’s terms stand for constructions, while they denote the objects v -constructed by the constructions. It holds

that every object is v -constructed by infinitely many v -congruent (logically equivalent if v -congruent on any v), but non-identical constructions. For example, the number eight is v -constructed by e.g. multiplying four by two or by the square root of sixty four, which are two distinct, yet v -congruent constructions of the number.

Using a common metalanguage five kinds of constructions can be described. Let

- $\xi_{(i)}$ (for $1 \leq i \leq m$) be any *type*, i.e., a set of objects of the same kind (cf. Section 2.2 for more details) that will be called $\xi_{(i)}$ -objects,
- v be any *valuation*, i.e., a function that supplies each variable of \mathcal{L} with a certain value from the field which consists of infinite sequences s^ξ of ξ -objects for each particular ξ ,
- $X_{(i)}$ be any construction or non-construction,
- x be a variable for constructions or non-constructions,
- $C_{(i)}$ be any construction,
- c be a variable for constructions,
- f be a variable for functions (as mappings),
- “_” represent partiality gap.

Where \mathcal{M} is a model (cf. Section 2.2 for more details), “ $\llbracket C \rrbracket^{\mathcal{M},v}$ ” can be read as “given \mathcal{M} and v , C v -constructs” or, alternatively, as “given \mathcal{M} and v , the denotational value of the term “ C ” is”.

1. *variables*

$$\llbracket V_j^\xi \rrbracket^{\mathcal{M},v} = \text{the only } x \text{ such that } s^\xi \in v \text{ and } x = s^\xi(j), \text{ where } j \in \mathbb{N}$$

2. *trivializations*

$$\llbracket {}^0X \rrbracket^{\mathcal{M},v} = X$$

3. *double executions*

$$\llbracket {}^2X \rrbracket^{\mathcal{M},v} = \begin{cases} \llbracket \llbracket C \rrbracket^{\mathcal{M},v} \rrbracket^{\mathcal{M},v} & \text{if } X \text{ is a certain construction } C \\ \text{and } \exists c(c = \llbracket C \rrbracket^{\mathcal{M},v} \wedge \exists x(x = \llbracket c \rrbracket^{\mathcal{M},v})) & \\ \text{—} & \end{cases}$$

4. *compositions*

$$\llbracket \llbracket CC_1 \dots C_m \rrbracket \rrbracket^{\mathcal{M},v} = \begin{cases} f(X_1, \dots, X_m) \text{ if} \\ \llbracket C \rrbracket^{\mathcal{M},v} = f \in (\xi_1 \times \dots \times \xi_m) \mapsto \xi, \\ \llbracket C_1 \rrbracket^{\mathcal{M},v} = X_1 \in \xi_1, \dots, \text{ and} \\ \llbracket C_m \rrbracket^{\mathcal{M},v} = X_m \in \xi_m \\ \text{—} \end{cases}$$

5. *closures*

$\llbracket [\lambda x_1 \dots x_m C] \rrbracket^{\mathcal{M},v} =$ the only $f \in (\xi_1 \times \dots \times \xi_m) \mapsto \xi$ which maps each possible $\langle \llbracket x_1 \rrbracket^{\mathcal{M},v^{(')}} , \dots , \llbracket x_m \rrbracket^{\mathcal{M},v^{(')}} \rangle$ to $\llbracket C \rrbracket^{\mathcal{M},v^{(')}}$ (if any), where C is a ξ -construction, $\llbracket x_i \rrbracket^{\mathcal{M},v^{(')}} \in \xi_i$ and each v' is like v except what it assigns to (some) variables other than x_1, \dots, x_m

Note that variables are genuine constructions, not letters; on the other hand, “ x_i ” only occurs in the ‘binding prefix’ “ $\lambda x_1, \dots, x_m$ ” as a letter, being thus a part of the formalism. Stipulations 1–5 specify the ‘raw terms’ of language of Tichý’s framework while its proper terms are obtained via filtration determined by the definition of types.

Precise definitions of the notions of *subconstruction* and *free variable*, both derivable from the above specification of constructions and the definition of types (see below) are omitted here for reasons of brevity. Constructions that do not v -construct anything at all (cf. “ $_$ ” in the above specification) will be called *v-improper*. Two constructions are called *v-congruent* iff they both v -construct the same object, or they are both v -improper.

Notational agreements. Symbols such as “ X^Y ”, where “ Y ” can be a string of letters of Greek alphabet, are simple symbols, while “ Y ” codes certain useful additional information about X . On the other hand, where w is a possible world variable, “ $[C w]$ ” will be abbreviated to “ C_w ”. “ 0 ” will be usually suppressed in records of trivializations of non-constructions; in some cases, upright boldface compensates for the suppressed “ 0 ”. For reasons of simplicity, I will sometimes loosely speak about the trivialization of some logical entity as the logical entity as such. Variables (but not meta-variables) will always be written as small italicized Latin letters. Constructions of well-known binary logical or mathematical operations will be written in the infix manner. Provided no confusion arises, pairs of brackets will be omitted; for a typical example, “ $\diamond \exists \lambda x [F x]$ ” is in its full form “ $[\diamond [\exists [\lambda x [F x]]]]$ ”. Below, round brackets delimiting certain propositional constructions will be introduced with a special meaning.

2.2. **Type theory**

THL utilizes an instance of Tichý’s *type theory* which is a substantial modification of Church’s simple theory of types [5]. As in other type

theories, the essential part of a frame is the so-called *base* B . Let B be a non-empty class of pairwise disjoint sets (i.e., domains) of primitive (i.e., unanalyzed) objects. In our case, we have $B_{\text{THL}} = \{\iota, o, \omega\}$, where

- ι is the type of individuals
- o is the type of truth values T and F (True and False)
- ω is the type of (an infinite number of) possible worlds W_1, W_2, \dots, W_n .⁴

The *hierarchy of types* is defined inductively as follows (the definition is adopted from [41]). Recall that types are directly understood as sets. Ramification excludes circular specification of constructions.

Let base B consist of any pairwise disjoint sets of primitive objects.

1. 1st-order types

- (t_1 i) Every member of base B is a *type of order 1 over B* .
- (t_1 ii) If $0 < m$ and ξ, ξ_1, \dots, ξ_m are types of order 1 over B , then the collection $(\xi\xi_1 \dots \xi_m)$ of all m -ary total and partial functions from $\xi_1 \times \dots \times \xi_m$ to ξ is also a *type of order 1 over B* .
- (t_1 iii) Nothing is a type of order 1 over B unless it so follows from (t_1 i) and (t_1 ii).

Let ξ be any type of order n over B .

2. n -order constructions

- (c_n i) Every variable ranging over ξ is a *construction of order n over B* . If X is of type ξ , then 0X and 2X are *constructions of order n over B* .
- (c_n ii) If $0 < m$ and C, C_1, \dots, C_m are constructions of order n , then $[CC_1 \dots C_m]$ is a *construction of order n over B* . If $0 < m$, ξ is a type of order n over B , C and the variables x_1, \dots, x_m are constructions of order n over B , then also $[\lambda x_1 \dots x_m C]$ is a *construction of order n over B* .
- (c_n iii) Nothing is a construction of order n over B unless it so follows from (c_n i) and (c_n ii).

Let $*_n$ be the collection of constructions of order n over B .

3. $(n + 1)$ -order types

⁴ THL normally utilizes a temporal parameter modelled by reals which are collected in the atomic type τ . For reasons of simplicity, I will suppress it. Members of ω should be thus understood rather as world–time points, cf. [32]. Note the assumption of a fixed domain.

- (t_{n+1} i) The type $*_n$ is a *type of order* $(n + 1)$ over B . Every type of order n is a *type of order* $(n + 1)$ over B .
- (t_{n+1} ii) If $0 < m$ a ξ, ξ_1, \dots, ξ_m are types of order $(n + 1)$ over B , then the collection $(\xi\xi_1 \dots \xi_m)$ of all m -ary total and partial functions from $\xi_1 \times \dots \times \xi_m$ to ξ is also a *type of order* $(n + 1)$ over B .
- (t_{n+1} iii) Nothing is a type of order $(n + 1)$ over B unless it so follows from (t_{n+1} i) and (t_{n+1} ii).

Then, a *frame* for THL is a tuple $\mathcal{F} = \langle \iota, o, \omega, *_1, *_2, \dots, *_n, (\xi_1 \times \dots \times \xi_m) \mapsto \xi \rangle$, where each particular type is a type over B_{THL} and “ $(\xi_1 \times \dots \times \xi_m) \mapsto \xi$ ” stands for all types of i -ary functions over B_{THL} (for $1 \leq i \leq m$). A *model* is a couple $\mathcal{M} = \langle \mathcal{F}, \mathcal{I} \rangle$, where \mathcal{I} is an interpretation function which maps each term “ X ” (unabbreviated: “ 0X ”) to the designated object X (if any) which is a member of a (typically) 1st-order type over B_{THL} .⁵

For the *logical analysis of natural language expressions* by means of THL one utilizes both extensions and intensions over B_{THL} , while their constructions serve as hyperintensional models of expressions’ *meanings*. *Intensions* involve total or partial functions from possible worlds, i.e., objects of type $(\xi\omega)$, which will be abbreviated in notation to “ $\xi\omega$ ”. The type o_ω will be abbreviated in notation to “ π ”. Important kinds of intensions include

- *propositions*, i.e., objects of type π ,
- ξ -*properties* (i.e., properties of ξ -objects), i.e., objects of type $(\pi\xi)$,
- ξ_1, \dots, ξ_m -*relations* (-in-intension; $1 < m$), i.e., objects of type $(\pi\xi_1, \dots, \xi_m)$,
- (*individual*) *offices* (‘individual concepts’), i.e., objects of type $\xi\omega$.

Properties and m -ary relations are thus in fact identified with monadic and m -adic *conditions*. Conditions are proposition-like entities: the fact that an entity X (or an m -tuple of entities) satisfies / counter-satisfies condition F in world W matches the fact that the corresponding proposition ‘ X IS F ’ holds / does not hold in W .

⁵ For example, let $\mathcal{I}(\text{“T”}) = \mathbf{T}$, where \mathbf{T} is a member of a 1st-order type o (according to the notational agreement from the preceding section, “ T ” on the left-hand side of $=$ is “ ${}^0\text{T}$ ”; similarly below). Further, let $\mathcal{I}(\text{“} = {}^o \text{”}) = ={}^o$, where $={}^o$ is the identity relation between members of o . Then, $\mathcal{I}(\text{“}[\text{T} = {}^o \text{T}] \text{”}) = \mathbf{T}$ and $\mathcal{I}(\text{“}[\text{T} = {}^o \text{T}] \text{”}) = [\mathbf{T} = {}^o \mathbf{T}]$ (i.e., the construction $[\text{T} = {}^o \text{T}]$, not what it v -constructs), since \mathcal{I} emulates our systematic construction of the language.

At least brief motivation for THL (due to Kuchyňka, p.c.) may be useful here. According to THL, “Fido is a dog” says that Fido satisfies the condition BE A DOG; and predication consists just in that. However, Montague and then Tichý (cf. e.g. [17]) were comfortable with a different picture of predication and verification. To ascertain the truth of an ordinary sentence such as “Fido is a dog” one inspects whether Fido belongs to the set of all actual dogs (this extension of the property BE A DOG in a given W is nothing but a bisection of the domain); and predication relates Fido to that set. This picture makes good sense in a model theory which deploys set-theoretic considerations, yet it is highly unrealistic in the context of use of a natural language in which one never verifies the sentence by investigating the whole world to ascertain which individuals are dogs in it and then checks for Fido’s membership in that set. That THL presents a more natural stance towards natural language is clearly visible when one compares advanced topics of analysis of natural language such as tenses or verb aspects, where TIL becomes rather heavy-handed. Such a comparison cannot be made here, however. At the moment, the greatest argument in favour of THL instead of TIL is the brevity of its analyses, the better readability of formulas and overall logical convenience. To illustrate, the TIL counterpart of the THL modal universal positive categorical statement $\Box\forall\lambda x[(\mathbf{F}x) \rightarrow (\mathbf{G}x)]$ is $\lambda w\Box\lambda w[\Pi\lambda x[(\mathbf{F}_w x) \supset (\mathbf{G}_w x)]]$ (assuming the same abbreviating conventions).

Below are types for ‘constants’ (i.e., trivializations), variables and even ‘metavariables’ which will often be the subject of discussion later. The intensional and extensional operators are defined below. Let “should v -construct an object of type” be abbreviated to “/” and “ $C_1/\xi; C_2/\xi$ ” be abbreviated to “ $C_1, C_2/\xi$ ”.

Intensions are chosen as the denotata of expressions whose reference varies across logical space, e.g. “the U.S. president”, “It rains in Paris”. For a simple example of logical analysis by means of THL consider the following sentences and the *propositional constructions* (i.e., constructions of π -objects) expressed by them:

“Max is a dog”	[Dog M]
“There exists a dog”	$\exists\lambda x$ [Dog x] ⁶
“Jane believes that Max is a dog”	[Bel J ⁰ [Dog M]]

where $\mathbf{M}, \mathbf{J}/\iota$; $\mathbf{Dog}/(\pi\iota)$; $\mathbf{Bel}/(\pi\iota^*1)$. Notice that ⁰[**Dog M**] v -constructs just [**Dog M**], i.e., the object of Jane’s belief, not the proposition

⁶ The construction is η -convertible to $[\exists \mathbf{Dog}]$.

<i>construction/type</i>	<i>description</i>
x/ι	variable for ι -objects, i.e., for individuals
$s/(o\iota)$	variable for classes of ι -objects, briefly: ι -classes, i.e., for characteristic functions of ι -objects
$w, w'/\omega$	variables for possible worlds (such as e.g. W, W')
$p, q/\pi$	variables for propositions (such as e.g. P, Q)
$\mathbf{P}, \mathbf{Q}/\pi$	where \mathbf{P} and \mathbf{Q} are any k -order constructions of the propositions P and Q , respectively
$f, g/(\pi\iota)$	variables for properties of ι -objects, i.e., for ι -properties
$\mathbf{F}, \mathbf{G}/(\pi\iota)$	where \mathbf{F} and \mathbf{G} are any k -order constructions of ι -properties F and G , respectively
$\Pi^\xi, \Sigma^\xi/(o(o\xi))$	trivialization of the $(o\xi)$ -class containing the universal ξ -class / all nonempty ξ -classes*
$c, d/*_k$	variables for k -order constructions
$o, o_1, o_2/o$	variables for truth values
$\mathbf{T}, \mathbf{F}/o$	trivializations of the two truth values
$\sim/(oo)$	trivialization of the classical negation
$\&, \supset/(ooo)$	trivialization of the classical conjunction / material conditional
$=^\xi/(o\xi\xi)$	trivialization of the classical identity relation between ξ -objects
$\neg/(\pi\pi)$	trivialization of the intensional negation
$\square, \diamond/(\pi\pi)$	trivialization of the operation of necessity / possibility
$\wedge, \rightarrow/(\pi\pi\pi)$	trivialization of the intensional conjunction / conditional
$\forall, \exists/(\pi(\pi\iota))$	trivialization of the intensional universal / existential ‘quantifier’

* The superscript “ ξ ” by “ Π ” or “ Σ ” will be suppressed because particular values of ξ can be easily gathered from surrounding context.

v -constructed by [Dog M]; this blocks the undesirable substitution that creates the hyperintensionality puzzle discussed in Section 2.1.⁷

For his simple type theory Tichý [39] proposed a system of *natural deduction in Gentzen style*,⁸ which has been extended for its ramified version in [34]. Since it is based on type theory, the system makes heavy use of *substitution* and α -, β -, η -*reductions*. Further main rules

⁷ According to THL’s strict doctrine, the constructions should be applied to w in order to v -construct a truth-value (if any); in this paper, I omit this to achieve greater notational economy.

⁸ As a system capturing a general notion of inference see [26]. For relation of Tichý’s approach to constructive type theory see [27].

characterize the ‘semantics’ of kinds of constructions, and some other rules characterize the behaviour of logical as well as non-logical ‘constants’. The deduction system cannot be reproduced here for reasons of brevity. Below, I consider *definitions* to be certain derivation rules of form $C_1 \Leftrightarrow C_2$, where C_1 and C_2 are two v -congruent (even logically equivalent) propositional constructions which are therefore *interderivable*.⁹ The interderivability of v -congruent constructions of truth values, “ \Leftrightarrow ”, is also employed.

Below are rules that specify meanings of our main non-primitive extensional and intensional operators in a type/proof-theoretic style, while the definiens even shows how to calculate their denotational values (the meanings of primitive operators \mathbf{T} , \sim , $\&$, $=^\xi$ are given by \mathcal{I}).¹⁰

$$\begin{array}{ll}
 [o_1 \supset o_2] & \Leftrightarrow \quad [[o_1 \& o_2] =^o o_1] \\
 [\Pi s] & \Leftrightarrow \quad [s =^{(ou)} [\lambda x.\mathbf{T}]] \\
 [\forall f] & \Leftrightarrow \quad \lambda w[\Pi [\lambda x[f x]_w]] \\
 [\Box p] & \Leftrightarrow \quad \lambda w[p =^\pi [\lambda w.\mathbf{T}]] \\
 [\neg p] & \Leftrightarrow \quad \lambda w[\sim p_w] \\
 [p \wedge q] & \Leftrightarrow \quad \lambda w[p_w \& q_w] \\
 [p \rightarrow q] & \Leftrightarrow \quad [[p \wedge q] =^\pi p]
 \end{array}$$

The interdefinability of Π with Σ and \Box with \Diamond needs a separate treatment: see the next section.

2.3. Determinate truth

It is a well-recognized fact that the adoption of *partiality* is useful for the analysis of our conceptual scheme (cf. e.g. [12]). And it is of especial importance for our topic: statements deploying properties denoted by *empty terms* (incl. empty predicates) are often *gappy*, i.e., without a truth-value in a given world W . Such statements thus typically have existential import, which I will discuss in detail in Section 3.2.

As an example of a gappy sentence consider

B_1 : “The King of France is bald”

⁹ Such an understanding of definitions is studied within proof-theoretic framework (cf. e.g. [36]).

¹⁰ For discussion of contemporary type theories from the viewpoint of model-theoretic/proof-theoretic distinction see e.g. [19, 14].

the negation of which

B₂: “The King of France is not bald”

is also without a truth-value in W . As famously argued by Strawson [37], “the King of France” is an empty term and so the common existential presupposition of the sentences, viz. that the King of France exists, is not satisfied.¹¹ To deny the sentence B₁ in such a way that our resulting statement is determinately true (not without a true value), one should use the sentence

B₂’: “It is not true that the King of France is bald.”

The expression “(is) not true” used in B₂’ is called *strong negation* and is contrasted with the *weak negation* “(is) false”.¹² Its reverse will be called the *strong truth predicate* (or: *determinate truth*); it is defined in the next paragraph.

Partiality as regards properties (or relations) consists in this: given W , property F can neither be possessed, nor counter-possessed, by an individual X . Consequently, the proposition that X is F is gappy in W (i.e., the property is a function that maps a given individual to such a partial proposition). Given a v which maps X to x and W to w , $[\mathbf{F} x]_w$ thus v -constructs nothing: the construction is v -improper. Consequently, the classical laws – from the Law of Excluded Middle to De Morgan’s Exchange Rule for Quantifiers – possibly involving such v -improper constructions do not hold. This is why we must amend them using the *strong truth predicate* which is definable by

$$[\mathbf{Tr}^{T\pi} p]_w \Leftrightarrow \Sigma\lambda o[[p_w =^o o] \& [o =^o \mathbf{T}]]$$

where $\mathbf{Tr}^{T\pi}/(\pi\pi)$.¹³ Thus, if P is without a truth-value in W (the value of w on v), both \mathbf{P}_w and $[\sim \mathbf{P}_w]$ are v -improper, whereas $[\mathbf{Tr}^{T\pi} \mathbf{P}]_w$

¹¹ Recently, several writers (e.g. [43]) have dismissed Strawson’s lesson by pointing to sentences such as “The King of France is having supper” which can’t be gappy. However, the writers have neglected the intuitive fact that, though sentences governed by episodic verbs (i.e., verbs describing an action) can never lack a definite truth-value, sentences governed by attributive verbs such as ‘(be) bald’ can.

¹² The two kinds of negations has been discussed in philosophical logic and philosophy of language circles repeatedly, e.g. in [10].

¹³ Cf. e.g. [30] for partiality issues and [31] for an analysis of truth in TIL. A description of the behaviour of the definiens might be welcome: If p_w v -constructs nothing, $=^o$ does not receive an argument and so $[p_w =^o o]$ v -constructs nothing as well; but then, $\&$ does not receive an argument and so $[p_w =^o o] \& [o =^o \mathbf{T}]$ v -constructs

v -constructs \mathbf{F} and $[\sim[\mathbf{Tr}^{T\pi} \mathbf{P}]_w]$ v -constructs \mathbf{T} . Where C/π , “ $[\mathbf{Tr}^{T\pi} C]$ ” will be abbreviated to “ (C) ”.

This method of ‘correction’ of partiality will only be applied to selected propositional constructions because sometimes such a correction is unnecessary (e.g., because all bottom subconstructions are already corrected), or even undesirable. For example, the *Exchange Rule for Quantifiers* in a version relevant for this paper becomes¹⁴

$$\neg\forall\lambda x[(fx) \rightarrow (gx)] \Leftrightarrow \exists\lambda x\neg[(fx) \rightarrow (gx)]$$

The determinateness of statements which ascribe possession of properties can be achieved by utilizing the notion of *instantiation* defined by

$$[\mathbf{Inst} x f] \Leftrightarrow (fx)$$

where $\mathbf{Inst}/(\pi\iota(\pi\iota))$. Because of the existence of the proposition undefined for all worlds, the *Exchange Rule for Modal Operators* has to be amended as follows:

$$\diamond\neg(p) \Leftrightarrow \neg\Box(p)$$

The firm notion of determinate truth is needed for definition of the notion of entailment. Since THL employs both intensional and hyperintensional levels, it harbours notions of entailment designated for each level.

Firstly, note that properties of constructions of certain objects ‘supervene on’ properties of those objects. One may therefore simply speak about properties of objects without a strict need to speak even about properties of the constructions of those objects. In consequence of this, the reader is assumed to be able to compile appropriate definitions of the notions applicable to constructions herself.

The truth ^{π} of a proposition P , for example, makes all its constructions true*. The truth* of constructions is thus easily definable with help of the notion of truth ^{π} of propositions:

$$[\mathbf{Tr}^{PT*k} c] \Leftrightarrow [\mathbf{Tr}^{T\pi} 2c]$$

nothing; as a result, $\lambda o[[p_w =^o o]\&[o =^o \mathbf{T}]]$ v -constructs an empty class – which Σ maps to \mathbf{F} .

¹⁴ A careful reader may observe that the construction on the right side of \Leftrightarrow can be simplified to $\exists\lambda x\neg[[fx] \rightarrow [gx]]$ and the law will still be valid. The above formulation of the Exchange Rule for Quantifiers has been chosen to preserve the similar form of the ‘bodies’ of various non-modal and modal categorical statements (sections 3.1 and 4.1).

where $\mathbf{Tr}^{PT*k}/(\pi*k)$.¹⁵ Similarly for entailment:

$$\begin{aligned} [p \vDash^\pi q] &\Leftrightarrow \Box[p \rightarrow q] \\ [c \vDash d] &\Leftrightarrow [{}^2c \vDash^\pi {}^2d] \end{aligned}$$

where $\vDash^\pi /(\pi\pi\pi)$; $\vDash /(\pi *k *k)$, which is the preferred of the two notions. Below, I will gradually omit brackets from “[$C_1 \vDash C_2$]” as well as the proper indications the particular constructions flanking \vDash have to be introduced, using ⁰, as constructions *per se*; e.g., [⁰**P** \vDash ⁰**Q**] will be written as “**P** \vDash **Q**”.

3. Categorical statements and their relations

3.1. Categorical statements and contradictoriness

That categorical statements can oppose each other, and that some entail some other, gives rise to the Square of Opposition; on a modern reading of the Square only some of the relations hold.

In my variant of the modern reading, specific versions of categorical statements occur. Each *categorical statement* is a propositional construction of the form “ $Q_j C_i$ ”, where Q_j (for $1 \leq j \leq 2$) is \forall or \exists and C_i (for $1 \leq i \leq 4$) is a construction of a property. C_i consists of \wedge or \rightarrow and two propositional constructions. It is important that these are amended by the strong truth predicate to prevent the undesirable impact of existential import – which I am going to discuss in detail in Section 3.3.

<i>Abbreviated form</i>	<i>full form</i>	<i>its usual verbal expression</i>
A	$\Leftrightarrow \forall \lambda x[(\mathbf{F}x) \rightarrow (\mathbf{G}x)]$	Every F is G .
E	$\Leftrightarrow \forall \lambda x[(\mathbf{F}x) \rightarrow \neg(\mathbf{G}x)]$	No F is G .
I	$\Leftrightarrow \exists \lambda x[(\mathbf{F}x) \wedge (\mathbf{G}x)]$	Some F is G .
O	$\Leftrightarrow \exists \lambda x[(\mathbf{F}x) \wedge \neg(\mathbf{G}x)]$	Some F is not G .

Table 1. Categorical statements

As in the familiar modern reading of the Square, my proposal does not fully preserve the traditional construal. The only important relation

¹⁵ If the value of c is a propositional construction, the definiens and so also definiendum v -constructs—if applied to w —T or F as determined by $\mathbf{Tr}^{T\pi}$; in all other cases, no truth-value is v -constructed.

that obtains is *contradictoriness* (see Section 3.3 for more discussion).¹⁶ I define it as an explicitly modal notion (similarly for subalternation, contrariety and subcontrariety):

$$[\mathbf{Contradictory} \ p \ q] \Leftrightarrow \Box \neg [[p \rightarrow q] \wedge [q \rightarrow p]]$$

where **Contradictory**/($\pi\pi\pi$).

As mentioned in Remark 14, particular categorical statements need not use the strong truth predicate (contradictoriness and conversions/obversions still hold). I deploy it only to achieve their closer structural similarity to universal categorical statements.

3.2. Existential import

It is well known that the modern reading of the Square does not preserve subalternation, contrariety and subcontrariety because of existential import.

Though there seems to be consensus as regards the intuitive notion, its detailed definitions vary – cf. e.g. [7, 4] (and e.g. [35, 8], for historical issues). I am going to propose a definition that is convenient for our purposes.

The informal wording of the definition comes in two variants, i. for meanings of predicates, ii. for statements. i. A ‘predicate’ **F** (or **G**) has *existential import in* a statement **P** of which it is a subconstruction iff there is a world W such that if there is no instance of F in W , **P** is not true in W ; ii. **P** *lacks existential import* iff there is no **F** such that **F** has existential import in **P**.

The precise forms of the two particular definitions employ variables c and d which range over the type $*_k$ to which the constructions **F** (and: **G**) and **P** belong; 2d is expected to v -construct the ι -property v -constructed by **F**.

¹⁶ Conversions and obversions remain valid too, cf. [33], where detailed proofs of contrariety and other relations are presented. A further remark: the reviewer objected that the contradictoriness of categorical statements does not hold, since e.g. both “All F are identical with the King of France” and “Some F are not identical with the King of France” may lack a truth-value. However, such an objection is misguided since it presupposes that the sentences are read as singular statements standardly expressed as “The King of France is such that all F are identical with him” and “The King of France is such that some F are not identical with him”, respectively. The logical analyses of the two singular statements are not v -congruent to the logical analyses of categorical statements “All F are G ” and “Some F is G ”, respectively, on their usual reading (cf. 3.2, point c. for their analyses).

$$\begin{aligned}
[\mathbf{HasExImportIn}^{(\pi\iota)} d c] &\Leftrightarrow \exists \lambda w [[\mathbf{Subc} d c] \& [{}^2d =^{(\pi\iota)} {}^2d] \\
&\quad \& \& [[\mathbf{Tr}^{PT^*k} c]_w \supset [\Sigma \lambda x [{}^2d x]_w]]] \\
[\mathbf{LacksExImport}^{(\pi\iota)} c] &\Leftrightarrow \neg \exists \lambda d [\mathbf{HasExImportIn}^{(\pi\iota)} d c] \\
[\mathbf{HasExImport}^{(\pi\iota)} c] &\Leftrightarrow \neg [\mathbf{LacksExImport} c]
\end{aligned}$$

where $\mathbf{HasExImportIn}^{(\pi\iota)}/(\pi *k *k)$; $\mathbf{LacksExImport}^{(\pi\iota)}$, $\mathbf{HasExImport}^{(\pi\iota)}/(\pi *k)$; and \mathbf{Subc} (subcontraction of)/($o *k *k$).

Remarks. a. Note that the definition is not limited to categorical statements. On the other hand, it defines the notion of existential import of constructions which only v -construct ι -properties for it is the only case I am interested in in this paper. The conjunct $[{}^2d =^{(\pi\iota)} {}^2d]$ ‘checks’ whether the value of d is a construction of a ι -property; if the checking fails, the value of d goes undecided as regards its existential import in the value of c .

To define the notion of existential import generated by meanings of (say) individual descriptions, as in “The King of France is bald”, the above definition must be adapted accordingly. Such a definition may utilize the fact that constructions of ξ -objects, where ξ equals (say) ι_ω , can be mapped to constructions of closely-related properties, so the above definitions can be enhanced to cover more cases of constructions. To show this, let us define the aforementioned ‘normalization function’ for constructions of individual offices first:¹⁷

$$\begin{aligned}
\lambda w [d =^{*k} [\mathbf{Norm}^{\iota_\omega} c]] &\Leftrightarrow [\neg [\mathbf{Tr}^{T\pi} [\lambda w [{}^2d =^{(\pi\iota)} {}^2d]]] \\
&\quad \rightarrow [\mathbf{Tr}^{T\pi} [\lambda w [{}^2d =^{(\pi\iota)} [\lambda x \lambda w [x =^\iota [{}^2c w]]]]]]] \\
[\mathbf{HasExImportIn}^{(\pi\iota)+\iota_\omega} d c] &\Leftrightarrow \exists \lambda w [[\mathbf{Subc} d c] \\
&\quad \& [[\mathbf{Norm}^{\iota_\omega} {}^2d] =^{(\pi\iota)} [\mathbf{Norm}^{\iota_\omega} {}^2d]] \\
&\quad \& [[\mathbf{Tr}^{PT^*k} c]_w \supset [\Sigma \lambda x [[\mathbf{Norm}^{\iota_\omega} {}^2d] x]_w]]]
\end{aligned}$$

where u/ι_ω ; $\mathbf{Norm}^{\iota_\omega}/(*k *k)$; $\mathbf{HasExImportIn}^{(\pi\iota)+\iota_\omega}/(\pi *k *k)$. The completion of definitions of $\mathbf{LacksExImport}^{(\pi\iota)+\iota_\omega}$ and $\mathbf{HasExImport}^{(\pi\iota)+\iota_\omega}$ is obvious.

b. Further, note that constructions of ι -properties within ‘opaque contexts’ are not allowed to cause the existential import of statements. As an example, consider the construction **Dog** that occurs in

¹⁷ $[\lambda x \lambda w [x =^\iota [{}^2c w]]]$ v -constructs the property whose only possible instance in W is the occupant (if any) of the individual office (if any) which is v -constructed by the value of c .

$[\mathbf{Bel} x^0[\mathbf{Dog} \mathbf{M}]]$, which is a so-called β -reduced form of the propositional construction $[[\lambda y[\mathbf{Bel} y^0[\mathbf{Dog} \mathbf{M}]] x]$ that may occur within a categorical statement. Surely the (non-)existence of dogs does not affect the truth-value of the corresponding belief sentence, or statements in which the belief sentence is embedded. Unlike other frameworks, TTT enables us to offer an adequate definition because of its capability to discriminate orders of constructions: the whole propositional construction as well as its main subconstructions are of order k , but the nested construction \mathbf{Dog} is of order $k - 1$ and so it does not satisfy the first conjunct $[\mathbf{Subc} d c]$ of the above definiens, since the notion of subconstruction involved in it only applies to k -order (sub)constructions.

c. My definition does not implement the conviction that existential import is only caused by the ‘subject’ term. One reason for this consists in the fact that the notion of ‘subject’ is debatable. Recall the familiar fact that categorical statements are logically equivalent to their variants which contain *generalized quantifiers* “all”, “some”, “none” (cf. e.g. [25]), which is evident from their definiens, e.g.

$$[[\mathbf{All} f] g] \Leftrightarrow \forall \lambda x[(fx) \rightarrow (gx)]$$

where $\mathbf{All}/((\pi(\pi\iota))(\pi\iota))$. The ‘subject’ of $[[\mathbf{All} \mathbf{F}] \mathbf{G}]$ seems to be \mathbf{G} , which contradicts the intuition that it is rather \mathbf{F} , as one might have perhaps concluded when inspecting “Every F is G ”. Anyway, a possible restriction of my definition to ‘subject’ terms is easy to implement.

d. Having (or: lacking of) existential import is a non-contingent property of statements: if \mathbf{P} has existential import, it has it in every world W . Moreover, having existential import in statements is also conceived of here as a non-contingent property, though of certain parts of statements. Arguably, a contingent variant of the notion – according to which \mathbf{F} has import only in some worlds – is intuitively admissible; but then, existential import of \mathbf{P} would be best defined in terms of \mathbf{F} ’s possibly having the existential import in \mathbf{P} ; the results of my paper thus would not change.

$$[\mathbf{HasCExImportIn}^{(\pi\iota)} d c]_w \Leftrightarrow \begin{array}{l} [[\mathbf{Subc} d c] \ \& \ [{}^2d =^{(\pi\iota)} \ 2d] \\ \& \ [[\mathbf{Tr}^{PT*k} c]_w \supset [\Sigma \lambda x[{}^2d x]_w]]] \end{array}$$

$$[\mathbf{LacksExImport}^{(\pi\iota)} c] \Leftrightarrow \neg \exists \lambda d \diamond [\mathbf{HasCExImportIn}^{(\pi\iota)} d c^k]$$

where $\mathbf{HasCExImportIn}^{(\pi\iota)}/(\pi *k *k)$. (The notion of existential import that is contingent, since a statement has such existential import dependently on given world, is extra-logical and thus not considered here.)

e. Finally, note that having existential import does not amount to having existential presupposition in the sense of [37].¹⁸ The definition of the latter notion: $\exists \lambda x[\mathbf{F}x]$ is an *existential presupposition* of \mathbf{P} of which it is a subconstruction iff it is entailed (\models) by \mathbf{P} as well as its negated form. Obviously, though $\exists \lambda x[\mathbf{F}x]$ is entailed by $\exists \lambda x[[\mathbf{F}x] \wedge [\mathbf{G}x]]$, i.e., \mathbf{I} , it is not entailed by its negated form (i.e., in fact \mathbf{E}). This means that $\exists \lambda x[\mathbf{F}x]$ is not an existential presupposition of \mathbf{I} or its negated form, despite being \mathbf{I} 's existential consequence (since \mathbf{I} has existential import).

3.3. Existential import of categorical statements and the Square of Opposition

Now let us examine why constructions such as

$$\forall \lambda x[[\mathbf{F}x] \rightarrow [\mathbf{G}x]]$$

cannot ‘decorate’ vertices of the Square of Opposition. The reason lies in the fact that the propositional construction has existential import because \mathbf{F} or \mathbf{G} can have existential import in it.

As an example of F consider the property

BE SUCH AN INDIVIDUAL THAT IS IDENTICAL WITH THE KING
OF FRANCE¹⁹

which is currently possessed by no individual because no individual is identical with the missing entity that is referred to by the definite description “the King of France”. No matter which individual one chooses as the value for x , $[\mathbf{F}x]$ as well as $[\mathbf{F}x] \rightarrow [\mathbf{G}x]$ v -construct gappy propositions. Thus, the closure $\lambda x[[\mathbf{F}x] \star [\mathbf{G}x]]$, where \star is \rightarrow or \wedge and $[\mathbf{G}x]$ is possibly negated, v -constructs a currently unsatisfiable property. Then,

$$\begin{array}{l} \forall \lambda x[[\mathbf{F}x] \rightarrow [\mathbf{G}x]] \\ \forall \lambda x[[\mathbf{F}x] \rightarrow \neg[\mathbf{G}x]] \\ \exists \lambda x[[\mathbf{F}x] \wedge [\mathbf{G}x]] \\ \exists \lambda x[[\mathbf{F}x] \wedge \neg[\mathbf{G}x]] \end{array}$$

¹⁸ For a brief overview of the debate concerning existential presupposition and categorical statements see [4].

¹⁹ The logical analysis of the predicate “be such an individual that is identical with the King of France” is $\lambda x \lambda w[x = {}^t [\mathbf{Sng} \lambda x[\mathbf{King} \ x \ \mathbf{Fr}]_w]]$, where $\mathbf{King}/(\pi \iota \iota)$; \mathbf{Fr}/ι ; $\mathbf{Sng}/(\iota(o\iota))$; the singularization function \mathbf{Sng} maps each singleton to its sole member and it is undefined otherwise. The logical analysis of “be such an individual that it is true^T that the individual is identical with the King of France” discussed below is $\lambda x \lambda w[\mathbf{Tr}^{T\pi} \lambda w[x = {}^t [\mathbf{Sng} \lambda x[\mathbf{King} \ x \ \mathbf{Fr}]_w]]_w$.

are all false (when applied to w , one gets \mathbf{F}). They all have existential import, since \mathbf{F} has existential import in them. Therefore, the quadruple of such statements cannot form a Square of Opposition since there is no contradictoriness relation present.

One may ask why this drawback of the standard modern formalization of the categorical statements within the Square — which does not employ the strong truth predicate — has gone unnoticed.²⁰ A very probable reason seems to be that unsatisfiable properties were ignored and/or implicitly considered in their ‘definite form’ such as

BE SUCH AN INDIVIDUAL THAT IT IS TRUE^T THAT THE INDIVIDUAL IS IDENTICAL WITH THE KING OF FRANCE

which is currently counter-instantiated by all individuals. By rectifying the above quadruple of statements with explicit help of the strong truth predicate — as I propose in Section 3.1 — the universal categorical statements \mathbf{A} and \mathbf{E} lack existential import and the quadruple forms a genuine Square of Opposition.

Let me add that to achieve subalternation of \mathbf{I} to \mathbf{A} some writers (e.g. [4, 35]) endow \mathbf{A} -statements with existential import by adding an appropriate existential statement:

$$\forall \lambda x [[\mathbf{F}x] \rightarrow [\mathbf{G}x]] \wedge \exists \lambda x [\mathbf{F}x]$$

The O -vertex is then ‘decorated’ by its negation (by application of familiar logical laws one obtains $\exists \lambda x [[\mathbf{F}x] \wedge \neg[\mathbf{G}x]]$). The resulting Square of Opposition is indeed a possibility, but it dismisses the familiar modern reading of the Square.

3.4. Lack of subalternation, contrariety and subcontrariety of categorical statements

As is well known, the lack of existential import of the universal categorical statements \mathbf{A} and \mathbf{E} affects subalternation, contrariety and subcontrariety. To demonstrate this, suppose there is no F in W ; the particular categorical statements \mathbf{I} and \mathbf{O} are thus false in W . But \mathbf{A} and \mathbf{E} are true in W , not false, since modern logic treats them as lacking existential import (sections 3.1 and 3.2). Consequently,

$$\mathbf{A} \not\equiv \mathbf{I} \qquad \mathbf{E} \not\equiv \mathbf{O}$$

²⁰ Exceptions exist, however. Geach [16], for example, wrote that if the categorical statements contain empty predicates such as “dragon”, they all are gappy and the Square of Opposition is inapplicable to them.

from which the invalidity of subalternation, contrariety and subcontrariety follow — as is easy to check.

Firstly, here are definitions of the three notions:

$$\begin{aligned} [\mathbf{Subaltern} \ q \ p] &\Leftrightarrow \Box[p \rightarrow q] \\ [\mathbf{Contrary} \ p \ q] &\Leftrightarrow \Box[p \rightarrow \neg q] \wedge \Diamond[\neg p \wedge \neg q] \\ [\mathbf{Subcontrary} \ p \ q] &\Leftrightarrow \Box[\neg p \rightarrow q] \wedge \Diamond[p \wedge q] \end{aligned}$$

where **Subaltern**, **Contrary**, **Subcontrary** / $(\pi\pi\pi)$.

The definiens of subalternation discloses the relationship of subalternation to entailment, viz. $[\mathbf{Subaltern} \ q \ p] \Leftrightarrow [p \vDash^\pi q]$, from which one easily concludes that subalternation does not hold: unlike traditional reading of the Square, **I** is not a subaltern of **A**; similarly for **O** with **E**.

The lack of subalternation invalidates contrariety and subcontrariety because the left conjuncts of their definiens assume $\mathbf{A} \vDash \mathbf{I}$ and $\mathbf{E} \vDash \mathbf{O}$. On the modern reading, there is no example of contrariety (traditional example: **A** with **E**), since $\mathbf{A} \not\vDash \neg\mathbf{E}$; and there is even no example of subcontrariety (traditional example: **I** with **O**), since $\neg\mathbf{I} \not\vDash \mathbf{O}$.

4. Modal categorical statements and their relations

4.1. Requisites, potentialities, and modal categorical statements

With categorical statements at hand, we are ready to articulate *modal (de dicto) categorical statements*. As indicated in the introduction, they are of the form “ $M_j P_i$ ”, where P_i (for $1 \leq i \leq 4$) is a categorical statement and M_j (for $1 \leq j \leq 2$) is \Box or \Diamond .

I am going to present modal categorical statements right away with statements logically equivalent to them. These statements can perhaps be viewed as their ‘intensional readings’ because they employ the notion of a *requisite relation* which is applied to couples of properties as such. It is defined thus:

$$[\mathbf{Requisite} \ g \ f] \Leftrightarrow \Box\forall\lambda x[(fx) \rightarrow (gx)]$$

where **Requisite** / $(\pi(\pi\iota)(\pi\iota))$.

The ‘intensional reading’ is derived from Tichý’s unpublished proposal²¹ to read universal categorical statements such as

²¹ It occurred in Tichý’s unpublished typescript called *Introduction to Intensional Logic* which was completed in 1976. In [38], Tichý published a variant of the notion for individual offices.

“Every horse is an animal”

as analytic because of the meaning connection between predicates. His motivation for the notion of requisite is this: to become a horse an individual has to be an animal, have legs, etc—the property BE AN ANIMAL is thus a property an individual must instantiate in order to instantiate the property BE HORSE, while BE AN ANIMAL is one of its many requisites.

The notion of a requisite is correlative with the notion of entailment between properties (which was considered by e.g. Plantinga [28]):

$$[\mathbf{Entails} \ f \ g] \Leftrightarrow [\mathbf{Requisite} \ g \ f]$$

where **Entails**/ $(\pi(\pi\iota)(\pi\iota))$. The entailment of a proposition Q from a proposition P , i.e., $P \subseteq Q$, is a medadic case of entailment between properties because in every world W the extension of $F \subseteq$ the extension of G .

By utilizing the above definition of the notion of requisite and familiar logical laws, one gets two equivalent forms of universal modal categorical statements:

	A^M	O^M
Modal categorical statement	$\Box\forall\lambda x[(\mathbf{F}x) \rightarrow (\mathbf{G}x)]$	$\Diamond\exists\lambda x[(\mathbf{F}x) \wedge \neg(\mathbf{G}x)]$
– its usual verbal expression	Necessarily, every F is G .	Possibly, some F is not G .
Its ‘intensional reading’	[Requisite G F]	\neg [Requisite G F]
– its verbal expression	G is a requisite of F .	G is not a requisite of F .

Table 2. Modal universal categorical statements.

To complete the quadruple of such statements one needs a notion comparable with the notion of a requisite. I call it *potentiality*; it is definable [33] by

$$[\mathbf{Potentiality} \ g \ f] \Leftrightarrow \Diamond\exists\lambda x[(fx) \wedge (gx)]$$

where **Potentiality**/ $(\pi(\pi\iota)(\pi\iota))$.

To explain: an individual who instantiates the property BE A HORSE has to be an animal, while BE A HORSE admits the individual being white or being a champion, etc. The properties BE WHITE and BE A CHAMPION are thus potentialities of BE A HORSE. On the other hand,

the meaning of the predicate “be a horse” excludes to affirm truly that a horse is a cat, i.e., BE A CAT is not a potentiality of BE A HORSE.

Note that the disjunction of the notions of potentiality and its negated form is a requisite of a given property. For example, BE A CHAMPION OR NOT A CHAMPION is a requisite of the property BE A HORSE, while both BE A CHAMPION and BE NOT A CHAMPION are its potentialities.

	\mathbf{I}^M	\mathbf{E}^M
Modal categorical statement	$\diamond\exists\lambda x[(\mathbf{F}x) \wedge (\mathbf{G}x)]$	$\Box\forall\lambda x[(\mathbf{F}x) \rightarrow \neg(\mathbf{G}x)]$
– its usual verbal expression	Possibly, some F is G .	Necessarily, no F is G .
Its ‘intensional reading’	$[\mathbf{Potentiality\ G\ F}]$	$\neg[\mathbf{Potentiality\ G\ F}]$
– its verbal expression	G is a potentiality of F .	G is not a potentiality of F .

Table 3. Modal particular categorical statements.

4.2. The Square of Opposition with modal categorical statements

With the four modal categorical statements \mathbf{A}^M , \mathbf{E}^M , \mathbf{I}^M and \mathbf{O}^M at hand one can ‘decorate’ vertices of the Square of Opposition:

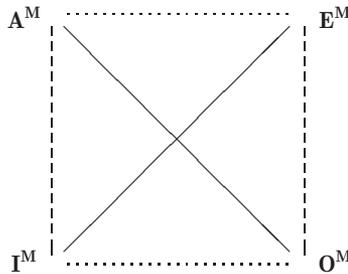


Figure 1. The Square of Opposition in modal reading.

Legend: the dotted line indicates that the relation of contrariety/subcontrariety does not hold, the dashed line indicates that the relation of subalternation holds with an exception, the full line indicates that the relation of contradictoriness holds; cf. Section 4.5 for discussion of the relations.

This Square enables us to quickly find relations that obtain between modal categorical statements and also between statements about requisites, potentialities, non-requisites and non-potentialities.

Note that this Square differs from the *ancient ‘modal’ Square of Opposition* attributed to Aristotle ([1], cf. also e.g. [13]) which concerns four statements of form “ M_iP ”, where M_i (for $1 \leq i \leq 4$) is (the meaning of) “necessarily”, “possibly”, “not necessarily”, “not possibly”. So the four statements share one and the same non-modal categorical statement P . In the ‘modal’ Square I propose, however, each statement M_jP_i (for $1 \leq j \leq 2$ and $1 \leq i \leq 4$) contains one of the four non-modal categorical statements P_i .

Moreover, all modal categorical statements of the above Square can be converted to their equivalent variants involving the notions of requisite and potentiality (cf. Section 4.1). After such a conversion we still have a genuine square, though not with modals. On the other hand, it is difficult to find such equivalent variants of statements such as “It is possible that no F is G ” of the ancient ‘modal’ Square. The two ‘modal’ Squares are clearly different.

4.3. Requisite/potentialities vs. essential/accidental properties

There is an obvious nexus between requisites and potentialities on the one hand and essential and accidental properties on the other, since the latter notions are likewise definable in terms of necessity and possibility. One may therefore ask whether statements such as

- a. ” “Being descended from apes is/is not essential for humans”
- b. ” “Being descended from apes is/is not accidental for humans”

can have existential import.

This has relevance even beyond purely modal discourse. As proved by Corcoran (e.g. [7]), the notion of **A**-statement in the modern reading is not completely devoid of existential import: **A** does entail $\exists \lambda x[\mathbf{F}x]$ provided $\exists \lambda x[\mathbf{F}x]$ is logically true. Though Corcoran did not say it, the existential statement is only logically true if F is essential for certain x . To elucidate this, I am going to define the notion of *property essential for something* which differs from that of *essential property*. The notion gets rid of Aristotelian essentialism; on the other hand, it covers even essential properties that are not as dull as the property BE SELF-IDENTICAL is.

Firstly note that the notions of *essential property* and *accidental property* overlap. For instance, BE AS TALL AS MADONNA is surely a contingent property: every individual instantiates it only accidentally, except the famous pop star, since it is essential for her. I will call such

properties *accidental–essential properties*.²² Setting apart accidental–essential properties, one then obtains *purely essential properties* (e.g. BE SELF-IDENTICAL, BE IDENTICAL WITH MADONNA) and *purely accidental properties* (e.g. BE A HORSE).

A quadruple of comparable notions is completed by adding the notion of *void properties* which are properties having no instance in every world W .²³ Examples of void properties are: i. BE NON-SELF-IDENTICAL, ii. BE A MAN OR NOT A MAN. i. is the only property whose constant extension is the total empty class. ii. has a variable range of extensions while each is an empty partial class – if an individual is (say) a quark in W , it instantiates neither the property BE A MAN, nor BE NOT A MAN (not to be confused with the property BE A NON-MAN),²⁴ since the two properties have among their requisites the property (say) BE AN ANIMAL, which is a non-potentiality of BE A QUARK.

Here are definitions (adapted from [29]):

$$\begin{array}{ll}
 [\mathbf{EssFor} \, fx] & \Leftrightarrow \square(fx) \\
 [\mathbf{AccFor} \, fx] & \Leftrightarrow \diamond(fx) \wedge \diamond\neg(fx) \\
 [\mathbf{Ess} \, f] & \Leftrightarrow \exists \lambda x [\mathbf{EssFor} \, fx] \\
 [\mathbf{Acc} \, f] & \Leftrightarrow \exists \lambda x [\mathbf{AccFor} \, fx] \\
 [\mathbf{PurAcc} \, f] & \Leftrightarrow \exists \lambda x [\mathbf{AccFor} \, fx] \wedge \neg \exists \lambda x [\mathbf{EssFor} \, fx] \\
 [\mathbf{PurEss} \, f] & \Leftrightarrow \neg \exists \lambda x [\mathbf{AccFor} \, fx] \wedge \exists \lambda x [\mathbf{EssFor} \, fx] \\
 [\mathbf{AccEss} \, f] & \Leftrightarrow \exists \lambda x [\mathbf{AccFor} \, fx] \wedge \exists \lambda x [\mathbf{EssFor} \, fx] \\
 [\mathbf{Void} \, f] & \Leftrightarrow \neg \exists \lambda x [\mathbf{AccFor} \, fx] \wedge \neg \exists \lambda x [\mathbf{EssFor} \, fx]
 \end{array}$$

where **EssFor** (essential for), **AccFor** (accidental for), **Ess** (essential), **Acc** (accidental), **PurAcc** (purely accidental), **PurEss** (purely essential) **AccEss** (accidental-essential), **Void**/ $(\pi(\pi\iota))$.

Requisites and potentialities are definable (with some caution) in terms of properties essential or accidental for someone. Our findings

²² The notion was proposed by Cmorej [6]; I use a slightly different definition and name. The notion should not be confused with Loux's [18] notion of an *impure property*, i.e., a property F such that an object X is F whenever X stands in a relation R to some object Y . The property LIVE WHEN MADONNA SINGS seems to be an example; however, it is not accidental–essential, since it is essential for nobody.

²³ The notion of void property is formally defined below. It should not be confused with the notion of an *empty property* which determines (a set of) properties that have empty extensions in given W : $[\mathbf{Empty} \, f]_w \Leftrightarrow \sim \Sigma \lambda x [\mathbf{Inst} \, x \, f]_w$, where **Empty**/ $(\pi(\pi\iota))$.

²⁴ The notion of the *strong property negation* is definable by $[[\mathbf{Non} \, f] \, x] \Leftrightarrow \neg(fx)$, where **Non**/ $((\pi\iota)(\pi\iota))$. Unlike the case with F and ‘be not F ’, the union of extensions (in any W) of F and non- F is the universal class of individuals.

concerning existential import of a'' and b'' thus apply even to a''' and b''' .

4.4. Usual lack of existential import

By utilizing modal categorical statements one immediately resolves the well-known puzzle according to which some intuitive **A**- and **O**-statements lack existential import. Consider the sentence

“Every unicorn is a beast.”

It is natural to understand it as saying that certain property is a requisite of another property. Regardless of the existence of unicorns, it is true in every W , and so it lacks existential import. It is thus adequate to parse it not as **A**-statement, but as **A^M**-statement.

As regards sentences such as

“Some unicorn is not a beast”

they are false in every W , since unicorns are, per definition, beasts. It is thus likewise natural to read it as an **O^M**-statement. On this reading, the sentence is false not because of existential import (though it has existential import if read as an **O**-statement).

From such examples one might perhaps conclude that no modal categorical statement has existential import. This is nearly the case. The exception to the rule concerns mainly **I^M**-statements that employ void properties. For example, the **I^M**-statement

“Possibly, some non-self-identical individual is G ”

is false because of the existential import of the term “non-self-identical individual” in the embedded **I**-statement, and so also in the whole **I^M**-statement. According to the above definition of existential import, thus both **I**- and **I^M**-statements have existential import. Similar considerations apply to some **O^M**-statements, e.g. “Possibly, some non-self-identical individual is not G ”.

4.5. Lack of subalternation, contrariety and subcontrariety

As in the Square of Opposition deploying categorical statements, subalternation does not generally hold in its ‘modal’ variant because

$$\mathbf{A}^{\mathbf{M}} \not\equiv \mathbf{I}^{\mathbf{M}} \qquad \mathbf{E}^{\mathbf{M}} \not\equiv \mathbf{O}^{\mathbf{M}}$$

This is caused by modal categorical statements that have existential import because they may involve properties.

To demonstrate it, consider the \mathbf{A}^M -statement

“Necessarily, everybody who shaves all and only those who do not shave themselves is a barber”

which is true because the property BE SOMEONE WHO SHAVES ALL AND ONLY THOSE WHO DO NOT SHAVE THEMSELVES — call it the “ F -property” — has the property BE A BARBER (“ G -property”) as a requisite. This \mathbf{A}^M -statement does not entail the corresponding \mathbf{I}^M -statement

“Possibly, somebody who shaves all and only those who do not shave themselves is a barber”

which is false because such barber cannot exist, i.e., the F -property is not a potentiality of the G -property. In sum, if the F -property is not a potentiality of the respective G -property, the subalternation of an \mathbf{I}^M -statement to the respective \mathbf{A}^M -statement does not hold. Similarly for \mathbf{E}^M with \mathbf{O}^M .

Consequently, subcontrariety and contrariety do not generally hold. To see this, consider an instance of the above definiens of contrariety,

$$\Box[\mathbf{A}^M \rightarrow \neg\mathbf{E}^M] \wedge \Diamond[\neg\mathbf{A}^M \wedge \neg\mathbf{E}^M]$$

If \mathbf{A}^M is true, there is no W in which it would be false and the right conjunct cannot be satisfied. Similarly for the case of subcontrariety.

5. Conclusions

To summarise: I have examined relations of four categorical statements “Every F is/is not G ” and “Some F is/is not G ”, and four modal (*de dicto*) categorical statements constructed from them, namely “Necessarily, every F is/is not G ” and “Possibly, some F is/is not G ”, as well as their ‘intensional’ variants that utilize the notions of requisite and potentiality. I focused on their existential import because it has a strong impact on the corresponding Squares of Opposition.

In Section 3.2, I defined i. existential import of (meanings of) predicates in statements and ii. existential import of statements, while ii. depends on i. The expressively rich higher-order hyperintensional partial type logic used in this paper makes the definitions rigorous, while fulfilling intuitive desiderata.

My construal of existential import follows the approach of modern logic and so universal categorical statements, whose enhanced versions have been proposed in Section 3.1, lack existential import. In Section 3.2, I discussed the details; and in Section 3.4, I restated the much-discussed lack of subalternation, contrariety and subcontrariety in the modern reading of the Square of Opposition. It is needless to stress that contradictoriness holds in it, as well as in its modal version, since no Square would be a Square of Opposition without contradictoriness.

As regards modal categorical statements—precisely formulated in Section 4.1—so-called void properties bring existential import to their particular affirmative and negative versions (Section 4.4). They are the only properties which invalidate subalternation; in consequence of this, contrariety and subcontrariety do not generally hold in the corresponding Square of Opposition (Section 4.5).

The fact that void properties make the only exception to a lack of existential import for modal particular categorical statements has a remarkable connection: *weakened modes of syllogisms*, e.g. Darapti

“All H are G ”, “All H are F ” / “Therefore, some F are G ”

are nearly generally valid on the modal reading of categorical statements which are involved in these schemes. There is a hypothesis, not examined here, that medieval logicians, who accepted the weakened modes, purposely ignored void properties because the properties simply never have instances. The investigation of a modal syllogistic that would utilize results of this paper is a task for future.

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