(in this possible world). All the sentences F1–F4 are temporally settled. $\mathbf{O}[\top]\mathsf{R}t_2k$, $\mathbf{O}[\mathsf{R}t_2k]\mathsf{R}t_3\neg a$ and $\mathbf{O}[\mathsf{R}t_2\neg k]\mathsf{R}t_3a$ are examples of sentences that are not temporally settled, as their truth values may vary from one moment of time to another (in one and the same possible world).

It is true on Monday that it is true on Friday that you do not keep your promise iff it is true on Friday that you do not keep your promise. $\mathsf{R}t_1\mathsf{R}t_2\neg k$ is equivalent to $\mathsf{R}t_2\neg k$. So, from now on, we will use $\mathsf{R}t_2\neg k$ as a symbolisation of N4. (We mention $\mathsf{R}t_1\mathsf{R}t_2\neg k$ because we want to know what is true on Monday, and according to our scenario, it is true on Monday that you will not keep your promise on Friday.) Note that it might be true on Monday that you will not keep your promise on Friday (in some possible world) even though this is not a settled fact, i.e., even though it is not historically necessary. In some possible worlds you will keep your promise on Friday and in some possible worlds you will not. F4 is true at t_1 (i.e. on Monday) in the possible worlds where you do not keep your promise at t_2 (i.e. on Friday).

Let F-CTD := $\{\text{F1}, \text{F2}, \text{F3}, \text{F4}\}$. We will show that F-CTD is consistent in all systems weaker than or deductively equivalent to the system **T** (see Section 4.3). Most philosophically interesting temporal alethic dyadic deontic systems (of the kind used in this paper) are included in this set.[11] (In Section 6.3 we will show that F-CTD has many other intuitively plausible properties, for instance that it is non-redundant.) To show this, it is enough to establish that this is true in the system **T**. Then it follows that the result holds in all weaker systems too. Hence, we can solve the contrary-to-duty paradox in temporal alethic dyadic deontic logic.

MODEL I. Consider the following supplemented temporal alethic dyadic deontic model. $W = \{\omega_1, \omega_2, \omega_3, \omega_4\}$. The model satisfies all conditions in tables 4 and 5 (and thus all conditions in tables 1–3), i.e., $>$ (better than) is transitive, etc. Hence, the ranking is the same in every possible world in the model: $\omega_1 > \omega_2 > \omega_3 > \omega_4$. This means, for instance, that ω_1 is better than ω_2 in every possible world, i.e., $\omega_1 > \omega_2$ is an

[11] It is possible to construct several systems that are stronger than **T** by adding temporal rules to this system, e.g. we can add rules that correspond to the fact that time is dense or that there is no first or last point in time. As far as we can see, the results in this section can be extended to all such systems that are philosophically interesting. However, we will not explicitly consider these examples since we want to keep our models as simple as possible, e.g. we want to avoid models with an infinite number of temporal points.

abbreviation of the following conditions: $\omega_1 >_{\omega_1} \omega_2$, $\omega_1 >_{\omega_2} \omega_2$, $\omega_1 >_{\omega_3} \omega_2$, $\omega_1 >_{\omega_4} \omega_2$, etc. Since ω_1 is better than ω_2 and ω_2 is better than ω_3, ω_1 is better than ω_3, etc. In other words, ω_1 is the best member of W, ω_2 the second best, etc. The alethic accessibility relation is an equivalence relation (i.e., it is reflexive, symmetric, transitive (at every moment in time) etc.). $T = \{\tau_1, \tau_2, \tau_3\}$, where $\tau_1 < \tau_2 < \tau_3$. The temporal relation is transitive, comparable, and it does not branch towards the future or the past. At τ_1 all possible worlds are alethically accessible from all possible worlds. At τ_2, ω_2 is alethically accessible from ω_1 and ω_1 is alethically accessible from ω_2. At τ_2, ω_3 can see ω_4 alethically and ω_4 can see ω_3 alethically. At all times every possible world is alethically accessible to itself. The deontic accessibility relations are defined by $C - D\gamma 0$. $v(t_1) = \tau_1$, $v(t_2) = \tau_2$, and $v(t_3) = \tau_3$. k is true in ω_1 and ω_2 at τ_2 and k is false in ω_3 and ω_4 at τ_2. a is false in ω_1 and ω_4 at τ_3 and a is true in ω_2 and ω_3 at τ_3. For our purposes, we do not need any further information about this model. Model I is a so-called **T**-model (see Section 3.3).

We are now in a position to prove the following theorem:

Theorem 5. *F-CTD is consistent in all systems weaker than or deductively equivalent to the system* **T**.

Proof. Every sentence in F-CTD is true in ω_3 at τ_1 in Model I. Hence, F-CTD is satisfiable in this model. By the definition of **T** and the soundness results in Section 7.1, it follows that F-CTD is consistent in **T**. Let us verify that every sentence in F-CTD is true in ω_3 at τ_1 in Model I.

F1. $\mathsf{R}t_1 O[\top]\mathsf{R}t_2 k$. $\mathsf{R}t_1 O[\top]\mathsf{R}t_2 k$ is true in ω_3 at τ_1 iff $\mathbf{O}[\top]\mathsf{R}t_2 k$ is true in ω_3 at τ_1. $\mathbf{O}[\top]\mathsf{R}t_2 k$ is true in ω_3 at τ_1 iff $\mathsf{R}t_2 k$ is true in all the best worlds that are alethically accessible from ω_3 at τ_1 where $\top$ is true at τ_1. Since $\top$ is true in every possible world at every moment in time, ω_1 is the best world that is alethically accessible from ω_3 at τ_1 in which $\top$ is true. Since k is true in ω_1 at τ_2, it follows that $\mathsf{R}t_2 k$ is true in ω_1 at τ_1. So, $\mathsf{R}t_2 k$ is true in all the best worlds that are alethically accessible from ω_3 at τ_1 where $\top$ is true. Consequently, $\mathbf{O}[\top]\mathsf{R}t_2 k$ is true in ω_3 at τ_1. In conclusion, $\mathsf{R}t_1 \mathbf{O}[\top]\mathsf{R}t_2 k$ is true in ω_3 at τ_1.

F2. $\mathsf{R}t_1 \mathbf{O}[\mathsf{R}t_2 k]\mathsf{R}t_3 \neg a$. $\mathsf{R}t_1 \mathbf{O}[\mathsf{R}t_2 k]\mathsf{R}t_3 \neg a$ is true in ω_3 at τ_1 iff $\mathbf{O}[\mathsf{R}t_2 k]\mathsf{R}t_3 \neg a$ is true in ω_3 at τ_1. $\mathbf{O}[\mathsf{R}t_2 k]\mathsf{R}t_3 \neg a$ is true in ω_3 at τ_1 iff $\mathsf{R}t_3 \neg a$ is true in all the best worlds that are alethically accessible from

ω_3 at τ_1 in which $\mathsf{R}t_2k$ is true at τ_1. $\mathsf{R}t_2k$ is true at τ_1 in ω_1 and ω_2. The best of these worlds is ω_1. It follows that $\mathbf{O}[\mathsf{R}t_2k]\mathsf{R}t_3\neg a$ is true in ω_3 at τ_1 iff $\mathsf{R}t_3\neg a$ is true in ω_1 at τ_1. $\mathsf{R}t_3\neg a$ is true in ω_1 at τ_1 iff $\neg a$ is true in ω_1 at τ_3. Since a is false in ω_1 at τ_3, $\neg a$ is true in ω_1 at τ_3. Hence, $Rt_3\neg a$ is true in ω_1 at τ_1. Consequently, $\mathbf{O}[\mathsf{R}t_2k]\mathsf{R}t_3\neg a$ is true in ω_3 at τ_1. It follows that $\mathsf{R}t_1\mathbf{O}[\mathsf{R}t_2k]\mathsf{R}t_3\neg a$ is true in ω_3 at τ_1.

F3. $\mathsf{R}t_1\mathbf{O}[\mathsf{R}t_2\neg k]\mathsf{R}t_3a$. $\mathsf{R}t_1\mathbf{O}[\mathsf{R}t_2\neg k]\mathsf{R}t_3a$ is true in ω_3 at τ_1 iff $\mathbf{O}[\mathsf{R}t_2\neg k]\mathsf{R}t_3a$ is true in ω_3 at τ_1. $\mathbf{O}[\mathsf{R}t_2\neg k]\mathsf{R}t_3a$ is true in ω_3 at τ_1 iff $\mathsf{R}t_3a$ is true in all the best worlds that are alethically accessible from ω_3 at τ_1 in which $\mathsf{R}t_2\neg k$ is true at τ_1. $\mathsf{R}t_2\neg k$ is true at τ_1 in ω_3 and ω_4. The best of these worlds is ω_3. Hence, $\mathbf{O}[\mathsf{R}t_2\neg k]\mathsf{R}t_3a$ is true in ω_3 at τ_1 iff $\mathsf{R}t_3a$ is true in ω_3 at τ_1. $\mathsf{R}t_3a$ is true in ω_3 at τ_1 iff a is true in ω_3 at τ_3. But a is true in ω_3 at τ_3. Accordingly, $\mathsf{R}t_3a$ is true in ω_3 at τ_1. Therefore, $\mathbf{O}[\mathsf{R}t_2\neg k]\mathsf{R}t_3a$ is true in ω_3 at τ_1. It follows that $\mathsf{R}t_1\mathbf{O}[\mathsf{R}t_2\neg k]\mathsf{R}t_3a$ is true in ω_3 at τ_1.

F4. $\mathsf{R}t_2\neg k$. $\mathsf{R}t_2\neg k$ is true in ω_3 at τ_1 iff $\neg k$ is true in ω_3 at τ_2. $\neg k$ is true in ω_3 at τ_2 iff k is false in ω_3 at τ_2. Since k is false in ω_3 at τ_2, it follows that $\mathsf{R}t_2\neg k$ is true in ω_3 at τ_1. $\dashv$

6.3. Reasons why the solution to the contrary-to-duty paradox in temporal alethic dyadic deontic logic is attractive

In this section, we will consider 12 reasons why the solution to the contrary-to-duty paradox suggested in Section 6.2 is attractive. Taken individually, each reason might not seem that impressive, but together they really show how powerful this solution is. Without further ado, let us turn to our reasons.

Reason 1 (F-CTD is consistent). N-CTD seems to be consistent, i.e., it does not seem to be the case that we can derive a contradiction from this set. Hence, we want our symbolisation of N-CTD to be consistent. We have already shown that this is the case (Theorem 5).

Reason 2 (F-CTD is non-redundant). N-CTD seems to be non-redundant, i.e., it seems to be the case that no member of this set is derivable from the others. Therefore, we want our symbolisation of N-CTD to be non-redundant. In monomodal deontic logic, for instance Standard Deontic Logic, we can solve the contrary-to-duty paradox by finding some other formalisation of the sentences in N-CTD. Instead of SDL2 we can use $k \to \mathbf{O}\neg a$ and instead of SDL3 we can use $\mathbf{O}(\neg k \to a)$. Then we obtain three consistent alternative symbolisations of N-CTD. However, these alternatives are not non-redundant. For $\mathbf{O}(\neg k \to a)$ follows from $\mathbf{O}k$ in every so-called normal deontic logic, including Standard Deontic Logic, and $k \to \mathbf{O}\neg a$ follows from $\neg k$ by propositional logic. But intuitively, N3 does not seem to follow from N1, and N2 does not seem to follow from N4. In temporal alethic dyadic deontic logic, we can avoid this problem since we can prove the following theorem:

Theorem 6. *F-CTD is non-redundant in all systems weaker than or deductively equivalent to the system* **T**.

To prove this theorem, we must establish propositions 1–4 below.

Proposition 1. F1 *is not derivable from* $\{F2, F3, F4\}$ *in* **T**. *To prove that* F1 *is not derivable from* $\{F2, F3, F4\}$ *it is sufficient to come up with a* **T**-*model* M, *a world* ω *in* M *and a time* τ *in* M *such that all members of* $\{F2, F3, F4\}$ *are true in* ω *at* τ *and* F1 *false in this world at this time. For* **T** *is strongly sound with respect to the class of all* **T**-*models. Consider the following model.*

Model II. This model is exactly like Model I except that we use the following ranking of possible worlds instead: $\omega_3 > \omega_1 > \omega_2 > \omega_4$. In world ω_4 at time τ_1 all members of $\{F2, F3, F4\}$ are true and F1 false. Hence, F1 is not derivable from $\{F2, F3, F4\}$ in **T** (or any weaker system).

Proposition 2. F2 *is not derivable from* $\{F1, F3, F4\}$ *in* **T**. *The following model proves this proposition.*

Model III. This model is exactly like Model I except that a is true in ω_1 at τ_3 and that we use the following ranking for the possible worlds: $\omega_1 > \omega_3 > \omega_2 > \omega_4$. In ω_4 at τ_1 all members of $\{F1, F3, F4\}$ are true and F2 false. Consequently, F2 is not derivable from $\{F1, F3, F4\}$ in **T**.

Proposition 3. F3 *is not derivable from* $\{F1, F2, F4\}$ *in* **T**. *To prove this claim we use the following model.*

Model IV. This model is exactly like Model I except that we use the same ranking as in Model III and that a is false in ω_3 at time τ_3. In world ω_4 at time τ_1 all members of $\{F1, F2, F4\}$ are true and F3 false. Accordingly, F3 is not derivable from $\{F1, F2, F4\}$ in **T**.

Proposition 4. F4 *is not derivable from* $\{F1, F2, F3\}$ *in* **T**. *To prove this proposition, we can use Model I. In world* ω_2 *at time* τ_1 *all members of* $\{F1, F2, F3\}$ *are true and* F4 *false.*

Reason 3 (F-CTD is dilemma free). One way of avoiding the contrary-to-duty paradox in monomodal deontic systems is to give up the axiom D, $\neg(\mathbf{O}A \wedge \mathbf{O}\neg A)$ ("It is not obligatory that A and obligatory that not-A"). Without this axiom (or something equivalent), it is no longer possible to derive a contradiction from SDL1–SDL4. In the so-called smallest normal deontic system K, for instance, SDL-CTD is consistent. Some might think that there are independent reasons for rejecting D since they think there are, or could be, genuine moral dilemmas. But even if this were true (which is debatable), rejecting D does not seem to be a good solution to the contrary-to-duty paradox for several reasons. Firstly, even if we reject axiom D, it is problematic to assume that a dilemma follows from N-CTD. We can still derive the sentence $\mathbf{O}a \wedge \mathbf{O}\neg a$ from SDL-CTD in every normal deontic system, which says that it is obligatory that you apologise and it is obligatory that you do not apologise. And this proposition does not seem to follow from N-CTD. Secondly, if there are any moral dilemmas of this kind, we can derive the claim that everything is both obligatory and forbidden in every normal deontic system, which is absurd (see Reason 4 below). Thirdly, such a solution might still have problems with the so-called pragmatic oddity (see Reason 5 below).

Our solution in temporal alethic dyadic deontic logic avoids this problem. From F1 and F2 we can derive the sentence $\mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_3\neg a$ (in some systems) (see Reason 10 below) and from F3b and F4 we can derive the sentence $\mathsf{R}t_2\mathbf{O}[\top]\mathsf{R}t_3 a$ (in some systems under some circumstances) (see Reason 11 below). And from this we can derive the following formula: $\mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_3\neg a \wedge \mathsf{R}t_2\mathbf{O}[\top]\mathsf{R}t_3 a$, from $\{\text{F1}, \text{F2}, \text{F3b}, \text{F4}\}$ (in certain systems). But this is not a moral dilemma. $\mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_3\neg a$ says "On Monday [when you have not yet broken your promise] it ought to be the case that you do not apologise on Saturday", and $\mathsf{R}t_2\mathbf{O}[\top]\mathsf{R}t_3 a$ says "On Friday [when you have broken your promise] it ought to be the case that you apologise on Saturday". But $\mathbf{O}[\top]\mathsf{R}t_3 a$ and $\mathbf{O}[\top]\mathsf{R}t_3\neg a$ are not true at the same time. Neither $\mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_3\neg a \wedge \mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_3 a$ nor $\mathsf{R}t_2\mathbf{O}[\top]\mathsf{R}t_3\neg a \wedge \mathsf{R}t_2\mathbf{O}[\top]\mathsf{R}t_3 a$ is derivable from F-CTD in **T** or any weaker temporal alethic dyadic deontic system. N-CTD seems to be dilemma free. So, we want our formalisation of N-CTD to be dilemma free too. The following theorem shows that F-CTD is dilemma free in **T** and any weaker temporal alethic dyadic deontic system:

Theorem 7. *F-CTD is dilemma free in* **T** *and any weaker temporal alethic dyadic deontic system.*

Proof. Consider Model I. Every sentence in F-CTD is true in ω_3 at τ_1 in this model. However, $\mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_3a$ is false in this world at this time. Hence, $\mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_3\neg a \wedge \mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_3a$ is also false in this world at this time. Since Model I is a **T**-model and **T** is sound with respect to the class of all **T**-models, it follows that $\mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_3\neg a \wedge \mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_3a$ is not derivable from F-CTD in **T** or any weaker temporal alethic dyadic deontic system. $\mathsf{R}t_2\mathbf{O}[\top]\mathsf{R}t_3\neg a$ is also false in ω_3 at τ_1. Hence, $\mathsf{R}t_2\mathbf{O}[\top]\mathsf{R}t_3\neg a \wedge \mathsf{R}t_2\mathbf{O}[\top]\mathsf{R}t_3a$ is false in this world at this time. It follows that $\mathsf{R}t_2\mathbf{O}[\top]\mathsf{R}t_3\neg a \wedge \mathsf{R}t_2\mathbf{O}[\top]\mathsf{R}t_3a$ is not derivable from F-CTD in **T**. ⊣

Reason 4 (It is not possible to derive the proposition that everything is both obligatory and forbidden from F-CTD). In every so-called normal deontic logic (even without the axiom D), we can derive the conclusion that everything is both obligatory and forbidden if there is at least one moral dilemma. This follows from the equivalence $\mathbf{F}A$ iff $\mathbf{O}\neg A$ and the fact that $\mathbf{O}a \wedge \mathbf{O}\neg a$ entails $\mathbf{O}r$ for any r. This is clearly absurd. N-CTD does not seem to entail that everything is both obligatory and forbidden. Hence, we do not want our symbolisation to entail this. Our solution in temporal alethic dyadic deontic logic has no such consequences. We have already seen that F-CTD is dilemma free (Reason 3 above). The following theorem shows that our solution avoids this problem:

Theorem 8. *F-CTD does not entail that for every A it is both obligatory that A and obligatory that not-A in* ***T*** *or any weaker temporal alethic dyadic deontic system.*

Proof. Again, we can use Model I to prove this. Every sentence in F-CTD is true in ω_3 at time τ_1 in this model. However, $\mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_3\neg a$ is true while $\mathsf{R}t_1\mathbf{O}[\top]\neg\mathsf{R}t_3\neg a$ and $\mathsf{R}t_1F[\top]\mathsf{R}t_3\neg a$ are false in this world at this time. From this our theorem follows easily. ⊣

Reason 5 (F-CTD avoids the so-called pragmatic oddity). Pragmatic oddity is a problem for many possible solutions to the contrary-to-duty paradox. In every so-called normal deontic logic (with or without the axiom D) it is possible to derive the following sentence from SDL-CTD: $\mathbf{O}(k \wedge a)$, which says that it is obligatory that you keep your promise *and* apologise (for not keeping your promise). Several solutions that use bimodal alethic-deontic logic or counterfactual deontic logic, for instance, also have this problem. The sentence $\mathbf{O}(k \wedge a)$ is not inconsistent, but it is certainly very odd and it does not seem to follow from N-CTD that you should keep your promise *and* apologise. Hence, we do not want our formalisation of N-CTD to entail this counterintuitive conclusion or anything similar to it. The following theorem shows that neither $\mathsf{R}t_1\mathbf{O}[\top](\mathsf{R}t_2k \wedge \mathsf{R}t_3a)$ nor $\mathsf{R}t_2\mathbf{O}[\top](\mathsf{R}t_2k \wedge \mathsf{R}t_3a)$ is derivable from F-CTD in **T** or any weaker system:

Theorem 9. *Neither* $\mathsf{R}t_1\mathbf{O}[\top](\mathsf{R}t_2k \wedge \mathsf{R}t_3a)$ *nor* $\mathsf{R}t_2\mathbf{O}[\top](\mathsf{R}t_2k \wedge \mathsf{R}t_3a)$ *is derivable from F-CTD in* **T**.

Proof. We have seen that all sentences in F-CTD are true in ω_3 at τ_1 in Model I. However, neither $\mathsf{R}t_1\mathbf{O}[\top](\mathsf{R}t_2k \wedge \mathsf{R}t_3a)$ nor $\mathsf{R}t_2\mathbf{O}[\top](\mathsf{R}t_2k \wedge \mathsf{R}t_3a)$ is true in this world at this time (in this model). Now our theorem follows easily from this fact. ⊣

Reason 6 (The solution in temporal alethic dyadic deontic logic is applicable to (at least apparently) actionless contrary-to-duty examples). It might be possible to solve some contrary-to-duty paradoxes by combining deontic logic with some kind of action logic, for instance some kind of Stit (“Seeing to it”) logic, or dynamic logic. However, there also seem to be examples of contrary-to-duty paradoxes that involve actionless contrary-to-duty obligations. And it is difficult to see how to solve these paradoxes in such systems.

Scenario II: Contrary-to-duty paradoxes involving (apparently) actionless contrary-to-duty obligations

Consider the following scenario. At t_1, you are about to get into your car and drive somewhere. Then at t_1 it ought to be the case that the doors are closed at t_2, when you are in your car. If the doors are not closed, then a warning light ought to appear on the car instrument panel (at t_3, a point in time as soon as possible after t_2). It ought to be that if the doors are closed (at t_2), then it is not the case that a warning light appears on the car instrument panel (at t_3). Furthermore, the doors are not closed (at t_2 when you are in the car). In this example, all of the following sentences seem to be true:

N2-CTD

AN1. (At t_1) The doors ought to be closed (at t_2).

AN2. (At t_1) It ought to be that if the doors are closed (at t_2), then it is not the case that a warning light appears on the car instrument panel (at t_3).

AN3. (At t_1) If the doors are not closed (at t_2) then a warning light ought to appear on the car instrument panel (at t_3).

AN4. (At t_1 it is the case that at t_2) The doors are not closed.

N2-CTD is similar to N-CTD. In this set, AN1 expresses a primary obligation (or ought), and AN3 expresses a contrary-to-duty obligation. The condition in AN3 is satisfied only if the primary obligation expressed by AN1 is violated. But AN3 does not seem to tell us anything about what you or someone else ought to do. AN3 seems to be an actionless contrary-to-duty obligation. It tells us something about what ought to be the case if the world is not as it ought to be according to AN1.

In temporal alethic dyadic deontic logic, we have no trouble symbolising such (apparently) actionless contrary-to-duty obligations. The logical form of the sentences in N2-CTD exactly parallels the logical form of the sentences in N-CTD. Contrary-to-duty paradoxes of this kind can therefore be solved in exactly the same way as we solved our original paradox.

Reason 7 (We can assign formal sentences with analogous structures to all conditional obligations in N-CTD in temporal alethic dyadic deontic logic). Some deontic logicians have suggested that a formalisation of N-CTD is adequate only if the formal sentences assigned to N2 and N3 have the same (or analogous) logical form (see e.g. [13]). Our solution in temporal alethic dyadic deontic logic satisfies this requirement, in contrast to many other solutions. F2 and F3 have the "same" logical form; both are formalised using dyadic obligation.

Reason 8 (We can express the idea that an obligation has been violated in temporal alethic dyadic deontic logic). It might be possible to solve some contrary-to-duty paradoxes by applying ordinary concepts of defeasibility from so-called non-monotonic logic. However, it is not obvious that such solutions can explain the difference between violation and defeat. If you will not see your friend and help her, the obligation to keep your promise will be violated. It is not the case that this obligation is defeated, overridden or cancelled. It is not the case that one of the conditional norms in N-CTD defeat or override the other. Nor is it the case that they cancel each other out.

In temporal alethic dyadic deontic logic, we can express the idea that an obligation has been violated. Nevertheless, we must be careful when we describe the facts. At τ_2, when it is already settled that you do not keep your promise, it is no longer obligatory that you keep your promise, since by then it is no longer possible to keep it (in the strong models, for instance the **T**-models, we are considering). However, the following sentence is still true (in ω_1 and ω_2 at τ_2): $\mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_2k$, i.e., it is still true at τ_2 (in the worlds where you do not keep your promise) that you should have kept your promise. So, when you do not keep your promise, you violate this earlier duty.

Reason 9 (We can symbolise higher order contrary-to-duty obligations in temporal alethic dyadic deontic logic). There are contrary-to-duty obligations of a higher order or degree. Consider the following variation of Scenario I:

Scenario III

One could claim that what you ought to do on Monday if you will not help your friend on Friday is call her on Wednesday, tell her that you will not keep your promise and apologise. If you neither keep your promise on Friday nor call your friend on Wednesday, then you ought to apologise (when you meet your friend on Saturday). If you keep your promise, then you ought not to apologise (when you meet your friend on Saturday) and you ought not to call her (on Wednesday). In this scenario all of the following sentences seem to be true:

N-HCTD

HN1. (On Monday it is true that) You ought to keep your promise (and see your friend on Friday).

HN2. (On Monday it is true that) It ought to be that if you keep your promise, you do not apologise (when you meet your friend on Saturday).

HN3. (On Monday it is true that) If you do not keep your promise (i.e., if you will not see your friend on Friday and help her out), you ought to call her (tell her that you will not keep your promise and apologise on Wednesday).

HN4. (On Monday it is true that) You do not keep your promise (on Friday).

HN5. (On Monday it is true that) If you do not keep your promise (on Friday) and you do not call your friend (on Wednesday), you ought to apologise (when you meet your friend on Saturday).

HN6. (On Monday it is true that) You do not call your friend (on Wednesday).

HN7. (On Monday it is true that) It ought to be that if you keep your promise, you do not call your friend (on Wednesday).

Let N-HCTD := {HN1, ..., HN7}. Here HN3 is an ordinary, first-order or first-degree contrary-to-duty obligation that tells us what ought to be the case if the primary obligation expressed by HN1 is violated. HN5 expresses a contrary-to-contrary to duty obligation, a second-order or second-degree contrary-to-duty obligation. The condition in this obligation is fulfilled only if the primary obligation expressed by HN1 is violated and the first-order contrary-to-duty obligation to call your friend is violated.

A reasonable solution to the contrary-to-duty paradox should be able to deal with higher-order contrary-to-duty obligations as well as ordinary first-degree contrary-to-duty obligations. In our temporal alethic dyadic deontic systems, we do not seem to have any trouble symbolising such higher-order contrary-to-duty obligations. N-HCTD can, for instance, be symbolised in the following way in temporal alethic dyadic deontic logic:

F-HCTD

HF1. $\mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_3k$
HF2. $\mathsf{R}t_1\mathbf{O}[\mathsf{R}t_3k]\mathsf{R}t_4\neg a$
HF3. $\mathsf{R}t_1\mathbf{O}[\mathsf{R}t_3\neg k]\mathsf{R}t_2c$
HF4. $\mathsf{R}t_1\mathsf{R}t_3\neg k \quad [\Leftrightarrow \mathsf{R}t_3\neg k]$
HF5. $\mathsf{R}t_1\mathbf{O}[\mathsf{R}t_3\neg k \wedge \mathsf{R}t_2\neg c]\mathsf{R}t_4a$
HF6. $\mathsf{R}t_1\mathsf{R}t_2\neg c \quad [\Leftrightarrow \mathsf{R}t_2\neg c]$
HF7. $\mathsf{R}t_1\mathbf{O}[\mathsf{R}t_3k]\mathsf{R}t_2\neg c$

Let F-HCTD := {HF1, ..., HF7}. F-HCTD is also consistent, non-redundant, etc. Not all solutions to the contrary-to-duty paradox seem to accommodate such examples.

Reason 10 (In temporal alethic dyadic deontic logic we can derive "ideal" obligations). It seems to follow that you ought not to apologise from N1 and N2. Ideally you ought to keep your promise, and ideally it ought to be that if you keep your promise, then you do not apologise (for not keeping your promise). Hence, ideally you ought not to apologise. We want our formalisation of N-CTD to reflect this intuition. In every tableau system that includes $T\text{-}D\alpha 0$ and $T\text{-}D\alpha 2$, $\mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_3\neg a$ is derivable from F1 ($\mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_2k$) and F2 ($\mathsf{R}t_1\mathbf{O}[\mathsf{R}t_2k]\mathsf{R}t_3\neg a$). To illustrate how to use our tableau systems we will now show this.

In our derivation below (and in the derivation in Reason 11), we will use two derived rules. According to the first derived rule, DR1, we may add $\neg A$, w_it to any open branch in a tree that includes $\neg\mathsf{R}tA$, w_it_j.

This rule holds in every system. According to the second derived rule, DR2, we may add $\mathbf{O}[\top](A \to B)$, $w_i t_j$ to any open branch in a tree that contains $\mathbf{O}[A]B$, $w_i t_j$. This derived rule holds in every system that includes the rules *T-Dα0* and *T-Dα2*.

(1) $\mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_2k, w_0t_0$
(2) $\mathsf{R}t_1\mathbf{O}[\mathsf{R}t_2k]\mathsf{R}t_3\neg a, w_0t_0$
(3) $\neg\mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_3\neg a, w_0t_0$
(4) $\neg\mathbf{O}[\top]\mathsf{R}t_3\neg a, w_0t_1$ [3, DR1]
(5) $\mathbf{P}[\top]\neg\mathsf{R}t_3\neg a, w_0t_1$ [4, $\neg\mathbf{O}$]
(6) $s_\top w_0w_1t_1$ [5, $\mathbf{P}$]
(7) $\neg\mathsf{R}t_3\neg a, w_1t_1$ [5, $\mathbf{P}$]
(8) $\neg\neg a, w_1t_3$ [7, DR1]
(9) $\mathbf{O}[\top]\mathsf{R}t_2k, w_0t_1$ [1, $\mathsf{R}t$]
(10) $\mathsf{R}t_2k, w_1t_1$ [9, 6, $\mathbf{O}$]
(11) k, w_1t_2 [10, $\mathsf{R}t$]
(12) $\mathbf{O}[\mathsf{R}t_2k]\mathsf{R}t_3\neg a, w_0t_1$ [2, $\mathsf{R}t$]
(13) $\mathbf{O}[\top](\mathsf{R}t_2k \to \mathsf{R}t_3\neg a), w_0t_1$ [12, DR2]
(14) $\mathsf{R}t_2k \to \mathsf{R}t_3\neg a, w_1t_1$ [13, 6, $\mathbf{O}$]

↙	↘
(15) $\neg\mathsf{R}t_2k, w_1t_1$ [14, $\to$]	(16) $\mathsf{R}t_3\neg a, w_1t_1$ [14, $\to$]
(17) $\neg k, w_1t_2$ [15, DR1]	(18) $\neg a, w_1t_3$ [16, $\mathsf{R}t$]
(19) * [11, 17]	(20) * [8, 18]

According to $\mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_3\neg a$, it is true at t_1, i.e., on Monday, that it ought to be the case that you will not apologise on Saturday when you meet your friend. For, ideally, you keep your promise on Friday. Note, however, that $\mathsf{R}t_2\mathbf{O}[\top]\mathsf{R}t_3\neg a$ does not follow from F1 and F2 (see Reason 3 above). On Friday, when you have broken your promise, and when it is no longer historically possible for you to keep your promise, then it is not obligatory that you do not apologise on Saturday. In fact, then it is obligatory that you apologise when you meet your friend on Saturday (see Reason 11). But on Monday it is not a settled fact that you will not keep your promise. Hence, it is reasonable to claim that it is true on Monday that it ought to be the case that you do not apologise on Saturday. For on Monday it is still possible for you to keep your promise on Friday, which you ought to do. All of these conclusions that follow from F-CTD correspond well with our intuitions about Scenario I.

Reason 11 (In temporal alethic dyadic deontic logic we can derive “actual” obligations in certain circumstances). It seems to follow that you ought to apologise from N3 and N4. Ideally you ought to keep your promise. But if you do not keep your promise, you ought to apologise. And in fact, you do not keep your promise. Hence, you should apologise. We want our formalisation of N-CTD to reflect this intuition. Accordingly, we will assume that the conditional (contrary-to-duty) obligation expressed by N3 is still in force at time t_2 (τ_2), i.e., we assume that the following sentence is true:

F3b. $\mathsf{R}t_2\mathbf{O}[\mathsf{R}t_2\neg k]\mathsf{R}t_3a$.

According to F3b it is true at t_2 (i.e., on Friday) that if you do not keep your promise on Friday, then you ought to apologise on Saturday. In every tableau system that includes *T-Dα0*, *T-Dα2*, *T-DMO* (the dyadic must-ought principle), and *T-BT* (backward transfer), $\mathsf{R}t_2\mathbf{O}[\top]\mathsf{R}t_3a$ is derivable from F4 ($\mathsf{R}t_2\neg k$) and F3b. According to $\mathsf{R}t_2\mathbf{O}[\top]\mathsf{R}t_3a$, it is true at t_2, i.e., on Friday when you have broken your promise to your friend, that it ought to be the case that you apologise to your friend on Saturday when you meet her.

Note that $\mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_3a$ is not derivable from F3 (or F3b or F3 and F3b) and F4 (see Reason 3). $\mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_3a$ says that it is true at t_1, i.e., on Monday, that you should apologise to you friend on Saturday when you meet her. But on Monday it is not yet a settled fact that you will not keep your promise to your friend. Hence, it is not true on Monday that you should apologise on Saturday. On Monday it is still open to you to keep your promise on Friday. And since you ought to keep your promise, and it ought to be that if you keep your promise then you do not apologise, it follows that it is true on Monday that it ought to be the case that you do not apologise on Saturday (see Reason 10). All of these facts correspond well with our intuitions about Scenario I.

Here is our derivation of $\mathsf{R}t_2\mathbf{O}[\top]\mathsf{R}t_3a$ from F3b and F4; this derivation also illustrates how to use our tableau systems:

(1) $\mathsf{R}t_2\neg k, w_0t_0$
(2) $\mathsf{R}t_2\mathbf{O}[\mathsf{R}t_2\neg k]\mathsf{R}t_3a, w_0t_0$
(3) $\neg\mathsf{R}t_2\mathbf{O}[\top]\mathsf{R}t_3a, w_0t_0$
(4) $\neg\mathbf{O}[\top]\mathsf{R}t_3a, w_0t_2$ [3, DR1]
(5) $\mathbf{P}[\top]\neg\mathsf{R}t_3a, w_0t_2$ [4, $\neg\mathbf{O}$]
(6) $s_\top w_0w_1t_2$ [5, $\mathbf{P}$]
(7) $\neg\mathsf{R}t_3a, w_1t_2$ [5, $\mathbf{P}$]
(8) $\neg a, w_1t_3$ [7, DR1]
(9) $rw_0w_1t_2$ [6, *T-DMO*]
(10) $\neg k, w_0t_2$ [1, $\mathsf{R}t$]

$$(11)\ \mathbf{O}[\mathsf{R}t_2\neg k]\mathsf{R}t_3 a, w_0 t_2\ [2, \mathsf{R}t]$$
$$(12)\ \mathbf{O}[\top](\mathsf{R}t_2\neg k \to \mathsf{R}t_3 a), w_0 t_2\ [11, \text{DR2}]$$
$$(13)\ \mathsf{R}t_2\neg k \to \mathsf{R}t_3 a, w_1 t_2\ [6, 12, \mathbf{O}]$$
$$\swarrow \qquad \searrow$$
$$(14)\ \neg\mathsf{R}t_2\neg k, w_1 t_2\ [13, \to] \qquad (15)\ \mathsf{R}t_3 a, w_1 t_2\ [13, \to]$$
$$(16)\ \neg\neg k, w_1 t_2\ [14, \text{DR1}] \qquad (17)\ a, w_1 t_3\ [15, \mathsf{R}t]$$
$$(18)\ k, w_1 t_2\ [16, \neg\neg] \qquad (19)\ *\ [8, 17]$$
$$(20)\ k, w_0 t_2\ [9, 18, T\text{-}BT]$$
$$(21)\ *\ [10, 20]$$

F3 and F3b are independent of each other (in **T** and weaker systems). But if this is true and we assume that the contrary-to-duty obligation to apologise, given that you do not keep your promise, is still in force at t_2, should not N3 be symbolised by a conjunction of F3 and F3b or something similar? It might be interesting to note that we can do this without affecting the main results in this section. $\{\text{F1}, \text{F2}, \text{F3}, \text{F3b}, \text{F4}\}$ is, for instance, consistent, non-redundant, etc. (in **T** and weaker systems). So, we can use such an alternative formalisation of N3 instead of F3. Furthermore, note that the symbolisation of N2 can be modified in a similar way.

Reason 12 (We can avoid the so-called dilemma of commitment and detachment in temporal alethic dyadic deontic logic). (Factual) Detachment is an inference pattern that allows us to infer or detach an unconditional obligation from a conditional obligation and this conditional obligation's condition. For example, if detachment holds for the conditional (contrary-to-duty) obligation that you should apologise if you do not keep your promise (if detachment is possible), then we can derive the unconditional obligation that you should apologise given that you do not keep your promise.

According to the so-called dilemma of commitment and detachment [52, p. 263]: (1) Detachment should be possible, for we cannot take seriously a conditional obligation if it cannot, by way of detachment, lead to an unconditional obligation. (2) Detachment should not be possible, for if detachment is possible, the following kind of situation would be inconsistent — A, it ought to be the case that B given that A; and C, it ought to be the case that not-B given C. But, such a situation is not necessarily inconsistent.

This dilemma seems to be a problem for solutions to the contrary-to-duty paradox in pure dyadic deontic logic. In pure dyadic deontic logic, we cannot derive the unconditional obligation that it is obligatory that A ($\mathbf{O}A$) from the dyadic obligation that it is obligatory that A given B ($\mathbf{O}[B]A$) and B. But how can we then take such conditional obligations seriously? Be that as it may, in temporal alethic dyadic deontic logic we can solve this dilemma. In Section 5, we saw that we cannot always detach an unconditional obligation from a conditional obligation and its condition, but that we can detach the unconditional obligation $\mathbf{O}B$ (in **STADDL**) from $\mathbf{O}[A]B$ and A if A is non-future or historically necessary. This seems to give us exactly the correct answer to our current puzzle. Detachment holds, but it does not hold unrestrictedly. We saw above that $\mathsf{R}t_2\mathbf{O}[\top]\mathsf{R}t_3a$, but not $\mathsf{R}t_1\mathbf{O}[\top]\mathsf{R}t_3a$, is derivable from $\mathsf{R}t_2\neg k$ and $\mathsf{R}t_2\mathbf{O}[\mathsf{R}t_2\neg k]\mathsf{R}t_3a$ in certain systems. In other words, we can detach the former sentence, but not the latter. From this it does not follow that a set of the following kind must be inconsistent: $\{A, \mathbf{O}[A]B, C, \mathbf{O}[C]\neg B\}$; this seems to be exactly what we want.

This concludes our discussion of the reasons for the solution to the contrary-to-duty paradox discussed in the present paper. All other solutions that have been suggested in the literature so far seem to lack at least some of the features mentioned above. This makes the symbolisation of N-CTD in temporal alethic dyadic deontic logic very attractive.

7. Soundness and completeness theorems

The soundness and completeness proofs in this section are modifications and extensions of proofs found in [43] and [44].

Let $S =$ "$aA_1...A_idB_1...B_j\alpha C_1...C_kadD_1...D_ltE_1...E_madtF_1...F_n$" be a temporal alethic dyadic deontic tableau system as defined above. Then we shall say that the class of models, $\mathcal{M}$, corresponds to S just in case $\mathcal{M} = \mathcal{M}(C\text{-}A_1, ..., C\text{-}A_i, C\text{-}B_1, ..., C\text{-}B_j, C\text{-}C_1, ..., C\text{-}C_k, C\text{-}D_1, ..., C\text{-}D_l, C\text{-}E_1, ..., C\text{-}E_m, C\text{-}F_1, ..., C\text{-}F_n)$.

S is strongly sound with respect to $\mathcal{M}$ iff for all Γ and A: $\Gamma \vdash_S A$ (i.e., A is derivable from Γ in S) entails $\mathcal{M}, \Gamma \Vdash A$ (i.e., A follows from Γ in $\mathcal{M}$). S is strongly complete with respect to $\mathcal{M}$ just in case $\mathcal{M}, \Gamma \Vdash A$ entails $\Gamma \vdash_S A$.

7.1. Soundness theorems

Let M be any (ordinary) model and b any branch of a tableau. Then b is satisfiable in M iff there is a function f from $w_0, w_1, w_2, \ldots$ to

W and a function g from $t_0, t_1, t_2, \ldots$ to T such that (i) A is true in $f(w_i)$ at $g(t_j)$ in M, for every node $A, w_i t_j$ on b, (ii) if $rw_i w_j t_k$ is on b, then $Rf(w_i)f(w_j)g(t_k)$ in M, (iii) if $s_A w_i w_j t_k$ is on b, then $S_A f(w_i)f(w_j)g(t_k)$ in M, (iv) if $t_i < t_j$ is on b, then $g(t_i) < g(t_j)$ in M, (v) if $w_i = w_j$ is on b, then $f(w_i) = f(w_j)$ in M, (vi) if $t_i = t_j$ is on b, then $g(t_i) = g(t_j)$ in M. If these conditions are fulfilled, we say that f and g show that b is satisfiable in M.

Lemma 10 (Soundness Lemma). *Let b be any branch of a tableau and M be any temporal alethic dyadic deontic model. If b is satisfiable in M and a tableau rule is applied to it, then it produces at least one extension, b', of b such that b' is satisfiable in M.*

Proof. The proof proceeds by going through all the tableau rules. Here are some steps to illustrate the method.

For *T-DdT′*: Assume that $s_A w_i w_j t_l$ is on b, and that we apply $T-DdT'$ to give an extended branch of b including $s_A w_j w_j t_l$. Since b is satisfiable in M, $S_A f(w_i)f(w_j)g(t_l)$. Accordingly, $S_A f(w_j)f(w_j)g(t_l)$, since M satisfies the condition *C-DdT′*.

For *T-Dα2*: Suppose that $s_A w_i w_j t_l$ and $B, w_j t_l$ are on b, and that we apply *T-Dα2* to give an extended branch of b containing $s_{A \wedge B} w_i w_j t_l$. Since b is satisfiable in M, $S_A f(w_i)f(w_j)g(t_l)$ and B is true in $f(w_j)$ at $g(t_l)$. Accordingly, $S_{A \wedge B} f(w_i)f(w_j)g(t_l)$, since M satisfies *C-Dα2*.

For *T-Dα4*: Suppose that $s_A w_i w_j t_l$, $B, w_j t_l$, and $s_{A \wedge B} w_i w_k t_l$ are on b, and that we apply *T-Dα4* to give an extended branch of b containing $s_A w_i w_k t_l$ and $B, w_k t_l$. Since b is satisfiable in M, $S_A f(w_i)f(w_j)g(t_l)$, $S_{A \wedge B} f(w_i)f(w_k)g(t_l)$ and B is true in $f(w_j)$ at $g(t_l)$. Accordingly, $S_A f(w_i)f(w_k)g(t_l)$ and B is true in $f(w_k)$ at $g(t_l)$, since M satisfies the condition *C-Dα4*. ⊣

Theorem 11 (Soundness Theorem I). *Let S be any of the tableau systems discussed in this essay and let $\mathcal{M}$ be the class of models that corresponds to S. Then S is strongly sound with respect to $\mathcal{M}$.*

Proof. Once Soundness Lemma is established, the proof is an easy modification of similar proofs found e.g. in [38, 43, 44, 46]. ⊣

From the above theorem and Theorem 1 we obtain:

Theorem 12 (Soundness Theorem II). 1. *α012 is sound with respect to the class of all supplemented models that satisfies C-Dγ0.*

2. *α0123 is sound with respect to the class of all supplemented models that satisfies C-Dγ0 and C-Dδ3.*
3. *α01234 is sound with respect to the class of all supplemented models that satisfies C-Dγ0, C-Dδ3, and C-Dδ4.*
4. *a45d4′5′α012ad45 is sound with respect to the class of all supplemented models that satisfies C-Dγ0, C-a4, C-a5 and C-Dδ6.*
5. *d7α012 is sound with respect to the class of all supplemented models that satisfies C-Dγ0, and C-Dδ7.* (Soundness results for other combinations of these conditions are easily obtained.)
6. *The tableau system* **T** (see Section 4.3) *is sound with respect to the class of all* **T***-models* (see Section 3.3).

7.2. Completeness theorems

Let b be an open complete branch of a tableau and let I be the set of numbers on b immediately preceded by a "t". We shall say that $i \rightleftharpoons j$ just in case $i = j$, or "$t_i = t_j$" or "$t_j = t_i$" occurs on b. $\rightleftharpoons$ is an equivalence relation and $[i]$ is the equivalence class of i. Furthermore, let K be the set of numbers on b immediately preceded by a "w". We shall say that $k \approx l$ just in case $k = l$, or "$w_k = w_l$" or "$w_l = w_k$" occurs on b. $\approx$ is an equivalence relation and $[k]$ is the equivalence class of k.

Definition 13 (Induced model). The temporal alethic dyadic deontic model, $M = \langle W, T, < R, \{S_A : A \in L\}, V, v\rangle$, induced by b is defined as follows. $W = \{\omega_{[k]} : k \in K\}$, $T = \{\tau_{[i]} : i \in I\}$, $\tau_{[i]} < \tau_{[j]}$ iff $t_i < t_j$ occurs on b, $R\omega_{[i]}\omega_{[j]}\tau_{[k]}$ iff $rw_iw_jt_k$ occurs on b, $S_A\omega_{[i]}\omega_{[j]}\tau_{[k]}$ iff $s_Aw_iw_jt_k$ occurs on b. If p, w_it_j occurs on b, then p is true in $\omega_{[i]}$ at $\tau_{[j]}$ (i.e., then $\langle\omega_{[i]}, \tau_{[j]}\rangle \in V(p)$); if $\neg p, w_it_j$ occurs on b, then p is false in $\omega_{[i]}$ at $\tau_{[j]}$ (i.e., then it is not the case that $\langle\omega_{[i]}, \tau_{[j]}\rangle \in V(p)$). If t_i occurs on b, then $v(t_i) = \tau_{[i]}$.

If our tableau system neither includes *T-FC*, *T-PC* (see [44]) nor *T-C* (see [46]), $\rightleftharpoons$ is reduced to identity and $[i] = i$. Hence, in such systems, we may take T to be $\{\tau_i : t_i$ occurs on $b\}$ and dispense with the equivalence classes. Likewise, if our tableau system does not include *T-Dd7*, $\approx$ is reduced to identity and $[k] = k$. Accordingly, in such systems, we may take W to be $\{\omega_i : w_i$ occurs on $b\}$ and dispense with the equivalence classes.

Lemma 14 (Completeness Lemma). *Let b be an open branch in a complete tableau and let M be a temporal alethic dyadic deontic model induced by b. Then*:

(i) *A is true in $\omega_{[i]}$ at $\tau_{[j]}$, if $A, w_i t_j$ is on b,*
(ii) *A is false in $\omega_{[i]}$ at $\tau_{[j]}$, if $\neg A, w_i t_j$ is on b.*

Proof. The proof is by induction on the complexity of A.

We will only go through one example to illustrate the method.

$A = \mathbf{O}[B]C$. Suppose $A, w_i t_k$ occurs on b, i.e., $\mathbf{O}[B]C, w_i t_k$ is on b. Since b is complete ($\mathbf{O}$) has been applied to $\mathbf{O}[B]C, w_i t_k$. Thus, for all w_j on b such that $s_B w_i w_j t_k$, $C, w_j t_k$ is on b. By the induction hypothesis, for all $\omega_{[j]}$ such that $S_B \omega_{[i]} \omega_{[j]} \tau_{[k]}$, C is true in $\omega_{[j]}$ at $\tau_{[k]}$. Hence, $\mathbf{O}[B]C$ is true in $\omega_{[i]}$ at $\tau_{[k]}$. Suppose that $\neg A, w_i t_k$ occurs on b, i.e., $\neg\mathbf{O}[B]C, w_i t_k$ is on b. Then $\mathbf{P}[B]\neg C, w_i t_k$ is on b (by $\neg\mathbf{O}$). For b is complete. Furthermore, since b is complete ($\mathbf{P}$) has been applied to $\mathbf{P}[B]\neg C, w_i t_k$. Thus, for some w_j, $s_B w_i w_j t_k$ and $\neg C, w_j t_k$ are on b. By the induction hypothesis, $S_B \omega_{[i]} \omega_{[j]} \tau_{[k]}$ and C is false in $\omega_{[j]}$ at $\tau_{[k]}$. Hence, $\mathbf{O}[B]C$ is false in $\omega_{[i]}$ at $\tau_{[k]}$. ⊣

Theorem 15 (Completeness Theorem). *Let S be any of the tableau systems discussed in this essay, not including T-Dα0, and let $\mathcal{M}$ be the class of models that corresponds to S. Then S is strongly complete with respect to $\mathcal{M}$.*[12]

Proof. The proof is a modification of similar proofs in [43, 44] (see also [38, 46]).

First we show that the weakest system is complete. Then we have to check that the model induced by the open branch, b, is of the right kind in every case. We only consider some cases to illustrate the method.

For *C-DdT′*: Suppose that $S_A \omega_{[i]} \omega_{[j]} \tau_{[k]}$. Then $s_A w_i w_j t_k$ occurs on b [by the definition of an induced model]. Since the tableau is complete, *T-DdT′* has been applied and $s_A w_j w_j t_k$ occurs on b. Hence, $S_A \omega_{[j]} \omega_{[j]} \tau_{[k]}$, as required (by the definition of an induced model).

For *C-DMO*: Suppose that $S_A \omega_{[i]} \omega_{[j]} \tau_{[k]}$. Then $s_A w_i w_j t_k$ occurs on b (by the definition of an induced model). Since b is complete, $(T-DMO)$

[12] We have excluded systems that contain *T-Dα0* because we have not been able to prove that these systems are complete. It should be noted that this does not mean that we have proven that they are incomplete. Our conjecture is that *all* systems in this paper are complete with respect to their (ordinary) semantics (including all systems that contain *T-Dα0*). Hopefully, someone will be able to prove this in the future or refute this conjecture.

has been applied and $rw_iw_jt_k$ occurs on b. Accordingly, $R\omega_{[i]}\omega_{[j]}\tau_{[k]}$, as required (by the definition of an induced model).

For *C-Dα3*: Assume that $R\omega_{[i]}\omega_{[j]}\tau_{[l]}$ and that A is true in $\omega_{[j]}$ at $\tau_{[l]}$. Then $rw_iw_jt_l$ [by the definition of an induced model]. Since the tableau is complete *CUT* has been applied and either A, w_jt_l or $\neg A, w_jt_l$ is on b. Suppose that $\neg A, w_jt_l$ is on b. Then A is false in $\omega_{[j]}$ at $\tau_{[l]}$ (by Completeness Lemma). But this is absurd. Hence, A, w_jt_l is on b. Since the tableau is complete *T-Dα3* has been applied. So, $s_Aw_iw_kt_l$ is on b. Accordingly, $S_A\omega_{[i]}\omega_{[k]}\tau_{[l]}$, as required (by the definition of an induced model).

For *C-Dα4*: Suppose that $S_A\omega_{[i]}\omega_{[j]}\tau_{[l]}$, $S_{A\wedge B}\omega_{[i]}\omega_{[k]}\tau_{[l]}$ and that B is true in $\omega_{[j]}$ at $\tau_{[l]}$. Then $s_Aw_iw_jt_l$ and $s_{A\wedge B}w_iw_kt_l$ occur on b (by the definition of an induced model). Since the tableau is complete *CUT* has been applied and either B, w_jt_l or $\neg B, w_jt_l$ is on b. Assume that $\neg B, w_jt_l$ is on b. Then B is false in $\omega_{[j]}$ at $\tau_{[l]}$ (by Completeness Lemma). But this is absurd. So, B, w_jt_l is on b. Since the tableau is complete *T-Dα4* has been applied and $s_Aw_iw_kt_l$ and B, w_kt_l occur on b. It follows that $S_A\omega_{[i]}\omega_{[k]}\tau_{[l]}$ and that B is true in $\omega_{[k]}$ at $\tau_{[l]}$, as required (by the definition of an induced model and Completeness Lemma). ⊣

PROBLEM. Whether any other system discussed in this paper is complete is left as an open question.

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References

[1] Alchourrón, C. E., and E. Bulygin, "Normative knowledge and truth", pages 25–45 in J. J. E. Garcia *et al.* (eds.), *Philosophical Analysis in Latin America*, D. Reidel Publishing Company, 1984.

[2] Anderson, A. R. (1956), "The formal analysis of normative systems", pages 147–213 in N. Rescher (ed.), *The Logic of Decision and Action*, Pittsburg: University of Pittsburg Press, 1967.

[3] Bailhache, P., "Les normes dans le temps et sur l'action. Essai de logique déontique", Université de Nantes, 1986.

[4] Bailhache, P., "Essai de logique déontique", Paris, Librarie Philosophique, Vrin, Collection Mathesis, 1991.

[5] Bailhache, P., "The deontic branching time: Two related conceptions", *Logique et Analyse* 36 (1993): 159–175.

[6] Bailhache, P., "Canonical models for temporal deontic logic", *Logique et Analyse* 149 (1995): 3–21.

[7] Bartha, P., "Moral preference, contrary-to-duty obligation and defeasible oughts", pages 93–108 in [37].

[8] Belnap, N., M. Perloff, and M. Xu, *Facing the Future: Agents and Choices in Our Indeterminist World*, Oxford, Oxford University Press, 2001.

[9] Brown, M. A., "Agents with changing and conflicting commitments: A preliminary study", page 109–125 in [37].

[10] Brown, M. A., "Conditional obligation and positive permission for agents in time", *Nordic Journal of Philosophical Logic* 5, 2 (2000): 83–112.

[11] Brown, M. A., "Rich deontic logic: A preliminary study", *Journal of Applied Logic* 2 (2004): 19–37.

[12] Burgess, J. P., "Basic Tense Logic", pages 89–133 and 1–42 in D. M. Gabbay and F. Guenthner (eds.), *Handbook of Philosophical Logic*, Vol. 2 and Vol. 7 (2nd Edition), Kluwer Academic Publishers, 1984 and 2002, respectively. DOI: 10.1007/978-94-009-6259-0_2 and 10.1007/978-94-017-0462-5_1

[13] Carmo, J., and A. J. I. Jones, "Deontic logic and contrary-to-duties", pages 265–343 in D. M. Gabbay and F. Guenthner (eds.), *Handbook of Philosophical Logic*, 2nd Edition, Vol. 8, Kluwer Academic Publishers, 2002. DOI: 10.1007/978-94-010-0387-2_4

[14] Chellas, B. F., *The Logical Form of Imperatives*, Stanford, Perry Lane Press, 1969.

[15] Chellas, B. F., *Modal Logic: An Introduction*, Cambridge, Cambridge University Press, 1980.

[16] Chisholm, R. M., "Contrary-to-duty imperatives and deontic logic", *Analysis* 24, 2 (1963): 33–36. DOI: 10.1093/analys/24.2.33

[17] Ciuni, R., and A. Zanardo, "Completeness of a branching-time logic with possible choices", *Studia Logica* 96, 3 (2010): 393–420. DOI: 10.1007/s11225-010-9291-1

[18] Cox, Azizah Al-Hibri, *Deontic Logic: A Comprehensive Appraisal and a New Proposal*, University Press of America, 1978.

[19] D'Agostino, M., D. M. Gabbay, R. Hähnle, and J. Posegga (eds.), *Handbook of Tableau Methods*, Dordrecht, Kluwer Academic Publishers, 1999. DOI: 10.1007/978-94-017-1754-0

[20] Danielsson, S., *Preference and Obligation: Studies in the Logic of Ethics*, Uppsala: Filosofiska föreningen, 1968.

[21] DiMaio, M. C., and A. Zanardo, "A Gabbay-rule free axiomatization of T × W validity", *Journal of Philosophical Logic* 27 (1998): 435–487.

[22] Feldman, F., *Doing the Best We Can: An Essay in Informal Deontic Logic*, Dordrecht: D. Reidel Publishing Company, 1986.

[23] Feldman, F., "A simpler solution to the paradoxes of deontic logic", *Philosophical Perspectives* 4 (1990): 309–341. DOI: 10.2307/2214197

[24] Fitting, M., *Proof Methods for Modal and Intuitionistic Logics*, Dordrecht, D. Reidel Publishing Company, 1983.DOI: 10.1007/978-94-017-2794-5

[25] Fitting, M., and R. L. Mendelsohn, *First-Order Modal Logic*, Kluwer Academic Publishers, 1998. DOI: 10.1007/978-94-011-5292-1

[26] Gabbay, D. M., and F. Guenthner (eds.), *Handbook of Philosophical Logic*, 2nd Edition, Vol. 3, Dordrecht, Kluwer Academic Publishers, 2001. DOI: 10.1007/978-94-017-0454-0

[27] Gabbay, D. M., A. Kurucz, F. Wolter, M. Zakharyaschev, *Many-Dimensional Modal Logics: Theory and Applications*, Amsterdam, Elsevier, 2003.

[28] Hansson, B., "An analysis of some deontic logics", *Noûs* 3 (1969): 373–398. DOI: 10.2307/2214372

[29] Hilpinen, R. (ed.), *New Studies in Deontic Logic: Norms, Actions, and the Foundation of Ethics*, Dordrecht, D. Reidel Publishing Company, 1981. DOI: 10.1007/978-94-009-8484-4

[30] Horty, J. F., *Agency and Deontic Logic*, Oxford, Oxford University Press, 2001. DOI: 10.1093/0195134613.001.0001

[31] Jeffrey, R. C., *Formal Logic: Its Scope and Limits*, McGraw-Hill, New York, 1967.

[32] Kracht, M., *Tools and Techniques in Modal Logic*, Amsterdam, Elsevier, 1999.

[33] Lenk, H., *Normenlogik*, Pullach bei München: Verlag Dokumentation, 1974.

[34] Lewis, D., *Counterfactuals*, Oxford: Basil Blackwell, 1973.

[35] Lewis, D., "Semantic analysis for dyadic deontic logic", pages 1–14 in S. Stenlund (ed.), *Logical Theory and Semantical Analysis*, D. Reidel Publishing Company, Dordrecht, 1974.

[36] Loewer, B., and M. Belzer, "Dyadic deontic detachment", *Synthese* 54, 2 (1983): 295–318. DOI: 10.1007/BF00869396

[37] McNamara, P., and H. Prakken (eds.), *Norms, Logics and Information Systems: New Studies in Deontic Logic and Computer Science*, Amsterdam, IOS Press, 1999.

[38] Priest, G., *An Introduction to Non-Classical Logic*, Cambridge, Cambridge University Press, 2008. DOI: 10.1017/CBO9780511801174

[39] Prior, A., "The paradoxes of derived obligation", *Mind* 63, 249 (1954): 64–65. DOI: 10.1093/mind/LXIII.249.64

[40] Prior, A., *Past, Present and Future*, Oxford, Clarendon, 1967. DOI: 10.1093/acprof:oso/9780198243113.001.0001

[41] Rescher, N., "An axiom system for deontic logic", *Philosophical Studies* 9, 1–2 (1958): 24–30. DOI: 10.1007/BF00797870

[42] Rescher, N., and A. Urquhart, *Temporal Logic*, Wien, Springer-Verlag, 1971. DOI: 10.1007/978-3-7091-7664-1

[43] Rönnedal, D., "Dyadic deontic logic and semantic tableaux", *Logic and Logical Philosophy* 18, 3–4 (2009): 221–252. DOI: 10.12775/LLP.2009.011

[44] Rönnedal, D., "Temporal alethic-deontic logic and semantic tableaux", *Journal of Applied Logic* 10, 3 (2012): 219–237. DOI: 10.1016/j.jal.2012.03.002

[45] Rönnedal, D., "Extensions of deontic logic: An investigation into some multi-modal systems", Department of Philosophy, Stockholm University, 2012.

[46] Rönnedal, D., "Quantified temporal alethic deontic logic", *Logic and Logical Philosophy* 24, 1 (2015): 19–59. DOI: 10.12775/LLP.2014.016

[47] Smullyan, R. M., *First-Order Logic*, Heidelberg, Springer-Verlag, 1968. DOI: 10.1007/978-3-642-86718-7

[48] Thomason, R., "Deontic logic as founded on tense logic", pages 165–176, Chapter 7, in [29]. DOI: 10.1007/978-94-009-8484-4_7

[49] Thomason, R., "Deontic logic and the role of freedom in moral deliberation", pages 177–186, Chapter 8, in [29]. DOI: 10.1007/978-94-009-8484-4_8

[50] Thomason, R., "Combinations of tense and modality", pages 135–165 and 205–234 in D. M. Gabbay and F. Guenthner (eds.), *Handbook of Philosophical Logic*, Vol. 2 and Vol. 7 (2nd Edition), 1984 and 2002, respectively. DOI: 10.1007/978-94-009-6259-0_3 and 10.1007/978-94-017-0462-5_3

[51] van Eck, J., "A system of temporally relative modal and deontic predicate logic and its philosophical applications", Department of Philosophy, University of Groningen, The Netherlands, 1981.

[52] van Eck, J., "A system of temporally relative modal and deontic predicate logic and its philosophical applications", *Logique et Analyse* 25, 99 (1982): 249–290.

[53] van Eck, J., "A system of temporally relative modal and deontic predicate logic and its philosophical application", *Logique et Analyse* 25, 100 (1982): 339–381.

[54] van Fraassen, B. C., "The logic of conditional obligation", *Journal of Philosophical Logic* 1, 3–4 (1972): 417–438. DOI: 10.1007/BF00255570

[55] van Fraassen, B. C., "Values and the heart's command", *The Journal of Philosophy* LXX (1973): 5–19. DOI: 10.2307/2024762

[56] von Kutschera, F., "Normative Präferenzen und bedingte Gebote", pages 137–165 in [33].

[57] von Kutschera, F., "T × W completeness", *Journal of Philosophical Logic* 26 (1997): 241–250.

[58] von Wright, G. H., "A new system of deontic logic", *Danish Yearbook of Philosophy* 1 (1964): 173–182. DOI: 10.1007/978-94-010-3146-2_4

[59] Wölfl, S., "Combinations of tense and modality for predicate logic", *Journal of Philosophical Logic* 28 (1999): 371–398.

[60] Zanardo, A., "Branching-time logic with quantification over branches: The point of view of modal logic", *The Journal of Symbolic Logic* 61, 1 (1996): 1–39. DOI 10.2307/2275595

[61] Åqvist, L., "Revised foundations for imperative-epistemic and interrogative logic", *Theoria* 37, 1 (1971): 33–73. DOI: 10.1111/j.1755-2567.1971.tb00060.x

[62] Åqvist, L., "Modal logic with subjunctive conditionals and dispositional predicates", *Journal of Philosophical Logic* 2, 1 (1973): 1–76. DOI: 10.1007/BF02115609

[63] Åqvist, L., "Deontic Logic", pages 605–714 and 147–264 in D. M. Gabbay and F. Guenthner (eds.), *Handbook of Philosophical Logic*, Vol. 2 and Vol. 8 (2nd Edition), 1984 and 2002, respectively. DOI: 10.1007/978-94-009-6259-0_11 and 10.1007/978-94-010-0387-2_3

[64] Åqvist, L., *Introduction to Deontic Logic and the Theory of Normative Systems*, Naples, Bibliopolis, 1987.

[65] Åqvist, L., "The logic of historical necessity as founded on two-dimensional modal tense logic", *Journal of Philosophical Logic* 28, 4 (1999) 329–369. DOI: 10.1023/A:1004425728816

[66] Åqvist, L., "Conditionality and branching time in deontic logic: Further remarks on the Alchourrón and Bulygin (1983) Example", pages 13–37 in K. Segerberg and R. Sliwinski (eds.), *Logic, Law, Morality: Thirteen essays in Practical Philosophy in Honour of Lennart Åqvist*, Uppsala philosophical studies 51, Uppsala: Uppsala University, 2003.

[67] Åqvist, L., "Combinations of tense and deontic modality: On the Rt approach to temporal logic with historical necessity and conditional obligation", *Journal of Applied Logic* 3 (2005): 421–460.

[68] Åqvist, L., and J. Hoepelman, "Some theorems about a "tree" system of deontic tense logic", pages 187–221 in [29]. 10.1007/978-94-009-8484-4_9

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