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A COMPOSITIONAL SEMANTICS FOR 'EVEN IF' CONDITIONALS

Abstract. This paper[1] presents the first possible world semantics for concessive conditionals (i.e., *even if A, C* conditionals) constructed in a compositional way. First, the meaning of *if* is formalized through a semantics that builds on the proposal given by Stalnaker [1968]. A major difference from Stalnaker's approach is that irrelevant conditionals (i.e., conditionals where the antecedent and the consequent have no connection) are false in this new setting. Second, the meaning of *even* is analyzed through a formal semantics based on the notion of *scale*. This analysis overcomes the problems arising in standard approaches, in which *even* is analyzed with the help of pragmatic presuppositions. Finally, the two particles are combined in order to provide a formal analysis of *even if*. This theory predicts the major phenomena concerning the behavior of concessive conditionals and without any call to pragmatic explanations. More generally, this approach creates the possibility of a compositional analysis of other conditionals such as *if then* or *only if* forms.

Keywords: Conditional logic; even; if; even if

1. Introduction

Consider the following reasoning:

(1.1) If it's windy, Mary goes to the beach. It is not windy.
(1.2) Therefore, Mary goes to the beach.

The conclusion (1.2) is invalid. Indeed, the conditional in (1.1) says nothing about what Mary's plan will be in the event that there is no

[1] Some of the ideas of this paper are issued from Vidal [2012].

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wind. For instance, this sentence could be uttered in a context in which Mary is a professional kite surfer who only trains outside if the weather conditions are favorable for her to practice her sport. We do not know what she will do when the wind speed is too low. Hence, the inference (1.1)–(1.2) is not supported. Now, consider the same reasoning with an *even if* conditional:

(1.3) Even if it's windy, Mary goes to the beach. It is not windy.
(1.4) Therefore, Mary goes to the beach.

The conclusion (1.4) is now correct. The conditional in sentence (1.3), which could be asserted, for instance, in a context in which Mary is a sunbathing aficionada, simply means two things. First, Mary will go to the beach whether or not it is windy. Second, she nonetheless prefers when there is no wind, for instance, because the temperature is higher. Hence, knowing the second premise in (1.3), that "it is not windy", we can safely conclude (1.4) "Mary goes to the beach". Notice that when we say that the inference from (1.3) to (1.4) is valid, we do not mean that it would be so forever and ever. Indeed, faced with new information, we might revise our judgment. For instance, if we learn soon after that Mary broke her leg this morning, we will conclude that she is more likely to be at the hospital than at the beach. Like many researchers in philosophical logic or in artificial intelligence, conditionals, I would argue, are non-monotonic and the conclusion obtained can always be revised when faced with new information. This non-monotonicity of the inference from (1.3) to (1.4) is shared with other inferences like the Modus Ponens (*If A, C. A. Therefore C*) and does not prevent them from being considered valid.

The reasoning from (1.3) to (1.4) can be schematized in the following way: *Even if A, C. Not A. Therefore C.* I call it the *Modus Ponens with Negated Antecedent* (MPNA hereafter) because like the Modus Ponens, the consequent can be derived, but beginning this time with the negation of the antecedent. Psychological experiments indirectly support this inference when an 'even if' conditional is used. Santamaría et al. [2005] showed that when people read first a conditional of the "even if A, C" form and not of the "if A, C" form, they subsequently read the negated-antecedent conjunction "not A and C" faster. The interpretation given to these results is that the negated-antecedent possibility (that not A leads to C) is part of the meaning of the "even if" conditional. Moreno-Ríos et al. [2008] examined the *denial of the antecedent* (DA), which follows the same reasoning as the MPNA, except that the consequence is the opposite. With an "if" conditional, its form is *if A, C. Not A. Therefore not C.* This is a classic logical fallacy, which is strongly supported.[2] However, the experiment shows that most subjects validate the DA for an *if* conditional and invalidate it for an *even if* conditional. This corroborates the idea that the negated-antecedent possibility is taken into account when an *even if* conditional is interpreted and blocks the possibility to support the DA. Ruiz-Ballesteros and Moreno-Ríos [2013] have further confirmed this result. More generally, all of these experiments show that the schemas of inference people consider valid differ between *if* and *even if* conditionals and that the addition of the particle *even* contributes to the general meaning of a hypothetical sentence. However, the most popular theories of conditionals given by Stalnaker [1968], Lewis [1973] or Adams [1975] do not provide a formal explanation of this phenomenon because they model these two types of conditionals with the same connective.

In this paper, I will try to overcome this limitation by offering a compositional analysis of the meaning of *even if* conditionals. More precisely, I will present a formal semantics for the particles *if* and *even* and show that their combination can explain the discrepancy between the *if A, C* and the *even if A, C* forms. I have chosen to conduct a formal analysis rather than give a pragmatic explanation of meaning for the following reasons. First, a formal analysis has the advantage of providing a more precise mathematical description. Second, such a theory can often be automated on a computer. Hence, given that both are identical in their power to predict as well a in other areas, a formal analysis remains preferable to a pragmatic explanation. Lycan [2001] has already offered a compositional theory of *even if* conditionals but his approach is based on an analysis of *even* as a *universal quantifier*. On the contrary, I am arguing here for a formalization of this particle using the notion of *scale*. Both approaches therefore differ in their explanations and predictions.

The literature on both *if* and *even* is very abundant. A considerably large number of puzzles exist, especially for conditionals. Since I cannot hope to solve them all in a unique paper, my goal is more modest. I would first like to argue for a semantics for both *if* and *even*, each on an independent basis, but without examining all the possible issues

[2] People consider this inference valid in this particular case because they adopt a biconditional interpretation.

attached to the two notions. Afterwards, I shall focus my efforts on the semantics obtained them, which makes it possible to formalize the *even if* conditional. The possibility of this reconstruction provides an important additional argument for the particular semantics each particle offers when taken alone. I hope to make a sufficiently strong case to alleviate concerns about the issues not touched upon here and which will be the subject of subsequent papers.

This article is divided into three main parts, with two additional appendixes. In Section 2, I offer a semantics for the *if* particle. This semantics can be seen as a reinforced version of Stalnaker's approach. The main difference is that this new proposal is able to deal with the problem of non-relevant conditionals. In Section 3, I introduce a formal semantics for the particle *even*, based on the notion of scale. Furthermore, I argue against the pragmatic approaches that are usually adopted in the field and which are based on presuppositions. Section 4 is devoted to the compositional semantics obtained for the *even if* conditional which is based on my preceding analysis of *if* and *even*. In particular, I will show that this approach offers better predictions than concurrent ones, such as those proposed by Pollock, Bennett and Lycan. Finally, Appendix A presents the formal semantics in detail and Appendix B offers the proof of each theorem presented in this paper.

2. If

The first task is to provide a general semantics for the word *if*. A good starting point is to ask under what conditions do we believe a conditional to be true. To answer this question, I shall detail an idealized epistemic process for evaluating such statements. The process I advocate bears some similarities to the Ramsey Test (see [Ramsey, 1990]), which requires that the antecedent be added to one's beliefs before one evaluates the consequent. As Stalnaker [1968] already noticed, we cannot simply add the antecedent to our stock of beliefs, because we sometimes already believe the negation of the antecedent. Stalnaker's solution to this issue was to support a selection function, of which the role was to adjust the inconsistent beliefs related to the antecedent and select the possible world most similar to the actual one. Later refinements of this semantics, such as that made by Chellas [1975], have allowed for the selection of not just one possible world, but a whole set of possible worlds. According to this approach, "if A, C" is true when the closest A-worlds (i.e., the worlds obtained through the selection function) are also C-worlds.

I think that, while Stalnaker's proposal heads in the right direction, it does not go far enough so I opt for a more sophisticated version of his semantics. Indeed, a second issue remains. The selection function offers no adjustment when the antecedent is already believed. In that case, the actual world is automatically selected. This has the unfortunate consequence of validating the following reasoning: in believing "A" and "C", one always believes "if A, C". This problem also exists for Lewis's favorite system VC and for Adam's probabilistic treatment. Nute [1980] calls this schema the Conjuctive Sufficiency (CS) and notices that its validity is problematic. To illustrate this issue, consider the following sentence:

(2.1) If Mickey Mouse has four fingers per hand, Mickey Mouse has big ears.

Despite the truth behind both its components, few people would agree with this conditional because there is no connection between the two. Douven [2015] made similar criticism against the validation of conditionals missing a link between their antecedent and consequent. Furthermore, Douven shows that pragmatic explanations of this phenomenon are at best elusive. For instance, according Grice [1975], it could be argued that the oddness of this conditional comes from the maxim of relevance, whereby each new piece of the conversation must be relevant to the previous one. But Douven remarks that, in the case of our conditional, the lack of relevance stems from the initial lack of connection between the antecedent and the consequent. The maxim of relevance therefore does not explain anything.[3]

To solve this issue from a semantic point of view, I would like to refine Stalnaker's semantics by dividing his selection function into two phases. From an epistemic point of view, I advocate that two moves are made during a correct evaluation of a conditional: an initial INHIBITION stage and a second RECONSTRUCTION stage. The inhibition stage is a phase during which some beliefs are blocked and no longer considered either true or false. In particular, this concerns the antecedent and its negation.

[3] Douven argues that Krzyżanowska et al. [2014] correctly analyze conditionals. However, this theory is only based on proof theory and not on semantics, so I do not see how it would be possible to obtain a compositional meaning of *even if* with this approach.

Thus, after this inhibition, the antecedent is simply considered as being hypothetical. After all, this is what is expected when the *if* locution is used. To illustrate this point, consider the sentence "If the sun is already up when I wake up, I'm late." If I believe the antecedent or its negation prior to the evaluation, neither of them holds after the inhibition stage. This allows us to correctly deal with cases in which the antecedent is already believed. Indeed, it is worth examining some slightly different circumstances in which the antecedent would have been true. Concerning my example, the sun rises at different hours depending on the season. Hence, a range of possibilities must be considered when evaluating the conditional (and thus a range of beliefs must be inhibited). Furthermore, by rejecting the negation of the antecedent and other conflicting beliefs, the process makes it possible to avoid inconsistency. Finally, the inhibition stage blocks all the beliefs that are not useful for the evaluation of the conditional at hand. For instance, the size of one's socks has no link with the previous conditional and is not taken into consideration. The final model of evaluation therefore represents only the facts that are deemed relevant for the issue at hand. In that way, we obtain a model that is easy to handle and which represents a more credible counterpart to epistemic and psychological models than full sets of complete possible worlds. The exact number of beliefs blocked during this phase is variable. In all cases, however, the antecedent is systematic inhibited. In the following discussion, the inhibition of a sentence will always be understood to also include the inhibition of its negation.

The second step of the process of judgment temporally occurs after the first step and is in line with the Ramsey test and Stalnaker's proposal. The various situations for evaluation are reconstructed by adding previously inhibited variations of the antecedent. Indeed, the protasis requires that we ultimately arrive at an epistemic model in which the antecedent is believed. The addition of the antecedent can be accompanied by the addition of other beliefs that explain why we believe it. For instance, if someone considers the protasis "if it rains tomorrow", he or she can imagine clouds blowing in from a particular direction, consequent changes in the air temperature, and so on, all of which will be the subject of additional beliefs. The level of detail of this reconstruction depends on the circumstances and the time that the person takes to consider the possibilities. Again, the process is approximate. But in all cases, the epistemic models will be considered as requiring a step during which the antecedent is believed. The final step is the evaluation, in which we consider whether the consequent is obtained in each of these reconstructed models.

This epistemic process can be translated on a semantic level. A possible world can be used to represent the initial factual beliefs of our idealized agent and semantic functions to model the epistemic phases of inhibition and reconstruction:

(i) The *neutralization* function, during which the antecedent is inhibited, must be successful. The result is a non-empty set of possible worlds.
(ii) The *expansion* function requires that, starting with these inhibited possible worlds, situations in which the antecedent is true are reconstructed.

The careful reader will already have noticed that, in order to inhibit some sentences, we have to consider them as neither true nor false. This justifies the need for partial possible worlds which allow a third truth-value.[4] This *Indeterminate* truth-value also makes it possible to construct abstract models in which superfluous details are eliminated. Nonetheless, the possible worlds considered at the end of the expansion stage will at least be true or false for the salient elements necessary for the judgment, among them the antecedent and the consequent. To enforce this requirement, what I call a *universe of projection* is used. This is a restricted set universe in which all the possible worlds are bivalent relative to the atoms of the conditional. It represents all the considered reconstructions for the antecedent and consequent.

This whole semantics is illustrated for the conditional "if A, C" in Figure 1, where w stands for the starting world of evaluation and the square for the universe of projection.

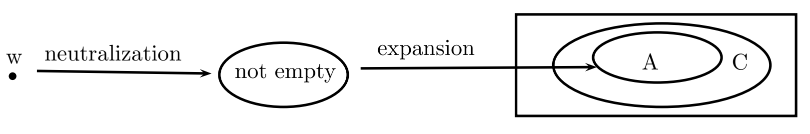

Figure 1. Basic semantics of the conditional

I would now like to turn to the issue of transforming this approach into a formal semantics by presenting the main elements. The exact

[4] The use of three or four truth-values in association with possible worlds is current in modern semantic theories. See for instance [Priest, 2008].

details are provided in Appendix A. Let us start with the neutrality and expansion functions.

In all the following definitions, let W be a set of possible worlds and let L be the formal language (a set of sentences) adopted.

Definition 2.1. A *neutrality function* $\mathfrak{n}$ is a mapping from the product $W \times 2^L$ into 2^W.

Notation. The first argument of $\mathfrak{n}$ will be a subscript and the second argument will be put between parentheses and without brackets, as in this example: $\mathfrak{n}_w(A)$, wherein w is the first argument (a possible world) and $\{A\}$ the second argument (a set of sentences).

Definition 2.2. A *expansion function* $\mathfrak{e}$ is a mapping from the product $2^W \times 2^L$ into 2^W.

Notation. The first argument of $\mathfrak{e}$ will be a subscript and the second argument will be put inside parentheses and without brackets, as in this example: $\mathfrak{e}_{\mathfrak{n}_w(A)}(A)$, wherein $\mathfrak{n}_w(A)$ is the first argument (the result of the neutrality function) and $\{A\}$ the second argument (a set of sentences).

I am continuing with the notion *of universe of projection* which enforces a limit about what is reconsidered when evaluating a conditional. As I have said, by hypothetically considering the antecedent, we envisage different ways in which how it could occur. However, we cannot inspect all the different possibilities, since this number is potentially infinite. Furthermore, some reconstruction of the antecedent and the consequent are too absurd to be taken into account. The exact limit cannot be completely fixed by the semantics, since this process is vague. However, using the notion of universe of projection, we can formally enforce the fact that both the antecedent and the consequent are true or false at the end of the judgment.

Definition 2.3. A world w of W is *bivalent for a set Σ* of sentences of L iff for any atomic sentence α composing any sentence in Σ, $\mathfrak{v}_w(\alpha) = \{0\}$ or $\mathfrak{v}_w(\alpha) = \{1\}$, where $\mathfrak{v}_w$ is a valuation associated with w. (For details see Definition A.2 and Footnote 8.)

Definition 2.4. The *universe of projection* for a conditional C relative to a possible world w is the union of all possible bivalent worlds obtainable by $\mathfrak{e}_{\mathfrak{n}_w(\Sigma')}(\Sigma'')$, where Σ' and Σ'' are any sets of subsentences of C.

DEFINITION 2.5. For any sentence A of L let $[A]^{\mathsf{W}}$ and $[A]^{\mathsf{U}}$ be respectively the set of all possible worlds of W in which A is true, and the set of all possible worlds in the universe of projection U. $[A]^{\mathsf{U}}$ is abbreviated to $[A]$, when U is evident. This set is called the *truth-set* of A (relative to U).

The first advantage of the notion of universe of projection is that by adding few constraints (see Appendix A), we obtain a set-theoretic behavior for the connectives inside conditionals. Negation, disjunction and conjunction become respectively the difference, the union and the intersection of set theory. Since all the worlds in the universe of projection U are bivalent for the atoms of the conditional, considering the negation of the antecedent or consequent A is the same as considering all the possible worlds of U that are not A-worlds. Always by bivalence, the same reasoning applies to show that in this context, the logical disjunction and conjunction behave respectively like the set-theoretic union and intersection. This is an interesting generalization because bivalent modal logics already give a set-theoretic semantics to these connectives when they are used outside intensional conditionals. For instance, if the truth-sets of A and B are respectively $[A]^{\mathsf{W}}$ and $[B]^{\mathsf{W}}$, the truth-set of $(A \wedge B)$ is $[A]^{\mathsf{W}} \cap [B]^{\mathsf{W}}$ and the truth-set of $(A \vee B)$ is $[A]^{\mathsf{W}} \cup [B]^{\mathsf{W}}$. In the same way, the truth-set of $\neg A$ is $\mathsf{W} \setminus [A]^{\mathsf{W}}$. Hence, with the universe of projection, we can postulate exactly the same semantics for these connectives, outside or inside intensional conditionals, except that the set universe is W in the first case and U in the second case.

Let us use the syntactic operator (if $\bullet$)$\bullet$ to represent the word *if* of natural language. To give its truth-conditions, we just have to formalize the inhibition and reconstruction phases of the epistemic process through the neutralization and expansion functions of the formal semantics. This is done in the following definition.

DEFINITION 2.6. Let w be a possible world, λ the lambda abstractor, X and Y some sentences, $\mathfrak{n}$ a neutrality function and $\mathfrak{e}$ an expansion function both governed by the universe of projection U used to evaluate the conditional at hand. Then, the *semantics of if* in w is

$$[if]^w = \lambda X \lambda Y \;\; \mathfrak{n}_w(X) \neq \varnothing \text{ and } \mathfrak{e}_{n_w(X)}(X) \subseteq [Y]$$

From this definition, we obtain the following truth-conditions for the sentence *if* A, C.

THEOREM 2.1 (Truth-conditions of *If*). $\vDash_w$ *(If A) C iff in the associated universe of projection* $\mathfrak{U}$:

(i) $\mathfrak{n}_w(A) \neq \varnothing$,
(ii) $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) \subseteq [C]$.

The neutrality function's main role is to distinguish between conditionals with different sets of inhibited sentences. Indeed, the neutralized set is the first argument of the expansion function. Hence, different sets of neutralized sentences potentially lead to different expansion sets inside different universes of projection and finally to different truth-values. An important consequence of this approach is the non-monotonic behavior of the conditional. Having different contexts of evaluation leads to the invalidation of the *Strengthening of the Antecedent* (SA). In the following formulas and for the sake of clarity, the *if* conditional will be symbolized by the '$\rightharpoonup$' connector.[5]

$$A \rightharpoonup C \nvDash (A \wedge B) \rightharpoonup C \tag{SA}$$

The invalidity of (SA) can be explained by stating that there are more beliefs to inhibit in the consequence than in the premise. The consequence '$(A \wedge B) \rightharpoonup C$' leads to the inhibition of the sentences A and B, while the premise '$A \rightharpoonup C$' leads to the inhibition of the only sentence A. Hence, the neutralization stage leads to two completely contrasting universes of projection. To illustrate this point, consider the classical example of non-monotonic logics: "If Tweety is a bird, Tweety flies, but if Tweety is a bird and a penguin, Tweety does not fly." In the first conditional, only ordinary birds, which are the prototypes of flying animals, are considered. The second conditional adds the important detail that Tweety is a penguin.

The favorite systems of Stalnaker, Lewis and Adams validate the Conjuctive Sufficiency (CS), which states that from a conjunction of facts, a conditional can always deduced. This is not the case in my semantics:

$$A, C \nvDash A \rightharpoonup C \tag{CS}$$

To explain its validity in Stalnaker's semantics, we must remember that the starting possible world of the evaluation models the agent's beliefs. Furthermore, Stalnaker's process requires that the antecedent be added

[5] All theorems are numbered to the right of the line, and their proof is given in Appendix B.

to the agent's initial beliefs before the evaluation of the consequent. Hence, if the antecedent is already believed, its addition does not change the possible worlds selected through the selection function. We remain in the initial world.[6] Furthermore, since the consequent is also part of the initial beliefs, the conditional is valid. According to Lewis's approach, the similarity between possible worlds is the central notion. We must select the most similar worlds to the actual one in which the antecedent is true. Hence, if the antecedent is already true in the actual possible world and because nothing is more similar to the actual world than itself, the evaluation of the consequent is conducted in the actual world. Finally, according to Adam's approach, if we believe A and C, the two of them have a subjective probability of 100%. Thus, their conditional probability is also maximal and we have to believe the conditional.

The first problem with the schema (CS) is that, from two initial beliefs, we will always believe a conditional constructed by taking the first one as the antecedent and the second one as the consequent. But there is another major problem. The order of the initial beliefs has no impact in this derivation. Thus, we can take the second belief to form the antecedent and the first one to form the consequent. Hence, from a conjunction of initial beliefs, we obtain a biconditional. From an epistemic point of view, this means that all our beliefs could be linked by biconditionals. For instance, with a stock of beliefs $\{A_1, A_2, \ldots, A_n\}$, we would have the complex belief $\{A_1$ iff A_2 iff $\ldots$ iff $A_n\}$. The consequence of the schema (CS) is a complete holism of beliefs.

I prefer to adopt a weaker relation between the initial possible world and the worlds selected through the process of judgment, when the antecedent is already true. This relation called (sec) has already been proposed by Nute [1980] and states that, in case the antecedent is true, the initial possible world is among the worlds selected.[7] This corresponds to the process of judgment I advocated. By inhibiting and reconstructing the beliefs, we are not forced to systematically return to the initial possible world. On the contrary, this process makes it possible to consider the variations on the original situation. But this original situation must at least be among the alternatives considered. With this position, we can understand that, when faced with two facts, two persons can postulate

[6] According to Chellas' semantics, in which several possible worlds can be selected, the following condition holds: if $w \in [A]$, then $f_w(A) = \{w\}$.

[7] In our system, if $w \in [A]$ and $\mathfrak{n}_w(A) \neq \varnothing$, then $w \in \mathfrak{e}_{\mathfrak{n}_w(A)}(A)$.

different relations between these facts. Indeed, starting with the same empirical phenomena, they can envisage different variations and obtain different conditional structures.

Let us apply this process to the irrelevant conditional (2.1) *If Mickey Mouse has four fingers per hand, Mickey Mouse has big ears.* In the first step of judgment, we must inhibit some beliefs. Among them, we are forced to inhibit our belief in the antecedent *Mickey Mouse has four fingers per hand.* But this process is also open to the inhibition of further beliefs — for instance, the consequent *Mickey Mouse has big ears.* Thus, one possibility is to inhibit both the antecedent and the consequent. The second step of the process is the reconstruction of situations in which the antecedent is true. We therefore now imagine that Mickey Mouse indeed has four fingers per hand and not five like humans. But since we see no connection between the number of his fingers and the size of his ears, we have no reason to add the belief in the consequent to all of these reconstructions. Admittedly, with the condition (sec), since the two components of the conditionals are initially true, one of these reconstructions will simply be the actual world. However, some other reconstructions could very well have a false consequent. Hence, we will not evaluate the whole conditional as being true, which is why the process of judgment I advocated together with the relation (sec) rule out the irrelevant conditionals that have true antecedents and consequents.

I would now like to turn to the Modus Ponens (MP). Imagine that both the antecedent and the conditional are true in the initial possible world. The question is whether the consequent is obtained. The relation (sec) is sufficiently strong to answer this question positively and validate this inference.

$$A, A \rightharpoonup C \vDash C \tag{MP}$$

Indeed, given that the antecedent is true in the initial possible world w, w is among the worlds selected through the process of inhibition and reconstruction by the relation (sec). Furthermore, by the truth of the conditional, for all these worlds selected and among them w, the consequent is true. Hence, it is also true in the initial possible world w. On the contrary, the Modus Ponens with Negated Antecedent is invalid.

$$\neg A, A \rightharpoonup C \nvDash C \tag{MPNA}$$

In that case, the negation of the antecedent is true in the initial world. Hence, the condition (sec) does not apply, and nothing forces the consequent to be true in this starting world. The invalidity of (MPNA) for *if* conditionals is consistent with the results of the psychological experiments presented in the introduction.

Lastly, with the exception of the schema (CS), this semantics is generally in line with the usual conditional theories. In particular, (CM) and (CC) are valid, contrary to Contraposition (CON) and Transitivity (TRAN). Finally, contrary to Stalnaker's theory and more in line with Lewis's approach, the Conditional Excluded Middle (CEM) is not valid.

$$A \rightharpoonup (B \wedge C) \vDash (A \rightharpoonup B) \wedge (A \rightharpoonup C) \tag{CM}$$

$$(A \rightharpoonup B) \wedge (A \rightharpoonup C) \vDash A \rightharpoonup (B \wedge C) \tag{CC}$$

$$A \rightharpoonup C \nvDash \neg C \rightharpoonup \neg A \tag{CON}$$

$$(A \rightharpoonup B) \wedge (B \rightharpoonup C) \nvDash A \rightharpoonup C \tag{TRAN}$$

$$\nvDash (A \rightharpoonup C) \vee (A \rightharpoonup \neg C) \tag{CEM}$$

3. Even

In this section, I will first look at the standard pragmatic analysis of *even* and discuss the main issues surrounding it. Then, I will present my formal analysis and show how it overcomes the usual difficulties associated with the meaning of *even*.

Karttunen and Peters [1979] offered what is known as the standard analysis of *even*. According to their theory, *even* does not change the truth-conditions of the underlying sentence but adds both an existential presupposition and a scalar presupposition.

(3.1) Even John sleeps.
(3.2) **truth-conditions:** John sleeps.
(3.3) **existential presupposition:** $\exists x(x \neq \textit{john} \wedge x \textit{ sleeps})$.
(3.4) **scalar presupposition:** $\forall x[(x \neq \textit{john} \wedge x \textit{ sleeps}) \Rightarrow \textit{likelihood}(x \textit{ sleeps}) > \textit{likelihood}(\textit{john sleeps})]$.

This theory faces three classical problems. The first one is that the term *likelihood* does not always qualify the scale correctly. It is sometimes better to talk about *expectedness*, *informativeness*, or some other close term [see Bennett, 1982; Kay, 1990; Barker, 1991; Lycan, 2001].

The second issue is that *the existential presupposition is sometimes explicitly negated*, such as in the following example given by Rullmann [1997]:

(3.5) We even invited Bill, although we didn't invite anyone else.

Furthermore, some scales contain exclusive alternatives. Here is an example taken from [Rullmann, 1997] and inspired by [Horn, 1972]:

(3.6) A: Is Claire an assistant professor?
(3.7) B: No, she's even an associate professor.

A professor cannot be both assistant and associate at the same time. This is at odds with the existential presupposition that Claire is an assistant professor. Thus, the presence of an existential presupposition is not always observed.

The third classical issue is that the *presuppositions associated with even are not kept intact under negation*. This contradicts the standard test to qualify a semantic condition as a presupposition, which is precisely its preservation in negative contexts. We can illustrate this problem by again considering (3.1), but this time preceded by the particle *not*.

(3.8) Not even John sleeps.
(3.9) **truth-conditions:** John sleeps.
(3.10) **existential presupposition:** $\exists x(x \neq john \wedge \neg\, x\ \ sleeps))$.
(3.11) **scalar presupposition:** $\forall x[(x \neq john \wedge x\ \ sleeps) \Rightarrow likelihood(x\ \ sleeps) < likelihood(john\ \ sleeps)]$.

The existential presupposition (3.3) becomes now (3.10). In (3.3), x sleeps but in (3.10), x does not sleep. Furthermore, the scalar presuppositions (3.4) and (3.11) differ on the direction of the ordering. Thus, the presuppositions changed. Two different types of answers are given for this issue. The first answer is the *scope theory*, defended by Karttunen and Peters [1979], Lahiri [1998], Guerzoni [2004], Nakanishi [2006]. Contrary to what is suggested by the direct syntactic analysis, *even* always takes a wider scope over negation. The second answer argues that there are two *evens* and is supported by Rooth [1985], Rullmann [1997], Schwarz [2005], and Giannakidou [2007]. The first *even* is the regular one, which appears in contexts without negation. The second *even* is the NPI one, which appears within the scope of negation. This last *even* has different presuppositions, which are the ones exposed in (3.10) and (3.11). It is used in all NPI-licensing environments.

Apart from these classical issues, there is another problem that is directly related to the aim of this paper: the *even if* conditional. Indeed, when *even* focuses on the antecedent of a conditional or on the particle *if*, it is difficult to see what existential presupposition could be conveyed.

This was not an issue when the focus was an individual or an object, but this is an altogether different matter when it is a protasis or a linguistic marker like *if*. An interesting suggestion made by an anonymous referee is that the existential presupposition in that case could concern another condition than the one expressed by the antecedent. However, even by admitting that the presupposition could focus on a proposition, a sentence, or whatever the exact linguistic means used to carry this other condition, the determination of what is exactly focused on is insufficient. Indeed, as I will explain, applying *even* to a conditional creates a scale that takes the antecedent as the first side on the scale and its *negation* as the opposite side. However, nothing in Karttunen and Peter's approach makes it possible to specify that the other condition that must exist is the *negation* of the antecedent. Hence, even by extending the presupposition approach in this way, the theory obtained is not sufficiently precise.

I would like to sum up the issues faced by the presupposition theory. First, the standard test to determine whether a semantic condition is a presupposition is to check its preservation under negation. The word *even* simply fails that test. Second, the term *likelihood* is not always adapted to the scale at hand. Third, the existential presupposition, which is a constitutive part of the theory is sometimes explicitly rejected in the discourse. Finally, this theory is not sufficiently precise when *even* is used with an antecedent of a conditional. All these elements cast high doubts on the credibility of the hypothesis that the additional semantic conditions carried by *even* are presuppositions.

My theory is the following. As usual, I consider semantics as dealing with the part of meaning that is universal and pragmatics with the part of meaning that is dependent on the context. I argue that the universal meaning of *even* is the expression of a scale and that this aspect must be put inside formal semantics. I also agree that what constitutes the precise points on the scale will sometimes vary according to the context and that these details can be left to pragmatics. An advantage of this shift is that the inferential power of sentences containing *even* is now directly computable. I would now like to explain the main elements of my approach. First, the whole meaning of *even* is given inside formal truth-conditions. Second, I do not keep the existential part of the meaning presupposed by Karttunen and Peter, because this aspect is unclear when *even* is associated with an antecedent of a conditional and is sometimes explicitly negated. This leaves us with the idea of *scale* originally theorized by Ducrot [1973] and which is a basic tool in modern linguistics. Fauconnier [1975] argues that the word *even* signals a low position on a scale of value. I take up this idea which was also partly defended by Karttunen and Peters [1979] among others, but I will implement it at the semantic level. To do so, I argue that both the focused element of *even* and its complement are parts of a more general set. As usual in the analysis of natural language, the determination of this complementary set is contextual and is left to pragmatics. The notion of scale also enforces an ordering between its elements. Here, we argue that the belonging of the focused element to the general set is less expected than the belonging of its complement. I note that A is less favorable to C than B by using a non-strict partial order: $[A] \leq_{[\mathrm{C}]} [B]$. This yields to the following formal definition.

DEFINITION 3.1. The meaning of *even* is a function noted $g[even]_{F,G}$ which transforms an input meaning containing a relation of form $[F] \subseteq [X]$ into an output meaning of form:

(i) $[F] \subseteq [X]$
(ii) $[G \setminus F] \subseteq [X]$
(iii) $[F] \leq_{[X]} [G \setminus F]$

These truth-conditions construct a scale with the members $[F]$ and $[G \setminus F]$. $[F]$, and its complement $[G \setminus F]$ entertains the same relation. They are a subset of X. Condition (i) expresses that this relation holds for F and condition (ii) that it holds for the remaining points, that is the set G minus the set F. Finally, condition (iii) states that F is the lowest point on the scale. An important point in this definition is that the input meaning must at least contain the relation $[F] \subseteq [X]$. But it can also contain further semantic conditions. In that case, the remaining input meaning is left untouched.

Let us examine how this semantics applies to the sentence *even John sleeps*. The base sentence *John sleeps* expresses the subset relation $\{john\} \subseteq \{sleep\}$. The scale is constructed with John and other persons. The whole group is denoted by G. Hence, we obtain the following truth-conditions:

(i) $\{john\} \subseteq \{sleep\}$
(ii) $G \setminus \{john\} \subseteq \{sleep\}$
(iii) $\{john\} \leq_{\{sleep\}} G \setminus \{john\}$

Even can be applied to many different groups of words in a sentence. In that case, the change of focus modifies the scale used. The following examples given by Lycan [1991] illustrate the shifts in focus:

(3.12) Even I hit him in the eye yesterday.
(3.13) I even hit him in the eye yesterday.
(3.14) I hit even him in the eye yesterday.
(3.15) I hit him even in the eye yesterday.
(3.16) I hit him in the eye even yesterday.

In these sentences, the word *even* focuses on the element just to its right. To analyze these sentences, we first give the sentence without the particle *even* and with the focused element highlighted in italics, all in order to establish the primitive subset relation. Then we repeat the sentence, this time with the particle *even*, and we give the truth-conditions.

(3.12′) *I* hit him in the eye yesterday.
[I] $\subseteq$ [persons who hit him in the eye yesterday]
(3.12″) Even *I* hit him in the eye yesterday.
[I] $\subseteq$ [persons who hit him in the eye yesterday] and
[Group $\setminus$ I] $\subseteq$ [persons who hit him in the eye yesterday] and
[I] $\leq_{\text{[persons who hit him in the eye yesterday]}}$ [Group $\setminus$ I]

Sentence (3.12) expresses that I was the last person among the whole group who was able to hit him. However, I joined the mob.

(.3.13′) I *hit* him in the eye yesterday.
[hit] $\subseteq$ [actions done by me on him in the eye yesterday]
(3.13″) I even *hit* him in the eye yesterday.
[hit] $\subseteq$ [actions done by me on him in the eye yesterday] and
[actions$\setminus$hit] $\subseteq$ [actions done by me on him in the eye yesterday]
and [hit] $\leq_{\text{[actions done by me on him in the eye yesterday]}}$ [actions $\setminus$ hit]

Sentence (3.13) means that hitting is the last expected action that I could perform. Anyway, I did it.

(3.14′) I hit *him* in the eye yesterday.
[him] $\subseteq$ [persons hit by me in the eye yesterday]
(3.14″) I hit even *him* in the eye yesterday.
[him] $\subseteq$ [persons hit by me in the eye yesterday] and [other *persons* $\setminus$ him] $\subseteq$ [persons hit by me in the eye yesterday] and
[him] $\leq_{\text{[persons hit by me in the eye yesterday]}}$ [other persons $\setminus$ him]

In (3.14), I hit several persons and among them, the most incredible is that I hit HIM (for instance my father).

(3.15′) I hit him *in the eye* yesterday.
[in the eye] $\subseteq$ [place where I hit him yesterday]

(3.15″) I hit him even *in the eye* yesterday.
[in the eye] $\subseteq$ [place where I hit him yesterday] and [other *parts*$\setminus$in the eye] $\subseteq$ [place where I hit him yesterday] and [in the eye] $\leq_{\text{[place where I hit him yesterday]}}$ [other parts $\setminus$ in the eye]

In (3.15), the harmfulness of touching the eye is under consideration. Despite the dangerousness of this action, this is the part of his body that I damaged.

(3.16′) I hit him in the eye *yesterday.*
[yesterday] $\subseteq$ [time when I hit him in the eye]

(3.16″) I hit him in the eye even *yesterday.*
[yesterday] $\subseteq$ [time when I hit him in the eye] and [other days $\setminus$ yesterday] $\subseteq$ [time when I hit him in the eye] and [yesterday] $\leq_{\text{[time when I hit him in the eye]}}$ [other days $\setminus$ yesterday]

Finally, sentence (3.16) means that yesterday was the least expected day for such an action. Perhaps it was his birthday or perhaps it was a day dedicated to peace and non-violence in my religion.

One interesting aspect of this theory is that by simply changing the focus in the underlying sentence, the basic subset relation that represents its central meaning is also modified. Indeed, sentences (3.12′), (3.13′), (3.14′), (3.15′), and (3.16′) are syntactically identical, apart from the focused element signalized by the italicized text. However, their associated subset relations differ. This comes as no surprise if we think about the semantics of the sentence *John sleeps.* If we adopt classical first-order logic, we obtain the relation $\{john\} \subseteq \{sleep\}$. But if we adopt Montague grammar in which an individual is identified with the set of its properties, we obtain a different relation: $\{sleep\} \subseteq \{john\text{'s } properties\}$. Hence, the same sentence can be analyzed by different subset relations. With *even*, the focus indicates the subsumed element and what is the intended subset relation.

The change of scale depending on the focus elicited by the word *even* makes it possible to explain the following important distributional property of the word *even.* In general, there is only one occurrence of this particle in a clause. As soon as two or more *even* are included, the sentence is difficult to unravel, as shown by the following example:

(3.17) Even words give trouble to even linguists.

As Kay [1990] already argued, this sentence is not ungrammatical but sounds odd because we have to interpret two different scales at the same time. The task to determine the exact relations between subsets is cognitively complex, and the context can greatly help us to find the exact interpretation. For instance, sentence (3.17) could be preceded by the following explanation: *Language is full of syntactic constructions that rebut even the specialists.* With this hint, the listener is more likely to understand the meaning of (3.17), which is that words constituting one of the simplest syntactic constructions are difficult to cope with, and this difficulty is faced even by linguists, who are language specialists. Without the context, the listener is simply unable to unravel the exact relations intended by the speaker. That explains why occurrences of two or more *even* in the same clause are very rare in everyday conversation.

I would now like to turn to the task of fully assessing my analysis. To do so, let us review each of the issues facing Karttunen and Peters' approach, which I have already exposed. The first problem concerns the ordering associated with the scale. This order cannot always be qualified as being a relation of *likelihood*. The theory proposed here does not force such a limitation. The ordering relation being defended says that the focused element is considered as less favorable for entertaining the subset relation than the other part on the scale. Hence, this order can be interpreted in different ways such as expressing a relation of *unexpectedness* or *likelihood*, depending on the circumstances. What is really important is that now this order is transferred from the presuppositions to the truth-conditions.

The second issue concerns the exclusive alternatives and the explicitly negated existential presupposition. Both of them can be resolved by carefully considerating the scale and the subset relation at hand. Let us start with the following exclusive alternatives:

(3.18) A: Is Claire an assistant professor?
(3.19) B: No, she's even an associate professor.

The focused element is *associate professor* and its opposite element on the scale is *assistant professor*. What is the relation between the points of this scale and Claire? This cannot be the relation *is* because it is explicitly negated in (3.19). The most straightforward explanation left is that this relation concerns *Claire's competences*. She has the competences of both an assistant professor and an associate professor because she is an associate professor. And this is less expected because associate professor is a higher-ranked position, which is more difficult to attain. More explicitly, the semantics of *she's even an associate professor* in this context is:

(i) [associate professor] $\subseteq$ [Claire's competences]
(ii) [assistant professor] $\subseteq$ [Claire's competences]
(iii) [associate professor] $\leq_{\text{[Claire's competences]}}$ [assistant professor]

The issue is resolved as soon as the right scale is taken into account. But this introduces a kind of flexibility in the choice of the relation expressed. This shows that there is an inherent difficulty in automatically defining the right scale for the example at hand. The same explanation can be adopted to analyze the example which illustrates the explicit denial of the existential presupposition. In sentence (5), *we even invited Bill, although we didn't invite anyone else*, the subset relation cannot be Bill's belonging to a group of persons because the possibility that these other persons were invited is explicitly denied. Hence, the focus of *even* is instead the action of inviting Bill, which is contrasted with a bunch of other actions, like the action of talking to Bill or simply ignoring him. For instance, we can imagine a situation in which the speaker wants to reconcile with Bill but not with some other people. The speaker wants to make the first move and is even ready to invite Bill for lunch. Hence, the two actions of talking to Bill and inviting Bill belong to the set of apologetic actions, and the invitation is unexpected.

Finally, the last issue concerns the behavior of *even* within a negative context. Our solution takes into account the ambiguity of applying the word *not* in front of a sentence. Indeed, the following sentence (20) can receive two main paraphrases that have a different meaning.

(3.20) Not even John sleeps.
(3.21) It is false that even John sleeps.
(3.22) Even John does not sleep.

In (3.21), the whole sentence is negated. Based on the analysis of the truth-conditions, this means that there are three possible reasons for this denial. First, John does not sleep. Second, a part of the other people in the group do not sleep. Third, John is not considered as being less able to sleep than the others.

(i) either [john] $\nsubseteq$ [sleep]
(ii) or [Group \ john] $\nsubseteq$ [sleep]
(iii) or [john] $\nleq_{\text{[sleep]}}$ [Group \ john]

Sentence (3.22) in which the verb is directly negated, is certainly the most common interpretation of (3.20). Here, the truth-conditions are different. It affirms that both John and the other persons in the group do not sleep. Furthermore, it is more unexpected for John not to sleep.

(i) $[\text{john}] \subseteq [\text{not sleep}]$
(ii) $[\text{Group} \setminus \text{john}] \subseteq [\text{not sleep}]$
(iii) $[\text{john}] \leq_{[\text{not sleep}]} [\text{Group} \setminus \text{john}]$

Our answer can be seen as a modification of the scope theory. But there is a difference. Classical scope theory treats *even* as assuming a wider scope over the negation. On the contrary, the version defended here argues that it is the negation that usually assumes a lower scope by being adjoined to the verb (sentence (3.22)) and more rarely focuses on the whole sentence (sentence (3.21)). The first advantage of this choice is to leave room for both interpretations. The second advantage is its application in interrogative contexts.

(3.23) Does not even Mary eat banana?
(3.24) Doesn't even Mary eat banana?
(3.25) # Does even Mary not eat banana?

In an interrogative sentence like (3.23), the auxiliary *does* precedes the negation and the *even* part. Sentence (3.24), which has exactly the same meaning, makes it explicit that the focus of the negation is the verbal part. In sentence (3.25), the *even* part assumes a wider focus on *not*, as argued by classical scope theory. But now, the whole construction is clumsy and difficult to grasp, to say the least. This offers a further argument for saying that a correct semantics of *even* in a negative context can be given as soon as we understand that the negation generally focuses on the verb, despite sometimes being away from the verbal phrase.

By adopting a unique theory of *even*, there is one last question to examine. How can we explain its meaning when it is used as an intensifier of comparatives? Consider this sentence from [Bennett, 1982]:

(3.26) Bill is even taller than John

As noticed by those who support the theory of the two *even*, languages other than English use a different word in this kind of context. For instance, French has the word *encore* for this precise case, which differs from the word *même* used in the preceding cases. Lycan [2001] remarks that (3.26) suggests that Bill and John are tall men, which is not directly supported by the semantics of *even*. This difficulty can be answered by carefully selecting the right scale at hand. We can consider that the scale concerns only the height of tall men (for instance, over 2 meters) whom the speaker knows. This scale only has two points: Bill's height and John's height. Finally, the taller one is Bill (for instance, 2.2 meters versus 2.1 meters for John). The distribution of height among humans is statistically normal (Gauss distribution). Hence, it is more surprising or unexpected to encounter a man of Bill's body size. With this scale, we obtain the following formal analysis of (3.26).

(i) [bill's height] $\subseteq$ [height of tall men]
(ii) [heights of the members of the group $\setminus$ bill's height] $\subseteq$ [height of tall men] = [john's height] $\subseteq$ [height of tall men]
(iii) [bill's height] $\leq_{[\text{height of tall men}]}$ [john's height]

Thus, the semantics proposed is also able to deal with the use of the word *even* as an intensifier of comparatives and we do not need to conduct special analysis for this case.

4. Even If

The main question arising when we want to combine the previous semantics given for *if* and *even* is to determine what scale is at work in an *even if* conditional. The antecedent of a simple *if* conditional establishes a set of possible worlds in which we check if the consequent is obtained. This first set of possible worlds will naturally be the lowest point on the scale and the focus of *even* in the *even if* construction. For the sake of simplicity, let us call this set [if A]. However, what will be the remaining points on the scale? We can already deduce that they will also be possible worlds. But they cannot be all the remaining possible worlds: $W \setminus [\textit{if } A]$. Indeed, the consequence would be that the antecedent is considered less favorable than all the remaining possible worlds. This would also mean that this relation holds for any other possibilities that we can imagine. This requirement is clearly too strong. For instance, let us consider a variation of our example from the introduction, in which Mary could say the following:

(4.1) Even if it's windy, I go to the beach.

In this case, Mary does not like windy weather because it lowers the temperatures and blows the sand, making the conditions less pleasant.

But we can imagine a different possible world in which this is no longer the case. In this situation, Mary is now a kite surfer who needs the wind to train. In that case, the presence of the wind is a favorable factor for Mary's visit to the beach. So the less favorable nature of the antecedent is not preserved in any other circumstances.

The issue of determining the remaining points for the scale of the *even if* construction is easily resolved by considering the notion of *universe of projection* already exposed. As a reminder, the universe of projection is the set of all considered reconstructions of the inhibited sentences obtained at the end of the epistemic process. It restricts the intended hypothetical situations to a set of reasonable alternatives for the context at hand. Hence, the remaining points on the scale will simply be the possible worlds inside the universe of projection minus the possible worlds selected through the antecedent: $\mathsf{U} \setminus [\mathit{if}\ A]$. Notice that no other semantics for conditionals offers an equivalent notion to the universe of projection and that they have difficulty resolving this issue in an elegant way. Admittedly, a similar notion could be added to their semantics. But this addition would be completely ad hoc. On the contrary, in the semantics proposed here, the universe of projection already serves to define the behavior of the conjunction, disjunction and negation and is a constitutive notion of the theory right from the start.

The use of the universe of projection has another advantage. By studying the semantics of *even*, we saw that the general set constituting the scale is not always directly determined and must often be adapted to the context. For instance, for the sentence "no, she's even an associate professor", the scale consists not of Claire's professions, but of Claire's competences. In the case of *even if*, this indetermination disappears because we will always adopt the universe of projection as the general set constituting the scale. Admittedly, this notion is partly vague, because we cannot say for each possible world whether it belongs to it or not. But at least, this notion is clearly defined from the point of view of formal semantics, making the truth-conditions of *even if* totally determined.

There are two ways to think about the combination of *if* and *even* in a conditional. The first one is to consider that *even* applies directly to the word *if*. Thus, we obtain the following construction of the signification of the sentence "even if A, C" in the world w.

$$[\mathit{if}]^w = \lambda X \lambda Y \;\; \mathfrak{n}_w(X) \neq \varnothing \text{ and } \mathfrak{e}_{\mathfrak{n}_w(X)}(X) \subseteq [Y]$$

$$[\mathit{even\ if}]^w = \lambda X \lambda Y \;\; \mathfrak{n}_w(X) \neq \varnothing,\; \mathfrak{e}_{\mathfrak{n}_w(X)}(X) \subseteq [Y],$$

$$
\begin{aligned}
&\mathfrak{e}_{\mathfrak{n}_w(\mathsf{U}\setminus X)}(\mathsf{U}\setminus X)\subseteq[Y] \text{ and}\\
&\mathfrak{e}_{\mathfrak{n}_w(X)}(X)\leq_{[Y]}\mathfrak{e}_{\mathfrak{n}_w(\mathsf{U}\setminus X)}(\mathsf{U}\setminus X)\\
[\textit{even if } A]^w = \lambda Y\ \ &\mathfrak{n}_w(A)\neq\varnothing, \mathfrak{e}_{\mathfrak{n}_w(A)}(A)\subseteq[Y],\\
&\mathfrak{e}_{\mathfrak{n}_w(\mathsf{U}\setminus A)}(\mathsf{U}\setminus A)\subseteq[Y] \text{ and}\\
&\mathfrak{e}_{\mathfrak{n}_w(A)}(A)\leq_{[Y]}\mathfrak{e}_{\mathfrak{n}_w(\mathsf{U}\setminus A)}(\mathsf{U}\setminus A)\\
[\textit{even if } A,\ C]^w = {}&\mathfrak{n}_w(A)\neq\varnothing,\ \mathfrak{e}_{\mathfrak{n}_w(A)}(A)\subseteq[C], \mathfrak{e}_{\mathfrak{n}_w(\mathsf{U}\setminus A)}(\mathsf{U}\setminus A)\subseteq[C]\\
&\text{and } \mathfrak{e}_{\mathfrak{n}_w(A)}(A)\leq_{[C]}\mathfrak{e}_{\mathfrak{n}_w(\mathsf{U}\setminus A)}(\mathsf{U}\setminus A)
\end{aligned}
$$

The other possible order of construction is to compute first *if* A and then to apply *even*, as follows:

$$
\begin{aligned}
[\textit{if}]^w &= \lambda X\lambda Y\ \ \mathfrak{n}_w(X)\neq\varnothing \text{ and } \mathfrak{e}_{\mathfrak{n}_w(X)}(X)\subseteq[Y]\\
[\textit{if } A]^w &= \lambda Y\ \ \mathfrak{n}_w(A)\neq\varnothing \text{ and } \mathfrak{e}_{\mathfrak{n}_w(A)}(A)\subseteq[Y]\\
[\textit{even if } A]^w &= \lambda Y\ \ \mathfrak{n}_w(A)\neq\varnothing,\ \mathfrak{e}_{\mathfrak{n}_w(A)}(A)\subseteq[Y],\\
&\qquad \mathfrak{e}_{\mathfrak{n}_w(\mathsf{U}\setminus A)}(\mathsf{U}\setminus A)\subseteq[Y] \text{ and}\\
&\qquad \mathfrak{e}_{\mathfrak{n}_w(A)}(A)\leq_{[Y]}\mathfrak{e}_{\mathfrak{n}_w(\mathsf{U}\setminus A)}(\mathsf{U}\setminus A)\\
[\textit{even if } A,\ C]^w &= \mathfrak{n}_w(A)\neq\varnothing,\ \mathfrak{e}_{\mathfrak{n}_w(A)}(A)\subseteq[C],\ \mathfrak{e}_{\mathfrak{n}_w(\mathsf{U}\setminus A)}(\mathsf{U}\setminus A)\subseteq[C]\\
&\qquad \text{and } \mathfrak{e}_{\mathfrak{n}_w(A)}(A)\leq_{[C]}\mathfrak{e}_{\mathfrak{n}_w(\mathsf{U}\setminus A)}(\mathsf{U}\setminus A)
\end{aligned}
$$

The two orders of constructions lead to the same result. At the end of the process, the two options have the same meaning, which is one of the other advantages of this semantics. There is no ambiguity surrounding the final meaning of the concessive conditional, regardless of the compositional order chosen.

The meaning of the concessive conditional can be further simplified. Remember that in the universe of projection, all worlds are bivalent toward the components of the conditional at hand. This means that the set $\mathsf{U}\setminus A$ is equal to the set $\neg A$ restricted to the universe of projection. Furthermore, inhibiting A is the same as inhibiting *not* A. Hence, $\mathfrak{n}_w(A)$ and $\mathfrak{n}_w(\neg A)$ are identical. The following simplified semantics are thus obtained.

THEOREM 4.1 (Truth-conditions of even if). $\vDash_w$ (*even if* A) C *iff*

(i) $\mathfrak{n}_w(A)\neq\varnothing$,
(ii) $\mathfrak{e}_{\mathfrak{n}_w(A)}(A)\subseteq[C]$,
(iii) $\mathfrak{e}_{\mathfrak{n}_w(A)}(\neg A)\subseteq[C]$,
(iv) $\mathfrak{e}_{\mathfrak{n}_w(A)}(A)\leq_{[C]}\mathfrak{e}_{\mathfrak{n}_w(A)}(\neg A)$.

The full meaning of the concessive conditional has three main parts. First, we can truly inhibit our belief in the antecedent and its negation (first condition). Second, both the antecedent and its negation lead to the same consequent (second and third conditions). Third, the antecedent is a factor, less favorable than its negation, for the realization of the consequent (last condition). Consider the following example:

(4.2) Even if it snows, the match won't be canceled.

With "normal" weather, that is to say without snow, an outdoor match will be played as usual. Sentence (4.2) expresses that in more extreme circumstances such as very bad weather, the sporting event will proceed. Despite the unfavorable conditions and perhaps unlike other sports, the game goes on. Notice that, as a simple conditional construction, the *even if* form is non-monotonic. The worsening of the situation can change the status of the consequent. In that case, the antecedent is no more adverse but becomes positive for the consequent. At this point, an "if then" form is used, as in the following example:

(4.3) But if it snows and the ground freezes, then the match will be canceled.

Let us now review the differences between our theory and the most well-known alternatives, beginning with [Pollock, 1976]. His formal semantics is as follows:

Pollock (*even if* A) C iff (i) C and
(ii) $\neg\exists B[(A \wedge B) \to \neg C$ and B might be true if A were true$]$

This definition has two parts. In part ii), the symbol $\to$ represents a conditional of necessitation, which is a conditional expressing a connection between its antecedent and its consequent. However, my analysis will focus on the first part of the definition, which simply says that the consequent is true. This proposal reaches the curious conclusion that the concessive conditional is not a hypothetical assertion about the truth of the consequent but a simple affirmation of this truth, supplemented by other factors. This position is supported when the scale of values described by the antecedent exhausts all the possibilities. In that case, the consequent is true in all circumstances. But the antecedent and its negation do not always cover all of the possible cases. Furthermore, the extent of this coverage is always relative to the circumstances of the assertion.

Pollock provides a counter-example (originally due to David Lewis) to his own theory. Imagine a puritanical boss who says the following:

(4.4) He would be fired even if he drank just a little.

This sentence does not claim that the employee will be fired regardless of the conditions. On the contrary, he will keep his place if he does not consume alcohol. This meaning comes from the application of the negation to the antecedent, which contains an adverb. By saying *he didn't drink just a little*, we mean that *he drank a lot*. So a theory like mine which says that both the antecedent and its negation lead to the occurrence of the consequent, is in a better position to solve this issue. Indeed, it is sufficient to supplement this proposal with a particular semantics for the negation when applied to a sentence containing an adverbial clause. For instance, in Montague grammar, a sentence like *John walks slowly* is translated as SLOWLY(ˆWALK(j)). Hence, we can say that the application of the negation to this sentence will lead to the negation of the adverb. But Pollock's theory cannot adopt this solution because it simply argues for the truth of the consequent.

Another issue facing the consequent-assertion phenomenon is due to Barker [1994]. Imagine that Lucy considers hypothetical alternatives at a party. "If anyone does something I don't like, I'll leave. If Mary starts arguing with me, I'll leave. If Fred starts screaming, I'll leave. Even if you leave, (though I always detested having you around), I'll leave." Now, if the only alternative that takes place is that the listener goes away, Lucy will stay, contrary to what is predicted by Pollock's theory and my own. However, this counterexample can be explained in the context of my theory by observing that the *even if* sentence occurs after several conditionals that, together, build a context. The first conditional details a general rule of behavior for the speaker ("If anyone does something I don't like"). The second and the third conditionals are particular instances of this rule ("If Mary starts arguing with me ... If Fred starts screaming"). The concessive conditional asserts that this rule will be observed despite possible opposite circumstances. Hence, the last conditional should be understood in the light of the first conditional: *If anyone does something I don't like and even if you leave, I'll leave.* In that context, the negation of the antecedent must be understood in the previously established context. In that case, it also implies the following consequent: *If anyone does something I don't like and if you don't leave, I'll leave.* Hence, a dynamic theory of the chaining of conditionals would solve this issue. I will not present a dynamic semantics of conditionals in this paper since it would lead us too far from our primary goal. Indeed, we would need to discuss in detail the embedding of conditionals and the different types of sequences of conditionals, such as reversed Sobel sequences. But a solution to this problem can be found, one inspired for instance by strategies presented by von Fintel [2001], Gillies [2007] or Moss [2012].

One aspect of the meaning of *even if* that is not present in Pollock's proposal is the different contribution of the antecedent and its negation to the truth of the consequent. Indeed, the antecedent is less favorable than its negation of the occurrence of the consequent. This aspect is present in Bennett's [1982] theory which argues that the antecedent brings a touch of surprise. Bennett [2003] is an amended version of this proposal:

Bennett

Truth: $(even\ if\ A)\ C$ is true iff $(if\ A)\ C$ is true

Assertion: (even if A) C is felicitously asserted iff

(i) a neighbor sentence S' is true and mutually believed by speaker and hearer, and salient for them;
(ii) the truth of "$(if\ A)\ C$" and that of S' can naturally be seen as parts of a single more general truth;
(iii) both "$(if\ A)\ C$" and S' involve some single scale, the focus item lies further along that scale than any items referred to in S', and for that reason the speaker and the hearers find it more surprising or striking or noteworthy that "$(if\ A)\ C$" is true than that S' is true.

Bennett also adopts the notion of scale as the underlying principle of his theory. Furthermore, he distinguishes between the truth of the concessive conditional based on truth-conditions and the felicity of its assertion based on pragmatic principles. This division makes it more difficult to compute the validity of inferential schemas. For instance, these truth-conditions invalidate the Modus Ponens with Negated Antecedent: $\neg A$, $(even\ if A)\ C \nvDash C$. Admittedly, by adding the sentence $(if\ not\ A)\ C$ to the neighbor sentences, the inference becomes pragmatically valid. But the notion of a neighbor sentence is too fuzzy to really be applicable. For instance, concerning the example (4.4) of the puritanical boss, Bennett chooses the sentence *he would be fired if he drank more* as the eligible neighbor. However, in the case of the concessive conditional *he would be fired even if he drank*, the selected sentence is now *he would be fired if he did not drink*. With this pragmatic principle, the theory can adjust its scale case by case. As Bennett says, a sentence "has countless 'neighbours"' and we can freely select one among them. The main problem of this position is that it is not falsifiable because no rule determines the choice of the chosen neighbor sentence. Furthermore, this furnishes no algorithm to compute the general meaning of *even if* sentences and its inferences, simply by lack of precision.

Another important issue for Bennett's theory is that it admits some false positives. Indeed, a sentence containing *even* is true as soon as there exists at least one true and less surprising neighbor sentence. But consider the following example given by Iten [2002]. "Everyone failed the exam. Sebastian and Neville are both more likely to fail than the others and Neville is more likely to fail than Sebastian." In that case, to utter the sentence "even Sebastian failed the exam" is infelicitous because this is more unexpected for a lot of other students. But all of Bennett's conditions for deeming this utterance correct are met. The neighbor sentence "Neville failed the exam" is true and is less surprising than "Sebastian failed the exam". The only way to save this approach would be to have clearer and more precise criteria for the selection of the neighbor sentence. Without it, this theory will admit a bunch of sentences that we deem false as being true.

The last theory I will examine is the one exposed by Lycan [1991]. This proposal is the first one to offer a formal semantics fully based on truth-conditions and not on pragmatic considerations.

Lycan

Ground Idea: (*even if* A) C iff C for all events, including all in which A.

Semantics: (*even if* A) C iff $(\forall e)\ e \in C \wedge (\forall f)(f \in A \supset f \in C)$, with e and f ranging over events.

Lycan's choice to represent the semantics of *even* through the universal quantification is motivated by an analogy with *only*. As the two particles can focus on different sentential groups of words in the same way, the two would be syntactical "soulmates". Furthermore, since Lycan contends that the meaning of *only* is governed by universal quantification, he concludes that the same applies for the meaning of *even*. The range of these events is not the whole space of possibilities but a restricted set. This is the class of all events believed to be real possibilities by the speaker in the present context. The first objection to this approach is that some uses of the word *even* do not exhibit any universal quantification, for instance, when there are only two values in the scale representing opposite positions. Lycan [2001] offers two refined versions of this theory in order to answer these types of counterexamples. But these improvements on the theory are not completely satisfactory, as Lycan himself has acknowledged:

> Worse, each of the theories faces objections that an impartial observer might reasonably consider fatal; at best, the universal quantification account is staggering under a weight of anomalies. (p. 137)

Another problem with Lycan's theory is that the antecedent is not considered as less favorable than its negation of the occurrence of the consequent. Indeed, both of the events in which the antecedent and its negation are true will be part of the restricted set universe for the utterance at hand. But no further difference is made between them. As Lycan [2001] has already noticed page 18, the truth-conditions given for the concessive conditional are equivalent to the following most simplified formulation: $(\forall e)\ e \in C$. Hence, there is no way to discriminate between the two conditionals *even if A, C* and *even if not A, C*. But this position is problematic. Consider the following example. Mary profoundly dislikes Cruella and generally manages to avoid her. Furthermore, she knows that Cruella will perhaps come to a very important party. She will certainly prefer to utter sentence (4.5) rather than sentence (4.6):

(4.5) Even if Cruella comes, I will go to this party.
(4.6) Even if Cruella does not come, I will go to this party.

To assert sentence (4.6) is simply to say something misleading because an uninformed listener could infer that Mary would have liked to meet Cruella at the party. Almost all linguistic theories of *even* recognize that a part of its signification concerns the unexpected, uninformative or surprising character of the relation described by the sentence. This aspect must also be present in the signification of *even if*. But it is absent from Lycan's theory. Thus, his attempt to formalize the whole signification of the *even if* conditional through truth-conditions is only partial and lacks a crucial element.

A consequence of this contrasting place on the scale for the antecedent and its negation is that we can infer *if A, C and if not A, C* from *even if A, C*. But the inverse inference is not always possible.

(4.5) Even if Cruella comes, I will go to this party.
(4.7) If Cruella comes, I will go to this party and if Cruella does not come, I will go to this party.

Knowing (4.5), someone can infer (4.7) without hesitation, even if this last formulation is clumsy. A better formulation would be *whether Cruella comes or not, I will go to this party.* On the contrary, the inference from (4.7) to (4.5) is not valid. Without knowing the relations between the speaker and Cruella, from (4.7), the listener does not know whether to deduce (4.5) or (4.8), which is as follows:

(4.8) Even if Cruella does not come, I will go to this party.

Hence, without knowing the speaker's preferences, the listener will prefer the following formulation: *whatever Cruella's choice, the speaker will go to this party.*

I would now like to examine the most salient inferential properties of this conditional construction in my theory. For the sake of simplicity, I will note the concessive connector with the help of the symbol '$\rightarrowtail$'. First of all, the Modus Ponens is valid.

$$A, A \rightarrowtail C \vDash C \tag{MP$\rightarrowtail$}$$

This comes as no surprise since this schema of inference is generally valid for any theory and any conditional. In fact, the primitive meaning of a conditional is that the consequent will be obtained as soon as the antecedent is realized. However, several counterexamples to this reasoning exist. I will only examine the one given by Lycan [2001] and which concerns the concessive.

(4.9) I'll be polite even if you insult me, but I won't be polite if you insult my wife.

A first formal representation of this sentence is $[(\textit{even if } A)\ C] \wedge [(\textit{if } B)\ \neg C]$. By applying Modus Ponens, it is therefore enough to insult the speaker and his wife to obtain the contradictory conclusion that the speaker will and will not be polite! Notice that the counterexample persists by replacing the *even if* with a simple *if* conditional in the first conjunct. The problem, then, is not linked to the semantics of the concessive in particular. It is just an example of non-monotonic reasoning. The first conditional sets a context in which the second one is uttered. The second antecedent is understood in the light of the acceptance of the first. Hence, the second conjunct can be more clearly expressed by *I won't be polite if you insult me and my wife*. Hence, the precise formal representation of this reasoning is $[(\textit{even if } A)\ C] \wedge [(\textit{if } A \wedge B)\ \neg C]$. The contradiction disappears because the second conditional details an exception to the rule stated by the first conditional. If both A and B are realized, we must only apply the modus ponens on $(\textit{if } A \wedge B)\ \neg C$ in order to deduce the consequent $\neg C$. We again come across the issue of how to formalize the chaining of conditionals. As I have already said, I am saving the formal treatment of these dynamic effects for another occasion.

The concessive conditional allows non-monotonic reasoning. Thus, the Strengthening of the Antecedent (SA$\rightarrowtail$) is not valid.

$$A \rightarrowtail C \nvDash (A \wedge B) \rightarrowtail C \qquad (\text{SA}\rightarrowtail)$$

Indeed, it is easy to find exceptions to a conditional formulation by adding a new antecedent.

(4.10) Even if it's sunny, I don't go out.
(4.11) Even if it's sunny and my house is on fire, I don't go out.

Contraposition is also invalid.

$$A \rightarrowtail C \nvDash \neg C \rightarrowtail \neg A \qquad (\text{CON}\rightarrowtail)$$

In the literature, some of the standard counterexamples against this schema of reasoning are *even if* conditionals. For instance, the two following sentences are respectively borrowed from Kratzer and cited by von Fintel [2001] and from Bennett [2003].

(4.12) (Even) if Goethe hadn't died in 1832, he would still be dead now.
(4.13) (Even) if the British and Israelis had not attacked the Suez Canal in 1956, the Soviets would (still) have invaded Hungary later in the year.

What is interesting in these examples is that *even* is written inside parentheses. This shows that this particle is possibly omitted in the sentence, without a change of meaning. In that case, the context helps the listener to understand that the intended conditional is not of the simple *if* form but rather an *even if* conditional.

Finally, I would like to return to the Modus Ponens with Negated Antecedent (MPNA$\rightarrowtail$). As explained in the introduction, this schema of inference displays a difference between *if* and *even if* sentences. I have already shown that my formal semantics invalidates this inference for *if* sentences. On the contrary, it validates it in the case of *even if* conditionals:

$$\neg A, A \rightarrowtail C \vDash C \tag{MPNA$\rightarrowtail$}$$

Hence, my theory complies with the empirical data and explains the differences between the two conditional forms.

To conclude, I would like to review the results obtained with this new compositional analysis of *even if* conditionals. The application of *even* to *if* sentences creates a scale compounded from the antecedent and its negation, both being cases in which the consequent is obtained. However, (MP$\rightarrowtail$) and (MPNA$\rightarrowtail$) do not entail the necessity of the consequent. As the concessive conditional is non-monotonic, we can find far-reaching circumstances in which the consequent is no longer obtained from either the antecedent or its negation. Hence, this theory complies with the judgment of validity supported by the subjects of psychological experiments concerning this conditional form. Furthermore, this theory is formulated without any call to pragmatic principles. Finally, formal semantics integrates the fact that the antecedent is an adverse factor for the occurrence of the consequent. In the future, I plan to use the same compositional strategy and the same semantics for *if* to explain the meaning of *if then* and *only if* conditionals.

A. Appendix: The Formal System

I give here the details of formal system which is used in this paper.

A.1. The Language

First, the formal language L is defined from a set of atomic sentences AT. The particles *if* and *even* are added to the usual propositional connectives. In this first presentation, a static approach is endorsed in order to limit the complexity of the semantics. As the embedding of conditionals entails dynamic effects, the last clause of the following definition forbids their construction.

Definition A.1. The *language* L is the closed set of sentences defined by the following conditions:

1. Every atomic sentence of AT is a sentence.
2. If A is a sentence, then $\neg A$ is a sentence.

3. If A and B are sentences, then $(A \vee B)$ and $(A \wedge B)$ are sentences.
4. If A is a sentence, then $(\mathit{if}\ A)$ is an antecedent.
5. If A is a sentence, then $(\mathit{even}\ A)$ is an antecedent.
6. if A is an antecedent and B is a sentence that does not contain any antecedent, then $(A,\ B)$ is a sentence.

As usual, different types of brackets are used to disambiguate complex formulas. The symbols '$\rightharpoonup$' and '$\rightarrowtail$' are also used to represent the *if* and *even if* connectors, respectively.

A.2. Models

Definition A.2. A *model* $\mathfrak{M}$ is a structure $\langle \mathsf{W}, \mathsf{B}, \{\mathsf{v}_w\}_{w \in \mathsf{W}}, \mathfrak{n}, \mathfrak{e}, \mathfrak{o} \rangle$, where:

- W is a set of *trivalent possible worlds*;
- B is a set of *bivalent possible worlds*, such that $\mathsf{B} \subseteq \mathsf{W}$;
- $\{\mathsf{v}_w\}_{w \in \mathsf{W}}$ is a family of *valuation relations* such that for any $w \in \mathsf{W}$, v_w is a binary relation included in $\mathrm{AT} \times \{0, 1\}$ such that for any $\alpha \in \mathrm{AT}$ the image $\mathsf{v}_w(\alpha)$ is equal either $\{0\}$, or $\{1\}$, or $\varnothing$; moreover, for any $w \in \mathsf{B}$ either $\mathsf{v}_w(\alpha) = \{0\}$ or $\mathsf{v}_w(\alpha) = \{1\}$;[8]
- $\mathfrak{n}$ is a *neutrality function* from $\mathsf{W} \times 2^{\mathrm{L}}$ into 2^{W};
- $\mathfrak{e}$ is an *expansion function* from $2^{\mathsf{W}} \times 2^{\mathrm{L}}$ into 2^{W};
- $\mathfrak{o}$ is an *ordering function* that associates with each sentence A from L a pair $\{\mathsf{E}_A, \leq_A\}$, where E_A is a set of subsets of W and $\leq_A$ is a proper partial order.[9]

Notation A.1. The following convention applies for the neutrality and expansion functions. The first argument is lowered and the elements of the second argument, which is a set of sentences, are put inside parentheses and without brackets. For instance, if w is the first argument and $\{A, C\}$ is the second argument of the function $\mathfrak{n}$, the notation is $\mathfrak{n}_w(A, C)$. Anyway, the brackets are kept around the elements of the second argument when some set theoretic operations are applied, as in $\mathfrak{n}_w(\{A, C\} \cup \{B\})$.

[8] In other words, for any $w \in \mathsf{B}$, the relation v_w is a function from AT into $\{0, 1\}$, but for $w \in \mathsf{W} \setminus \mathsf{B}$ the relation v_w may be only a partial function from AT into $\{0, 1\}$.

[9] That is, $\leq_A$ bear the following properties: *partial*, i.e. $\neg\forall x \forall y (x \leq_A y \vee y \leq_A x)$; *reflexive*, i.e. $\forall x\ x \leq x$; *antisymmetric*, i.e. if $\forall x \forall y (x \leq_A y \wedge y \leq_A x \Rightarrow x = y)$; *transitive*, i.e. $\forall x \forall y \forall z (x \leq_A y \wedge y \leq_A z \Rightarrow x \leq_A z)$.

In order to correctly model the evaluation process of conditional sentences, conditions are added on the neutralization, expansion and ordering functions. Let us start with the neutralization function.

Properties of neutralization functions. The first condition (neut) says that the neutralization of a complex sentence is equal to the neutralization of its atomic components. The second condition (neutsimp) states that neutralizing a set of sentences implies the neutralization of all its subsets.

Notation A.2. The set of atomic sentences composing sentences from a set Σ is noted $\mathcal{E}(\Sigma)$.

Definition A.3 (Conditions for the neutrality function). Every neutrality function $\mathfrak{n}$ satisfies the following conditions for all sets Σ and Σ' of sentences and any $w \in \mathsf{W}$:

(neut) $\mathfrak{n}_w(\Sigma) = \mathfrak{n}_w(\mathcal{E}(\Sigma))$.

(neutsimp) If $\Sigma' \subseteq \Sigma$ and $\mathfrak{n}_w(\Sigma) \neq \varnothing$, then $\mathfrak{n}_w(\Sigma') \neq \varnothing$.

Properties of expansion functions. Some conditions are also added on the expansion function. The main objective is to obtain a set-theoretic behavior for classical connectives inside the antecedent or the consequent of conditional expressions. The notion of *universe of projection* is first introduced. This notion makes it possible to obtain a contextualized set universe, depending on the sentences previously inhibited. Furthermore, the sentences previously inhibited become again true or false at the possible worlds constituting the universe of projection.

Definition A.4. A possible world $w \in \mathsf{W}$ is *bivalent for a set of sentences* Σ iff for all atomic sentence p composing any sentence in Σ, $\nu_w(p) = \{0\}$ or $\nu_w(p) = \{1\}$.

Definition A.5. Let w a possible world, C a conditional, Σ the set of its subsentences, $\mathfrak{n}$ the neutrality function and $\mathfrak{e}$ the expansion function used to evaluate this conditional. The *universe of projection* for $\mathbf{C}$ relative to w is the union of all possible bivalent worlds for Σ obtainable by $\mathfrak{e}_{\mathfrak{n}_w(\Sigma')}(\Sigma'')$ with $\Sigma' \subseteq \Sigma$ and $\Sigma'' \subseteq \Sigma$.

Notation A.3. For a conditional C, Σ its subsentences and U an universe of projection for C, for all $\sigma \in \Sigma$, the truth-set of σ inside U is noted $[\sigma]^{\mathsf{U}}$. We abbreviate it $[\sigma]$, when U is evident. For $\Sigma' \subseteq \Sigma$, we have $\mathfrak{e}_{\mathfrak{n}_w(\Sigma')}(\sigma) = \mathfrak{e}_{\mathfrak{n}_w(\Sigma')} : [\sigma]$. The truth-set of σ inside W is noted $[\sigma]^{\mathsf{W}}$.

The additional conditions on the expansion function are the following ones. The conditions (expneg), (expdisj) and (expconj) state that the negation, disjunction and conjunction behave like their set-theoretic counterparts (difference, union and intersection). Finally, the intuitive meaning behind the condition (sec) is that when an antecedent is true at the starting world of evaluation, this world is among the set of possible worlds obtained by reconstruction of the antecedent.

DEFINITION A.6 (Conditions on the expansion function). Let w be a possible world, Σ' be a set of sentences containing A and B, $\mathfrak{U}$ their universe of projection, $\Sigma \subseteq \Sigma'$, $\mathfrak{n}$ be a neutrality function, and $\mathfrak{e}$ be an expansion function. Then:

(expneg) $\mathfrak{e}_{\mathfrak{n}_w(\Sigma)}(\neg A) = \mathfrak{e}_{\mathfrak{n}_w(\Sigma)} : [\neg A] = \mathfrak{e}_{\mathfrak{n}_w(\Sigma)} : \mathfrak{U} \setminus [A]$.
(expdisj) $\mathfrak{e}_{\mathfrak{n}_w(\Sigma)}(A \vee B) = \mathfrak{e}_{\mathfrak{n}_w(\Sigma)} : [A \vee B] = \mathfrak{e}_{\mathfrak{n}_w(\Sigma)} : [A] \cup [B]$.
(expconj) $\mathfrak{e}_{\mathfrak{n}_w(\Sigma)}(A \wedge B) = \mathfrak{e}_{\mathfrak{n}_w(\Sigma)} : [A \wedge B] = \mathfrak{e}_{\mathfrak{n}_w(\Sigma)} : [A] \cap [B]$.
(sec) If $\vDash_w A$ and $\mathfrak{e}_{\mathfrak{n}_w(\mathcal{E}(A)\cup\ldots)}(A) \subseteq [C]$, then $\vDash_w C$.

Ordering function. Ordering functions are used to state the truth-conditions of the *even if* conditional and meets four conditions. The condition (o1) states that the disjunction in an antecedent is equivalent to the conjunction of two conditionals with each antecedent taken in turn. The condition (o2) states that the behavior of conjunction in an antecedent is equivalent to that of disjunction. The condition (o3) states that to be unfavorable to the disjunction of two sentences is to be unfavorable at least to one of these sentences. The condition (o4) says that to be unfavorable to the conjunction of two sentences is to be unfavorable to each in turn.

DEFINITION A.7 (Conditions for the ordering). Let w be a possible world, Σ' be a set of sentences containing A, B and C, $\Sigma \subseteq \Sigma'$, $\mathfrak{n}$ be a neutrality function, $\mathfrak{e}$ be an expansion function, and $\leq_\sigma$ be an ordering function for $\sigma \in \Sigma'$. Then:

(o1) $\mathfrak{e}_{\mathfrak{n}_w(\Sigma)} : [A \vee B] \leq_C [\neg(A \vee B)]$ iff $\mathfrak{e}_{\mathfrak{n}_w(\Sigma)} : [A] \leq_C [\neg A]$ and $\mathfrak{e}_{\mathfrak{n}_w(\Sigma)} : [B] \leq_C [\neg B]$.
(o2) $\mathfrak{e}_{\mathfrak{n}_w(\Sigma)} : [A \wedge B] \leq_C [\neg(A \wedge B)]$ iff $\mathfrak{e}_{\mathfrak{n}_w(\Sigma)} : [A] \leq_C [\neg A]$ and $\mathfrak{e}_{\mathfrak{n}_w(\Sigma)} : [B] \leq_C [\neg B]$.
(o3) $\mathfrak{e}_{\mathfrak{n}_w(\Sigma)} : [A] \leq_{C_1 \vee C_2} [B]$ iff $\mathfrak{e}_{\mathfrak{n}_w(\Sigma)} : [A] \leq_{C_1} [B]$ or $\mathfrak{e}_{\mathfrak{n}_w(\Sigma)} : [A] \leq_{C_2} [B]$.
(o4) $\mathfrak{e}_{\mathfrak{n}_w(\Sigma)} : [A] \leq_{C_1 \wedge C_2} [B]$ iff $\mathfrak{e}_{\mathfrak{n}_w(\Sigma)} : [A] \leq_{C_1} [B]$ and $\mathfrak{e}_{\mathfrak{n}_w(\Sigma)} : [A] \leq_{C_2} [B]$.

Truth-conditions. Truth-conditions are only given from a bivalent world. Indeed, this study is limited to the simplest case in which the starting world of evaluation is a world where every sentence is either true or false, in order to keep the classical semantics for negation, disjunction and conjunction when they are used outside conditionals. Notice however that this semantics could easily extended to a full trivalent logic.

The intuitive ideas behind these truth-conditions are first briefly reviewed. The semantics of $(if\ A)\ C$ is a formal representation of the following process of judgment. The antecedent A is first neither believed nor disbelieved. From this suspension of judgment, the next step is to add A. The final step is to check whether C is obtained. The semantics of $(even\ if\ A)\ C$ is obtained by combining the formal semantics of *even* given in Section 2 and the notion of universe of projection. We thus obtain that $(even\ if\ A)\ C$ is true when $(if\ A)\ C$ and $(if\ \neg A)\ C$ are true and that A is less favorable than its negation in order to conduct to C.

Definition A.8 (Truth-conditions in a bivalent world). Let $\mathfrak{M} = \langle W, \mathsf{B}, \{\mathsf{v}_w\}_{w \in W}, \mathfrak{n}, \mathfrak{e}, \mathfrak{o} \rangle\rangle$ be any model, w a member of B, and A and C sentences of L. $\vDash_{\mathfrak{M},w} A$ (or simply $\vDash_w A$ if $\mathfrak{M}$ is evident) means that $\mathsf{v}_w(A) = \{1\}$ for w in W. Then we have the following truth-conditions for conditionals:

- $\vDash_w (if\ A)\ C$ iff $\mathfrak{n}_w(A) \neq \varnothing$ and $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) \subseteq [C]$.
- $\vDash_w (even\ if\ A)\ C$ iff $\mathfrak{n}_w(A) \neq \varnothing$, $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) \subseteq [C]$, $\mathfrak{e}_{\mathfrak{n}_w(A)}(\neg A) \subseteq [C]$ and $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) \leq_C \mathfrak{e}_{\mathfrak{n}_w(A)}(\neg A)$.

The notion of semantic consequence also starts from a bivalent world.

Definition A.9. A formula A is a *semantic consequence* of a set of formulas Σ, noted $\Sigma \vDash A$, iff for any model $\mathfrak{M}$ and for any $w \in \mathsf{B}$: if for any $\sigma \in \Sigma$ we have $\vDash_w \sigma$, then also $\vDash_w A$.

B. Appendix: Proofs

In this appendix, proofs are given for the different inferences stated in the text. For valid inferences, it is shown that the consequence holds in every model in which the premises are true. For invalid inferences, a counter-model is provided. Notice that the transformation by (exp) is explicitly used only in the first proofs. The symbols '$\rightharpoonup$' and '$\rightarrowtail$' are used to represent the *if* and *even if* connectives, respectively.

$$A \rightharpoonup C \nvDash (A \wedge B) \rightharpoonup C \tag{SA}$$

Let $\vDash_w A \rightharpoonup C$ with $\mathfrak{n}_w(A) = \{w_1\}$, $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) = \{w_2\}$, $w_2 \in [C]$. We can construct a model with $\mathfrak{n}_w(A, B) = \{w_3\} \neq \{w_1\}$ and $\mathfrak{e}_{\mathfrak{n}_w(A,B)}(A \wedge B) = \{w_4\} \neq \{w_2\}$ with $w_4 \notin [C]$.

$$A, C \nvDash A \rightharpoonup C \tag{CS}$$

We can construct a model in which $w \in [A \wedge C]^W$, $\mathfrak{n}_w(A) = \{w_1\}$, and $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) = \{w, w_2\}$. In that case, we can have $w_2 \in [A]$, but $w_2 \in [\neg C]$.

$$A, A \rightharpoonup C \vDash C \tag{MP}$$

Let $\vDash_w A$ and $A \rightharpoonup C$. Then $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) \subseteq [C]$. So $\vDash_w C$, by (sec).

$$\neg A, A \rightharpoonup C \nvDash C \tag{MPNA}$$

We can construct a model in which $\mathfrak{n}_w(A) = \{w_1\}$, $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) = \{w_2\}$ with $w \in [\neg A]^W$ and $w \notin [C]$.

$$A \rightharpoonup (B \wedge C) \vDash (A \rightharpoonup B) \wedge (A \rightharpoonup C) \tag{CM}$$

From the premise, we have $\mathfrak{n}_w(A) \neq \varnothing$ and $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) \subseteq [B \wedge C]$. By (expconj), we obtain $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) \subseteq [B] \cap [C]$. Hence, $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) \subseteq [B]$ and $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) \subseteq [C]$.

$$(A \rightharpoonup B) \wedge (A \rightharpoonup C) \vDash A \rightharpoonup (B \wedge C) \tag{CC}$$

From the premises, $\mathfrak{n}_w(A) \neq \varnothing$, $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) \subseteq [B]$ and $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) \subseteq [C]$. So $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) \subseteq [B \wedge C]$, by (expconj).

$$A \rightharpoonup C \nvDash \neg C \rightharpoonup \neg A \tag{CON}$$

Let $\vDash_w A \rightharpoonup C$ with $\mathfrak{n}_w(A) = \{w_1\}$ and $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) = \{w_2\}$. Then we can construct a model with $\mathfrak{n}_w(\neg C) = \{w_3\} \neq \{w_1\}$ and $\mathfrak{e}_{\mathfrak{n}_w(\neg C)}(\neg C) = \{w_4\} \neq \{w_2\}$ with $w_4 \notin [\neg A]$.

$$(A \rightharpoonup B) \wedge (B \rightharpoonup C) \nvDash A \rightharpoonup C \tag{TRAN}$$

Let $\vDash_w A \rightarrow C$ with $\mathfrak{n}_w(A) = \{w_1\}$, $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) = \{w_2\}$, $w_2 \in [B]$, $\mathfrak{n}_w(B) = \{w_3\}$, and $\mathfrak{e}_{\mathfrak{n}_w(B)}(B) = \{w_4\}$. Then we can construct a model with $w_2 \notin [C]$.

$$\nvDash (A \rightharpoonup C) \vee (A \rightharpoonup \neg C) \tag{CEM}$$

We can construct a model with $\mathfrak{n}_w(A) = \varnothing$. Or we can construct a model in which $\mathfrak{n}_w(A) = \{w_1\}$, $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) = \{w_2, w_3\}$ with $w_2 \in [C]$ and $w_3 \in [\neg C]$.

$$A, A \rightarrowtail C \vDash C \qquad \text{(MP}\rightarrowtail\text{)}$$

Let $\vDash_w A$ and $A \rightarrowtail C$. Then $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) \subseteq [C]$. So $\vDash_w C$, by (sec).

$$A \rightarrowtail C \nvDash (A \wedge B) \rightarrowtail C \qquad \text{(SA}\rightarrowtail\text{)}$$

Let $\vDash_w A \rightarrowtail C$ with $\mathfrak{n}_w(A) = \{w_1\}$, $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) = \{w_2\}$, $\mathfrak{e}_{\mathfrak{n}_w(A)}(\neg A) = \{w_3\}$. Then we can construct a model with $\mathfrak{n}_w(A, B) = \{w_4\} \neq \{w_1\}$ and $\mathfrak{e}_{\mathfrak{n}_w(A,B)}(A \wedge B) = \{w_5\} \nsubseteq \{w_2, w_3\}$ with $w_5 \notin [C]$.

$$A \rightarrowtail C \nvDash \neg C \rightarrowtail \neg A \qquad \text{(CON}\rightarrowtail\text{)}$$

Let $\vDash_w A \rightarrowtail C$ with $\mathfrak{n}_w(A) = \{w_1\}$, $\mathfrak{e}_{\mathfrak{n}_w(A)}(A) = \{w_2\}$, $\mathfrak{e}_{\mathfrak{n}_w(A)}(\neg A) = \{w_3\}$. Then we can construct a model with $\mathfrak{n}_w(\neg C) = \{w_4\} \neq \{w_1\}$ and $\mathfrak{e}_{\mathfrak{n}_w(\neg C)}(\neg C) = \{w_5\} \nsubseteq \{w_2, w_3\}$ with $w_5 \notin [\neg A]$.

$$\neg A, A \rightarrowtail C \vDash C \qquad \text{(MPNA}\rightarrowtail\text{)}$$

From $A \rightarrowtail C$, we have $\mathfrak{e}_{\mathfrak{n}_w(A)}(\neg A) \subseteq [C]$ which is equivalent by (neut) to $\mathfrak{e}_{\mathfrak{n}_w(\neg A)}(\neg A) \subseteq [C]$. From $\vDash_w \neg A$ and (sec), we conclude that $\vDash_w C$.

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