



## BOOK REVIEWS

### Overview of results presented in *Trends in Logic XIII*

ANDRZEJ INDRZEJCZAK, JANUSZ KACZMAREK, and MICHAŁ ZAWIDZKI (editors), **Trends in Logic XIII. Gentzen’s and Jaśkowski’s Heritage 80 Years of Natural Deduction and Sequent Calculi**, Wydawnictwo UŁ, Łódź (Poland), 2014, 269 pages, ISBN 978-83-7969-161-6.

In 2014 we had an opportunity to celebrate the 80th anniversary of Gerhard Gentzen’s and Stanisław Jaśkowski’s first publications on natural deduction and also first Gentzen’s works on the sequent calculus. One of the events focusing on the great contribution of Gentzen and Jaśkowski was the conference Trends in Logic XIII, which took place on 2–5 July, 2014 in Łódź. The book under review contains extended abstracts of the lectures given by invited speakers and of the papers presented during the conference. We concentrate on the latter. It should be emphasized that the aim of this review is to introduce and briefly summarise the results of the papers rather than comment on or discuss them. We hope that the reader finds some of them interesting and worth pursuing further.

MASSIMILIANO CARRARA and ENRICO MARTINO, “Arbitrary reference through acts of choice: A constructive view of reference in logic”, pp. 43–54. The paper presents the role of arbitrary reference in logic and mathematics. The authors introduce the Principal of Arbitrary Reference (PAR). They argue that PAR is essential for deduction and also that PAR is essential in the semantics of first-order logic (in short: FOL). They also refer to Hintikka’s game theoretical semantics for FOL and compare it with their approach. The paper gives a way in which one can understand arbitrary reference and a specification of PAR, namely the Principal of Arbitrary Choice (see p. 51).

FERNANDO FERREIRA and GILDA FERREIRA, “The faithfulness of atomic polymorphism”, pp. 55–65. The paper presents a relationship between the intuitionistic propositional logic (IPL) and the atomic polymorphic system  $F_{\text{at}}$ . The latter is a calculus with only two logical constants: the conditional and the second-order quantifier. The main result of the paper is a proof of the faithfulness of the embedding of IPL into  $F_{\text{at}}$  (see Theorem 2.8, p. 64). In order to determine the embedding, the authors use a translation due to Prawitz. The embedding relies on the property of instantiation overflow (see p. 58). With regards to the faithfulness, the authors call the embedding faithful if any formula which (after translation) is a thesis of  $F_{\text{at}}$  is also a thesis of IPL.

JOSEP MARIA FONT and TOMMASO MORASCHINI, “On the logics associated with a given variety of algebras”, pp. 67–80. The paper presents some results in abstract algebraic logic. The authors begin with considerations that concern three classes of algebras associated with a given propositional logic  $L$ : the Leibniz-reduced algebras of  $L$  (the algebraic reducts of the reduced matrix models of  $L$ ), the algebraic counterpart of  $L$  (the Tarski-reduced algebras of  $L$ ) and the intrinsic variety of  $L$  (the variety generated by the Lindenbaum–Tarski algebra of  $L$ ). Let  $V$  be a given variety of algebras of an arbitrary language. Three questions concerning variety of algebras are raised and answered by the authors:

1. Is there a logic  $L$  such that the Leibniz-reduced algebras of  $L$  is equal to the variety  $V$ ?
2. Is there a logic  $L$  such that the algebraic counterpart of  $L$  is equal to the variety  $V$ ?
3. Is there a logic  $L$  such that the intrinsic variety of  $L$  is equal to the variety  $V$ ? (see p. 68)

For the questions (2) and (3) the authors give positive answers. The paper also concerns the variety of algebras generated by a primal algebra and the variety of algebras of semilattices. According to Corollary 2.3 for a given primal algebra  $\mathbb{A}$  there is exactly  $|\mathcal{P}(\mathbb{A})| - 2$  ( $\mathbb{A}$  is an universum set of  $\mathbb{A}$ ) algebraizable logics whose equivalent algebraic semantics is the intrinsic variety of  $\mathbb{A}$  (see Corollary 2.3, p. 73). Thanks to certain facts concerning the variety of semi-lattices, the authors are able to show that the poset is atomless and its maximal element is the conjunctive and the disjunctive fragment of classical propositional logic (see Theorem 4.1, p. 75). That leads to new characterizations of the conjunctive fragment of classical propositional logic.

SABINE FRITTELLA, GIUSEPPE GRECO, ALEXANDER KURZ, ALESSANDRA PAMIGIANO, and VLASTA SIKIMIĆ, “Multi-type sequent calculi”, pp. 81–93. The paper presents a basic idea and some motivation for multi-type calculi. It mainly concerns technical aspects of a proof-theoretic methodology of certain logics: in a proof-theoretic setting, the authors generalize Belnap’s display logic and Sambin’s basic logic and thereby introduce a new proof-theoretic treatment of logics such as multi-modal logics and dynamic logics. As the authors point out the generalization makes it possible to account for logics which admit connectives which are neither adjoints nor residual or that are not closed under uniform substitution. The authors also obtain the cut-elimination for the introduced calculi (see Theorem 4.1, p. 89).

TOMASZ JARMUŻEK and ANDRZEJ PIETRUSZCZAK, “Decidability methods for modal syllogisms”, pp. 95–112. The paper gives results concerning two related topics: some tableau system for modal syllogistic and the decidability of modal syllogisms. The authors consider modal syllogisms in Johnson-like models for categorical propositions with modalities *de re*. By introducing a complete tableau system for modal syllogism the authors determine two methods of decidability for modal syllogisms (see soundness and completeness theorems, Section 3, pp. 106–110). They address so-called strong decidability, which means that they give methods which enable one to check whether a given syllogism is logically valid or not in a finite and estimated number of steps. Briefly, the authors explain that the first method estimates the limit of the length of a tableau for a given syllogism, while the second one gives the upper bound of the cardinality of possible counter-models.

MARCIN KACPROWICZ and ADAM NIEWIADOMSKI, “On dedicated fuzzy logic systems for emission control of industrial gases”, pp. 113–129. The paper presents some applications of fuzzy logic to control filters reducing air pollution. The authors introduce some fuzzy logic systems to efficiently control so called Selective Catalytic Reduction and to limit human participation in the process. They point out that because of the non-linearity of the process their ideas might be particularly useful. Some possible future directions of the research are given.

MAKOTO KANAZAWA, “Almost affine lambda terms”, pp. 131–148. The paper presents some new results concerning  $\lambda$ -terms, namely that a  $\lambda$ -term that has a negatively non-duplicated typing is always  $\beta\eta$ -equal to

an almost affine  $\lambda$ -term (Theorem 4.3, p. 147). As a corollary of certain important lemmas the author shows that all inhabitants of a negatively non-duplicated sequent are  $\beta\eta$ -equal (see Theorem 3.7, p. 141). As a consequence of the main theorem the author shows that the  $\lambda$ -term  $M$  in long normal form  $\beta$ -expands to an almost affine  $\lambda$ -term iff the principle typing of  $M$  is negatively non-duplicated (see Corollary 4.4, p. 147).

MAREK NASIENIEWSKI and ANDRZEJ PIETRUSZCZAK, “Aximatisations of minima modal logics defining Jaśkowski-like discussive logics”, pp. 149–163. The paper presents some results concerning minimal modal logics defining Jaśkowski-like discussive logics. The system of discussive logic  $D_2$  is determined by a translation  $t$  from a language of discussive logic into the language of modal logic. It was originally determined by a translation into the formulas of system  $S5$ . However, one could consider other modal systems. With regards to the original translation determined by Jaśkowski, discussive conjunction  $A \wedge^d B$  is translated as  $t(A) \wedge \diamond t(B)$ , but it might be also translated as  $\diamond t(A) \wedge t(B)$ . It has been proved that the latter enables one to get a different system than  $D_2$ . But there is a question: how does this translation influence on the weakest modal systems defining Jaśkowski-like discussive logics? By introducing axiomatizations of such modal systems the authors present their answer.

LUIZ CARLOS PEREIRA and EDWARD HERMANN HAEUSLER, “Cut-elimination and consistency: Variations on a Gentzen–Prawitz theme”, pp. 165–179. The paper presents a syntactic proof of cut-elimination for a sequent calculus formulation of Peano Arithmetic, that is for system LKA (see Theorem 4.2, p. 177). As a corollary of the cut-elimination the authors prove a consistency of LKA (see Proposition 5.1, p. 179). The authors briefly describe a steps of the proof of the cut-elimination as follows:

1. Define an assignment of ordinals  $< \epsilon_0$  to proofs in LKA;
2. Define reductions whose aim is to eliminate non-essential cuts and unnecessary applications of induction (reducible inductions);
3. Show by inductions on the ordinals assigned to proofs that every proof can be transformed/reduced to a normal proof, i.e., to a proof without non-essential cuts and reducible inductions. (see p. 168)

KRZYSZTOF RENKAS and ADAM NIEWIADOMSKI, “Hierarchical fuzzy logic systems and their extensions based on type-2 fuzzy sets”, pp. 181–197. The paper concerns an extension of Hierarchical Fuzzy

Logic Systems (HFLS) — which is introduced by the authors — and its application. A basis for a such extension is to model input and output variables for systems by type-2 fuzzy sets. The application of such a system concerns controlling vehicles in a single player computer game. Furthermore, the authors compare HFLS with non-hierarchical fuzzy systems and type-2 with type-1 HFLS. Some considerations concerning future directions of the research are given.

UMBERTO RIVIECCIO, “Algebraic semantics for bilattice public announcement logic”, pp. 199–215. The paper presents an algebraic interpretation of a bilattice-based version of public announcement logic. The author describes logic of public announcement as a dynamic logic that enables one to model the epistemic change brought about to the cognitive state of a group of agents once a given proposition has become publicly known (see pages 199–200). The presented logic is proved to be sound and to be complete with respect to the algebraic semantics (see Theorem 5.5, p. 213). The author identifies his main aim as giving a semantically-grounded analysis of epistemic updates in the presence of incomplete and/or inconsistent information. It is also important for him to introduce some methods of algebraic logic to the study of mathematical foundations of dynamic logic (see p. 201).

KATSUHIKO SANO and YUICHIRO HOSOKAWA, “Gentzenization of dynamic topological hybrid logics”, pp. 217–232. The paper presents a hybridization of dynamic topological logics and determines how one can obtain a sequent calculus for hybrid topological dynamic logics from Hilbert-style axiomatizations. Such axiomatizations are also provided by the authors. They prove, inter alia, the admissibility of cut for the constructed calculus (see Lemma 4.5 and Theorem 4.6, pp. 227–228) as well as a semantic completeness theorem (see Section 4.1, pp. 224–227).

PETER SCHROEDER-HEISTER, “Frege’s sequent calculus”, pp. 233–245. The paper concerns an interpretation of Frege’s logical system developed in *Grundgesetze der Arithmetik* as a kind of sequent calculus and also justifies a claim that Frege was aware of the fundamental structural issue that was developed by Jaśkowski and Gentzen. The author compares the Frege-Hilbert calculus with the Jaśkowski-Gentzen calculus and presents connections between Frege’s formal concept of implication and the sequent arrow. There is also an explanation of the sense in which the inference rules presented by Frege correspond to structural rules in

Gentzen's style and in what sense Frege's system could be recognized as a sequent-style natural deduction system.

PAWEŁ URZYCZYN, "Games for intuitionistic logic", pp. 247–269. The paper presents a game interpretation of intuitionist logic. More specifically, the author provides a duality of proof construction and counter-model for intuitionistic logic in the form of a game. As the author points out game interpretations are usually model-oriented or proof-oriented. In this case there is one game and two players. One of them,  $\exists$ Edward, tries to construct a proof while another one,  $\forall$ lexis, tries to construct a counter-model. The determination of the game implies completeness and cut-elimination. The author considers intuitionistic zero and first order logic.

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