Logic and Logical Philosophy
Volume 25 (2016), 259–283
DOI: 10.12775/LLP.2015.012

Peter Forrest

THE MEREOLOGY OF STRUCTURAL UNIVERSALS

In memory of David Armstrong (1926–2014)

Abstract. This paper explores the mereology of structural universals, using the structural richness of a non-classical mereology without unique fusions. The paper focuses on a problem posed by David Lewis, who using the example of methane, and assuming classical mereology, argues against any purely mereological theory of structural universals. The problem is that being a methane molecule would have to contain being a hydrogen atom four times over, but mereology does not have the concept of the same part occurring several times. This paper takes up the challenge by providing mereological analysis of three operations sufficient for a theory of structural universals: (1) Reflexive binding, i.e. identifying two of the places of a universal; (2) Existential binding, i.e. the language-independent correlate of an existential quantification; and (3) Conjunction.

Keywords: mereology; conjunction; existential binding; David Lewis; reflexive binding; structural universals; unique fusion

Introduction

This paper explores the mereology of structural universals, and in doing so it also explores some of the structural richness of mereology, provided we do not assume the uniqueness of fusion. The paper focuses on a problem posed by David Lewis in [9]. Using the example of methane, he argued against any purely mereological theory of structural universals. For on a mereological theory, the universal *being a methane* (CH4) *molecule*

Special Issue: Mereology and Beyond (II). Edited by A. C. Varzi and R. Gruszczyński
 Published online May 27, 2015

should be the sum of its parts, namely the universals *being a carbon atom*, *being a hydrogen atom*, and *being bonded*. As Lewis points out, *being a methane molecule* would have to contain *being a hydrogen atom* four times over, but mereology does not have the concept of the same part occurring several times. Hence, he argues, there cannot be a purely mereological account of structural universals, which he takes to be a serious objection to realism about universals. This problem may be reformulated as the challenge to distinguish the universal *being a methane molecule* from the universal *being a methylene* (CH2) *molecule*, and to make this distinction using mereology alone. An obvious thought is that *being a methane molecule* has a part, *being the sum of 4 hydrogen atoms* that *being a methylene molecule lacks*. But this just shifts the challenge to providing a mereological distinction between *being the sum of 4 hydrogen atoms* and *being the sum of 2 hydrogen atoms*. This paper takes up the challenge by rejecting the classical mereology that Lewis assumed. To do so, I provide mereological analyses of three operations sufficient for a theory of structural universals.

1. Reflexive binding: identifying two of the places of a universal to obtain another universal. In the dyadic case this is forming Rxx out of Rxy. (E.g. *x's loving self* is the reflexive binding of *x loving y*.) I stipulate the no universal is a reflexive binding of itself. Thus if $Rxx = Rxy$ then we do not call Rxx a reflexive binding of Rxy. (Suppose it was a deep metaphysical truth that necessarily if you truly love yourself you love everyone. Then *x's loving self* and *x's loving y* turn out to be the same universal, *x loves*.)
2. Existential binding: the language-independent correlate of an existential quantification that binds one free variable, so as to form another universal (E.g. Both *x's loving someone* and *someone's loving x* are existential bindings of *x loving y*.) I stipulate that no universal is an existential binding of itself. An n-adic universal can have up to n successive existential binding. The result of the n of them is the total existential binding.
3. Conjunction.

I am considering a sparse theory of universals, inspired by [1][1]. Therefore three further operations, disjunction, negation and subtrac-

[1] [9] distinguishes sparse from abundant conceptions of universals, citing Armstrong as the proponent of the former.

tion do not apply automatically. It is nonetheless interesting to decide when, if at all, they can be given a mereological analysis.

I hypothesise that if we start with some universals without structure then using the three operations repeatedly we can derive all the structural universals.[2]

The way our predicates apply is, in non-fictional cases, determined by the universals and their instances. There are more and less complicated ways linking a univocal predicate to one or more universals. The most straightforward is the way the predicate Ux applies to the instances of the corresponding property Ux. Greater complexity is illustrated by disjunctions. If Ux and Vy have no disjunction, the disjunctive predicate 'Ux and/or Vx' applies both to instances of Ux and of Vy. But where there is a disjunctive universal the disjunctive predicate also applies directly to instances of the disjunctive universal.

The core of this paper consists of a section in which the mereology of structural universals is explored and a section in which I show how the various structure-forming operations can be characterised mereologically. Of the three sections preceding the core, the first will deal with some preliminaries such as notation. Then I expound the non-classical mereology that I employ. This theory is a refinement of classical mereology, in the sense that there is an equivalence relation (overlapping the very same items) and the equivalence classes satisfy the axioms of classical mereology. Next I provide (nominal) definitions of the operations. After the mereological analyses of these operations, I reply to some objections.

This paper is part of a larger project of Ontology Within the Bounds of Mereology, something I have provided a preliminary sketch elsewhere (see [7]).[3] In this paper I shall assume that universals are somehow distinguished from particulars, and I shall not rely on the mereological characterisation of *instantiation in*. Likewise, I shall ignore second and higher order universals until I reply to the final objection. So the

---

[2] In this paper I shall stick to the finite case, so the problem of infinite complexity does not arise.

[3] I there distinguished a relation from its converse, which I now think is an unwarranted projection onto universals of the distinction among predicates. More important, I attempted a mereological characterization of the *exemplification* of a universal by one or more particulars. This was too complicated. I now consider it best to concentrate on *instantiation in*, which I distinguish from *exemplification by* in Section One below. The complexities are thus shifted to semantics, which as a human construct should not be expected to be simple.

part/whole and other mereological operations are restricted to first order universals.

1. Preliminaries

1.1. Stipulation concerning the terms 'property' and 'relation'

By a *property* I mean a monadic universal, by a *relation* a universal of adicity greater than one. I shall also be considering 0-adic universals. These are neither properties nor relations.

1.2. A sparse platonist theory of universals[4]

Following Armstrong I analyse similarity in a respect as the sharing of a universal. Similarity in a respect is something we have quite strong intuitions about, extrapolating from perceived resemblance. Intuitively, being of a precise hue of colour such as scarlet is a respect of similarity, and so intuitively we expect there to be a universal *x's being scarlet*. As this example indicates, such intuitions are not infallible. For there is a lively debate between realists and anti-realists about colour, conducted independently of debates about universals So a realist about universals who is an anti-realist about colours will treat *x's being scarlet* as a fictional universal.

Intuitions about similarity are not the only data for a sparse theory. The statements of laws of nature and the truths about causation distinguish various predicates that should correspond to universals. In addition, a theory of structural universals should provide enough (first and higher-order universals) to form the topic of mathematics (see [8]).

On a sparse theory we have a list of basic first-order universals, ones lacking structure. There is a range of operations on them that does not imply that every property has a negation, and does not imply that any two properties have a disjunction. Nor are these restrictions merely consequences of the requirement that no property belongs to everything possible and no property belongs to nothing possible. In this context, the mark of the sparse conception is the reluctance to admit disjunctive

[4] 'Platonist' only in the sense of granting the existence of uninstantiated universals, including uninstantiated 0-adic universals, which may be said to obtain (in something like Roderick Chisholm's sense) if and only if they are actually instantiated in some particular.

or negative properties, rather than the positing of some minimal list of basic universals. I shall, therefore, help myself to illustrative examples, such as the relation of *x's loving y*, which might not meet the strict standards of a sparse theory.

There are two respects in which my proposed theory is sparser than Armstrong's. The first is that there are no impure relational properties, such as *x's being within a billion kilometres from the Sun*. There are, however pure relational properties, such as *x′s being within a billion kilometres from a star*. The second respect in which I am sparser than Armstrong is that I do not treat as distinct a dyadic relation (e.g. *x loving y*) and its converse (*x being loved by y*).[5] And generalizing, I do not accept a multiplicity of n-adic universals obtained by permuting the places. As a consequence we do not need the operation of permuting places.

I require every universal to be possibly instantiated. In addition, to avoid trivial universals I require every universal to have a possible non-instance.[6] For example, the putative relation of *x's being identical to y* collapses into the property of *x's being identical to x*, which I reject as trivial.

1.3. Some notation

To avoid cumbersome notation I rely heavily on two conventions. The first is that universals are referred to using italics, but predicates are not. The second is that the same letters will be used for predicates and the corresponding universals, where there are any. Consequently referring expressions for a universal have free variables, as would the corresponding predicate. In this way there is no confusion with other uses of italics.

Thus the metathety (between-ness) universal would be referred to not as *being between* but as *y being between x and z*. If considering a monadic universal I write Ux or Uy, Vx or Vy etc. For a dyadic universal, I write Uxy or Uyz, Vxy or Vyz etc. When considering universals of arbitrary adicity, I write $Uxy\ldots$, $Vxy\ldots$, etc.

[5] This non-distinction is intuitive provided we scrupulously refrain from projecting the structure of language onto our ontology.

[6] Because of my reliance on *instantiation in*, this amounts to requiring that for every universal it be possible that there be some particular in which the universal is not instantiated.

1.4. Correspondence

On a sparse theory only some predicates correspond to universals. On the other hand, for any universal $Uxy\ldots$, we can form the following predicate: u, v etc. exemplify $Uxy\ldots$. This is an especially straightforward case of the correspondence of some predicates and some universals. I shall use 'Uxy...' (or with any distinct letters replacing x and y) ambiguously for any predicate corresponding to $Uxy\ldots$.

Correspondence could easily be defined if we assumed *Modal Extensionality*, the principle that if $Uxy\ldots$ and $Vxy\ldots$ are necessarily co-extensive, then $Uxy\ldots$ is identical to $Vxy\ldots$. For in that case, a predicate corresponds to a universal just in case the predicate and universal are necessarily co-extensive; that is, necessarily the predicate applies to all and only the (n-tuples) of particulars that exemplify the universal. I shall not, however, assume Modal Extensionality. The case provided by Elliott Sober in [12] against it might persuade some readers. He uses the putative counter-example of *triangularity* and *trilaterality*. In addition, the operations on universals that I consider may themselves generate counter-examples as I show below. I shall, therefore, assume we know what correspondence amounts to and consider my need to define it as one of the objections I consider at the end.

Because I shall not be distinguishing a relation from its converses, correspondence is ambiguous up to the order of the n-tuples in the predicate. Thus the same relation corresponds to 'x loves y' and 'y loves x', but not, alas, to the symmetric 'x and y love each other'. More alarming, if, for instance, Px corresponds to a universal Px then the predicates, Px & Py, Px & Py & Pz etc. all correspond to the same universal Px. So by the adicity of a universal I mean the minimum number of variables in a corresponding predicate.

1.5. Instantiation in

If a universal $Uxy\ldots$ is exemplified by u, v etc, and if u, v etc. are all parts of w, then I say that $Uxy\ldots$ is *instantiated in* w. So, obviously if w is part of w' then $Uxy\ldots$ is also instantiated in w'. Although I mark the contrast with the words 'exemplify' and 'instantiate' the contrast is between being exemplified *by* and being instantiated *in*.

My reason for considering *instantiation in* is that the mereological theory of properties is part of the more ambitious program of Ontology

Within the Bounds of Mereology, in which a 0-adic universal is instantiated in a particular just in case it is part of that particular.[7] Although I shall not be assuming that more general program, I propound nothing incompatible with it. This motivates the basic hypothesis of my theory:

> *Parthood Instantiation Principle*: If universal $Uxy\dots$ is part of universal $Vxy\dots$ then: necessarily, for all u, if $Vxy\dots$ is instantiated in u so is $Uxy\dots$.

I take it that this provides an explanation of the situation in which necessarily for all u, if $Vxy\dots$ is instantiated in u so is $Uxy\dots$ I shall be guided by this and, where it coheres well with the other hypotheses assume that this is *the* correct explanation. Hence mutual parthood is the default explanation of fact that universal $Uxy\dots$ and $Vxy\dots$are necessarily instantiated in the same particulars. And, in accordance with the assumed mereology, mutual parthood implies identity. An example is the above mentioned case of $Px = Px \,\&\, Py = Px \,\&\, Py \,\&\, Pz$.

For the remainder of the paper, I shall ignore *exemplification* and concentrate on *instantiation in*.

1.6. More notation

The symbols '&', '∃', '∨' and '¬' are reserved for operations on predicates. Hence the nominal definition of the (narrow) conjunction of two monadic universals Ux and Vy is $Uz \,\&\, Vz$, which is equivalent to:

> The conjunction of two monadic universals Ux and Vy is the universal corresponding to the predicate 'z exemplifies U and z exemplifies V'.

A theory of the structure of universals should provide characterizations of such operations as conjunction without using correspondence. To use the same symbols for operations on predicates and universals might suggest that we are assimilating universals to predicates. My restriction of the symbols '&', '∃', '∨' and '¬' to operations on predicates helps to avoid this suggestion. If, for instance, I wrote the (narrow) conjunction

[7] To avoid the paradox that the 0-adic universal *there being something simple* is a proper part of any simple particular, I claim that although the predicate 'is simple' applies univocally to particulars and to universals there is no corresponding universal. Instead there is a universal of *x's being simple qua particular*, that is, having no other particular as a part.

of Ux and Vy as Uz & Vz, the nominal definition would be: Uz & $Vz =$ Uz *&* Vz, where the left and right hand sides differ only in whether the symbol '&' is in italics.

2. Mereology without unique fusion

As a reminder that the mereology being considered holds for universals as well as particulars I use upper case letters for the variables. I take the primitive predicate to be the transitive reflexive dyadic predicate 'X is a part of Y' ('$X \leqslant Y$'). We may define identity ($X = Y$ iff $X \leqslant Y$ and $Y \leqslant X$), proper parts ($X < Y$ iff $X \leqslant Y$ and not $Y \leqslant X$), overlapping ($X \circ Y$ iff for some Z, $Z \leqslant X$ and $Z \leqslant Y$) and being disjoint: X and Y are disjoint just in case they do not overlap)[8]. X is a maximal proper part of Y if $X < Y$ but for no Z do we have $X < Z < Y$. X and Y are mutual fusions iff: for all Z, $X \circ Z$ iff $Y \circ Z$. Mutual fusion is obviously an equivalence relation. The equivalence classes will be called *fusion classes*.

An alternative development of mereology is based on the concept of mereological *sum*. Then we may define $X \leqslant Y$ as $Y = X + Y$. This is I think inferior for two reasons. First, the paradigms of things with parts are not mereological sums but rather structured things with essential relations between the parts. The idea of a mereological sum is an abstraction away from these relations. For artefacts this abstraction is easy. A piece of flat pack furniture might consist of, say, 97 parts to banged and screwed together. The resulting piece of furniture consists of 96 parts with various essential geometric relations plus the left-over screw. It is easy to abstract away from these relations and consider the sum of the 96 parts, which existed before assembly and would continue to exist given a gentle enough disassembly. Such abstraction is harder for the case of an animal composed of distinct organs. Thus Aristotle's unwillingness to think of the organs as parts is understandable if we take summation as primitive. He was a brilliant zoologist but it would have been unfair to ask him to take his sea urchin apart and put it back together again — alive. The lesson is to prefer '$\leqslant$' as the primitive predicate, and take an animal's organs as paradigm parts.

[8] Or, if identity is taken as a distinct primitive predicate, proper parthood may be taken as primitive, and parthood characterised as $X \leqslant Y$ if either $X = Y$ or $X < Y$. In that case, we require that proper parthood is transitive and strictly anti-symmetric (i.e. for no X and Y do we have $X < Y$ and $Y < X$).

Another reason for preferring '$\leqslant$' as the mereological primitive, is that, in the interests of intellectual economy, we should try to give an account of structure in terms of mereology and hence avoid a structural theory of mereology. I admit that it is not that clear just what counts as structure, but X being the sum of Y and Z seems more like structure than U being part of V.

I conclude that the idea of summation as composition relies too much on the use of artefacts as paradigms, and I draw the corollary that we should not rely upon intuitions based on the summation-as-composition approach to mereology.

How then do we analyse summation? Elsewhere I have used examples to argue that our intuitive idea of a sum is of a least upper bound that is also a fusion (see [6]). If readers disagree they may take that as a stipulative definition of summation. So I have the following definitions.

1. An *upper bound* of some things is something that has all of them as parts. A *lower bound* of some things is something that is part of them all.
2. A least upper bound or *join* of some things is an upper bound of them that is part of any other upper bound of them. A greatest lower bound or *meet* of some things is an lower bound of them of which any other lower bound is part.
3. A *fusion* of some things, the Fs, is something that overlaps precisely those things that overlap some F. (Hence, as expected, X and Y are fusion equivalent iff X is a fusion of Y and vice versa.)
4. A *sum* of some things, the Fs, is a join of the Fs that is also a fusion. A proper sum is a proper join that is also a fusion.

By anti-symmetry there cannot be more than one join of some things, and we may talk of *the* sum of X and Y, $X+Y$. I shall assume a general principle governing the mereology of universals, namely that any universals X and Y have a join and this join is the sum $X+Y$. I have no objection to assuming that even infinitely many universals have a sum, which is required for a theory of infinitely complex universals, but that is not my present concern.

Mereology in Leśniewski's sense (Classical Mereology) is obtained by assuming the existence and uniqueness of arbitrary fusions. This paper is compatible with the existence of arbitrary fusions although some readers might want to restrict fusion to things of the same kind. Uniqueness of fusion is, however, more problematic and I reject it along with Weak Supplementation. The case for their rejection is that putative counter-examples occur in several different areas of metaphysics. Here are some instances.

First, suppose we follow David Lewis and take the subsets of a set to be parts. If we then allow the empty set as a genuine subset of every set, all sets overlap. Assuming also that pure sets have no parts other than sets, it follows that every pure set is a fusion of every other pure set. In particular $\emptyset$ and $\{\emptyset\}$ are mutual fusions, as well as providing a counter-example to Weak Supplementation. 'So much the worse for the empty set', you might retort; and if this was the only putative counter-example to unique fusion that retort would be appropriate.

My next case is that some of us find it intuitive that s and all its parts are among the parts of $\{s\}$. In that case, $\{\{s\}\}$ and $\{s\}$ are mutual fusions, and a counter-example to Weak Supplementation[9].

Finally, there is the lump of clay and the statue — of 'The Maid of All Work' presumably. Here is a simplified case. Suppose b and c are distinct particular atoms that are adjacent. Then we may consider both the analog of the clay, their sum $(b + c)$, and the analog of the statue, b-adjacent-to-c. First suppose we are nominalists and deny the existence of a relation of x *being adjacent to* y, as well as derived properties such as x *being adjacent to something*. Then $(b+c)$ and b-adjacent-to-c are mutual fusions and provide a counter-example to Uniqueness of Fusion. Judith Jarvis Thompson (1998: 155) takes the statue and the lump of clay as mutual parts, on the basis that if at some time the location of x is part of that of y then at that time x is itself part of y. Aaron Cotnoir ([4]) would say that because they are mutual parts they provide a counter-example to Mereological Extensionality.[10] Both on grounds of conservativism and intuition even a nominalist should concede that $(b + c)$ is a proper part of b-adjacent-to-c, and not vice versa. So this is a counter-example to Weak Supplementation rather than Mereological Extensionality.

If, as I advocate, however, we are realists about the relation x *being adjacent to* y, holding between b and c, then we could say that b-adjacent-

[9] Ben Caplan, Chris Tillman, and Pat Reeder (see [3]) share my intuition. They assume that $\emptyset < \{s\} < \{\{s\}\}$, but not $\emptyset \leqslant s$, unless s is itself a set.

[10] The supposed failure of antisymmetry for parthood without identity is also discussed by Cotnoir and Andrew Bacon in [5] and Lida Obojska in [11]. Chris Tillman and Gregory Fowler in [14] argue for this failure on the grounds that the sum of everything is a part of propositions about this sum of everything, which in turn is part of the sum of everything.

to-c overlaps that relation, but $(b + c)$ does not, so they are not mutual fusions. But now consider three atoms, b, c and d each adjacent to the other two, and compare ($b + c$-adjacent-to-d) with (b-adjacent-to-$c + d$). If the adjacency relations are the same in the two cases this is another counter-example to Uniqueness of Fusion. So classical mereology drives us to taking the relations as relation-instances ('tropes').

In all three examples we could reject otherwise fairly promising metaphysical speculations by an appeal to the uniqueness of fusions. In the third case our conclusion would not merely be negative; we would have a positive argument for the 'trope' theory. In none of the three examples, however, is the uniqueness intuitive in the circumstances. In addition, I hold that the uniqueness of sums, and, I suspect, the uniqueness of fusions, is held largely as a result of identifying fusions with sums. I take these considerations to undermine the Uniqueness of Fusions.

Given arbitrary summation, for any given Y, we may consider the sum Y^{Σ} of the fusion class containing Y. It is itself fusion equivalent to Y. So we may think of the Y^{Σ} obtained this way as the members of a classical mereological system. Hence we may analyse the mereology into (a) the classical mereology of the classical representatives, the Y^{Σ}, and (b) the structure of the fusion classes. I see no need however to reject the anti-symmetry of proper parthood, as Cotnoir ([4]) and others have. (That would result in a further refinement of classical mereology.)

This digression is relevant because it shows that concentrating, as I shall, on fusion classes is not some technical trick but a natural way of thinking of non-classical mereology as a refinement of classical mereology without going as far as Cotnoir. It is also explains why classical mereology fails to provide a theory of structural universals. For such a theory requires operations for 'forming' a universal out of another fusion equivalent one.

3. Operations on universals, with examples

My method is to consider the operations used to 'compose' structural universals. I shall then show how these operations may be analysed in purely mereological terms.

The first operation I consider is reflexive binding whereby two of the places of a universal are identified. For instance, if Rxy is a dyadic relation then often there is a monadic universal, Rxx. So if there is a relation Rxy of *x being a mirror image of y*, Rxx is the property *x's being its own mirror image.* The exceptions occur when Rxx either for no x or for all. If, for instance, we take *x being a part of y* to be a universal then it has no reflexive binding because everything is part of itself, and if we take *x being a proper part of y* to be a universal then it has no reflexive binding because nothing is a proper part of itself. In general, if $n \geqslant 0$ and $Ux_1x_2\dots x_{n+1}$ is $(n+1)$-adic, then there may be reflexive bindings to an n-adic universal obtained by replacing x_i and x_j by z, for any $i \neq j$.

Next there is existential binding. If Uxy is an $(n+1)$-adic universal, where $n \geqslant 1$ then there universals such as $\exists x\, Uxy$ and $\exists y\, Uxy$ in which a free variable is bound by an existential quantifier. This operation applies, I submit, even to properties. For I hypothesize that for any property Ux there is an existential state of affairs of the form $\exists x\, Ux$, which obtains in a particular z just in case Ux is instantiated in z. Because they can obtain in many disjoint particulars these are 0-adic universals not particulars.

These two operations of reflexive and existential binding generate the counter-example to modal extensionality that I mentioned above. Consider a transitive symmetric relation Rxy that is not reflexive. In that case Ryy and $\exists x\, Rxy$ are necessarily co-extensive. And such relations cannot be excluded by appeal to the sparse character of the realism about universals. For consider Sxy, a symmetric irreflexive relation, for example *x being adjacent to y without overlapping.* Then the ancestral Rxy of Sxy will be symmetric and transitive but perhaps not reflexive. In the example, Sxy is the property of *y being adjacent to something* (*without overlap*).

The conjunction of two monadic universals Ux and Vy is usually—I say narrowly—defined as Uz & Vz, using the convention that the symbol '&' connects predicates. This definition of narrow conjunction extends straightforwardly to two universals of the same adicity:

> If $Ux_1x_2\dots x_n$ and $Vy_1y_2\dots y_n$ are both n-adic, then their narrow conjunctions are the n-adic universals obtained from the predicate $\mathrm{Ux_1x_2\dots x_n}$ & $\mathrm{Vy_1y_2\dots y_n}$ by identifying the $\mathrm{x_i}$ with the $\mathrm{y_j}$, in a 1 to 1 fashion.

For example, we may consider the universal $Mxyz =$ *y being (geometrically) between x and z*, and $Bxyz =$ *y being of mass between x and z.* Then there is a narrow conjunction $Mxyz$ & $Bxyz$. It holds between the

Sun, the Earth and the Moon during an eclipse of the Moon but not during an eclipse of the Sun. Another narrow conjunction is $Mxyz$ & $Bzyx$, which holds between Sun, Moon and Earth during an eclipse of the Sun.

To introduce some order into the proliferation of kinds of conjunction, I introduce the *broad conjunction* of universals $Uxy\ldots$ and $Vxy\ldots$. This is the least universal whose instantiation in w implies that both $Uxy\ldots$ and $Vxy\ldots$ are instantiated in w. Later I shall characterise it as the join of $Uxy\ldots$ and $Vxy\ldots$. Often the broad conjunction of the m-adic $Ux_1x_2\ldots x_m$ and the n-adic $Vy_1y_2\ldots y_n$ is the $(n+m)$-adic universal $Ux_1x_2\ldots x_m$ & $Vy_1y_2\ldots y_n$. The broad conjunction may, however, have adicity less than $n+m$. For instance, I shall argue that every universal is the broad conjunction of it with itself.

Any other conjunction of universals is the result of one or more reflexive bindings of a broad conjunction. Thus if $n = m$ the narrow conjunctions (e.g. $Uz_1z_2\ldots z_m$ & $Vz_1z_2\ldots z_m$) are obtained from the broad conjunction $Ux_1x_2\ldots x_m$ & $Vy_1y_2\ldots y_n$ by reflexive bindings that identify x_i with y_i in some order (e.g. $x_i = y_i = z_i$).

The broad conjunction of a universal with itself is, I now argue, the same universal. For simplicity consider a monadic universal Ux. Then the broad conjunction is Ux & Uy. There is no requirement that $x \neq y$, so Ux is instantiated in precisely the same particulars as Ux & Uy. That does not, I admit, prove that they are the same but it supports the identification. In addition, they make the same contribution to resemblance, causal powers and any other roles played by universals. Likewise, if Rxy is not itself a conjunction Ux & Vy, it corresponds to the $2k$-ary predicates Ruv, Ruv & Rwx, Ruv & Rwx & Ryz, etc. In general a non-conjunctive n-adic universal corresponds to nk-ary predicates.

There will not, in general be a unique narrow conjunction of universals. For example, if Rwx and Syz are distinct and neither is symmetric then we have two dyadic narrow conjunctions Rxy & Sxy and Rxy & Syx, as well as the four triadic conjunctions Rxy & Sxz, Rxy & Szx, Rxy & Syz, and Rxy & Szy. The triadic conjunctions are distinct results of reflexive binding of the broad conjunction and the two dyadic conjunctions further reflexive bindings.

Because the n-adic $Rx_1x_2\ldots x_n$ is also the $2n$-adic $Rx_1x_2\ldots x_n$ & $Ry_1y_2\ldots y_n$, there can be non-trivial narrow conjunctions of a universal with itself, such as Rxy & Ryz. Hence the adicity of a reflexive binding can be greater than that of the relation being bound. Thus if Rxy is not transitive Rxy & Ryz has adicity 3.

Some other non-trivial narrow conjunctions of a universal with itself are the various symmetrisations such as the symmetrisation Rxy & Ryx of a non-symmetric non-antisymmetric dyadic relation Rxy.

A narrow conjunction of special importance in a theory of structural universals is Ux & Vy & Rxy, if Rxy is dyadic, and its analogs for relations of higher adicity. For suppose Ux and Vy are the natures of particulars u and v, which, I further suppose, are the unique exemplifications of those universals in w. In that case the instantiation of Ux & Vy & Rxy in w is a surrogate for the particular state of affairs Ruv. Now I am not considering particulars or exemplification, so this surrogate is a way of characterising the structural universals as if we could describe them in terms of complex particulars. The universal Ux & Vy & Rxy may be treated as a reflexive binding of the broad conjunction of Ux & Rxy with Vu & Ruv.

If two universals of the same adicity n, $Uxy\ldots$ and $Vwz\ldots$ have no narrow conjunction $(U \mathbin{\&} V)xy\ldots$, then they are incompatible. Because I am not distinguishing relations from their converses, incompatibility is in general stronger than inconsistency between a pair of corresponding predicates. The resulting problems do not, however, arise with monadic universals or totally symmetric universals. So we may use incompatibility to define their negations:

> If universal $Uxy\ldots$ is monadic or totally symmetric then it has a negation $Vxy\ldots = \neg Uxy\ldots$, only if $Vxy\ldots$ is the disjunction of all those universals of the same adicity as $Uxy\ldots$ that are incompatible with $Uxy\ldots$.

On a sparse theory not all universals have negations. Some plausible examples of negation nonetheless holding between universals are the monadic *x is finite* and *x is infinite*; and the symmetric dyadic *x overlapping y*, and *x and y being disjoint*.

Relative negation or subtraction results in universals of the form $Uxy\ldots$ & $\neg Vxy\ldots$, where $Vxy\ldots$ is ether monadic or totally symmetric. These, if they exist, are the disjunctions of all those universals that are narrow conjunctions of $Uxy\ldots$ with a universal incompatible with $Vxy\ldots$. One important example is the irreflexive strengthening of a dyadic relation Uxy, which, if it exists, is the subtraction of the symmetrisation of Uxy from Uxy itself. For example if Uxy is *x loving y*, the irreflexive strengthening is *x loving y but not vice versa*. Another important example is the universal *x having as parts exactly n disjoint*

Us, which is the result of subtracting *x having as parts at least n disjoint Us* from *x having as parts at least $n-1$ disjoint Us*. The universal *x having as parts at least n disjoint Us* will be analysed below.

The broad and other disjunctions of universals, if they exist, are defined much as for conjunctions, by replacing '&' by 'and/or'. It should be noted, however, that three or more universals might have a disjunction without two of them having one. This is the typical situation with determinables of a determinate: there is no disjunction of *x's being dove grey* and *x's being charcoal grey* but there is a disjunction of all the (50?) shades of grey, namely the determinable *x's being grey*. A theory of sparse universals should tell us some of the circumstances in which universals have disjunctions. I submit that determinable properties may well be such cases of systematic disjunction. We have an intuition, which should be endorsed by those who are realist both about colours and about universals, that *being red* is a respect of resemblance, and hence that the predicate 'is red' corresponds to a determinable property rather than referring to several precise red hues. Likewise, enthusiasts for speculative physics (a.k.a. mathematical metaphysics) who are also realists about universals might well wonder whether the determinable universal *x's being of hypervolume less than one Planck unit* has any (actual) instances. It is natural, although not compulsory, to think of determinables whose determinates are first order properties as themselves first order properties, namely the disjunction of the, perhaps infinitely many, determinates. The alternative is to treat determinables as higher order properties. In that case 'red' applies to x just in case x is an instance of some universal X that is itself an instance of *X's being a shade of red*. Without settling this, I shall provide a theory that treats a determinable as the disjunction of its determinates.

The determinates of a determinable are pairwise inconsistent, which would cause me problems if there were polyadic determinables that were not totally symmetric. A putative example would be *x being earlier than y*, whose determinates would be of the form *x being earlier than y by t seconds* for any positive real number t. I submit, however, that we can analyse this as the conjunction of *x being earlier than y* with *x being separated from y by t seconds*. The determinable *x being separated from y by some positive number of seconds* is, then, symmetric. Quite generally given any antisymmetric determinable we may analyse it in terms of its qualitative structure and a symmetric quantitative determinable.

I shall conclude this section with a couple of examples, starting with *x having as parts at least n disjoint Us*. This is, I say, one relatum of the higher order dyadic relation, of *cardinality at least n*. The other relatum is Ux itself. Clearly *x having as parts at least n disjoint Us* is to be characterised recursively. The recursion requires the dyadic relation *x and y are disjoint*. Then *x having as parts at least $n+1$ disjoint Us* is the result of existential binding (of x and y) of repeated reflexive bindings (putting $x = x_1 = x_2 = x_3$, and $y = y_1 = y_2 = y_3$) of the broad conjunction of:

(1) *x_1 is part of z*,
(2) *y_1 as part of z*,
(3) *x_2 having as parts at least n disjoint Us*,
(4) *y_2 having a U as a part* and
(5) the relation of *x_3 being disjoint from y_3*.

Of these (4) *y_2 having a U as a part*, is itself obtained by existential binding (of w) of the repeated reflexive binding (putting $w = w_1 = w_2$) of the broad conjunction of *w_1 is part of y_2* and Uw_2. The case $n = 1$, which starts the recursion does not require (3).

Finally, in honour of Lewis' objection, *u is a methane molecule* is the narrow conjunction of:

(1) *u has one part that is a carbon atom*,
(2) *u has four parts that are hydrogen atoms* and
(3) the existential binding (of v, w, x, y, and z) of the repeated reflexive binding (obtained by ignoring subscripts) of the external conjunction of: *v_1 is part of u, w_1 is part of u, x_1 is part of u, y_1 is part of u, z_1 is part if u; v_2 is a carbon atom, w_2 is a hydrogen atom, x_2 is a hydrogen atom, y_2 is a hydrogen atom, z_2 is a hydrogen atom; v_3 is bonded to w_3, v_4 is bonded to x_3, v_5 is bonded to y_3,* and *v_6 is bonded to z_3*.

4. The mereology of structural universals

In this section I explore the mereology of structural universals, a topic that is both of intrinsic interest (to me anyway) and a preliminary to the mereological characterisation of the operations considered above.

4.1. Broad conjunction

I identify the broad conjunction of two universals with their join, which I am assuming is also their sum, noting that the rival hypothesis (the identification of narrow conjunction with the sum) violates the theoretically elegant principle that any two things (of the same kind) have a sum. For inconsistent universals have no narrow conjunction. Moreover the rival requires the distinction between a dyadic relation and its converse. Consider for instance the narrow conjunctions of the anti-symmetric relations:

$$Ewx = w \textit{ being earlier than } x$$

and

$$Syz = y \textit{ being shorter (i.e. of less duration) than } z.$$

Then Exy & $Sxy \neq Exy$ & Syz, but if narrow conjunctions were sums both would have equal claim to be the sum.

4.2. Existential binding

Relying on the Parthood Instantiation Principle, I take a universal to be part of any of its reflexive bindings. Existential bindings are not so straightforward. For $Uxy\ldots$ and its existential binding $\exists x\, Uxy\ldots$ are instantiated in precisely the same particulars. Consider the case of the monadic Vx and its 0-adic existential binding $\exists x\, Vx$. They differ in their negations, if they have them. For example, take Vx to be *x's being infinite*. Its negation is *x's being finite* but the negation of $\exists x\, Vx$ is the state of affairs of *there being nothing infinite*, that is of *everything being finite*, which is instantiated in a region w just in case w has no infinite parts and so w is itself finite. The property *x's being finite* is, however, instantiated in w just in case w has some finite part. Relying on the Parthood Instantiation Principle, *x's being finite* is plausibly taken, therefore, as a proper part of *everything being finite*. Given the way negation was characterised $\exists x\, Vx$ should therefore be a proper part of Vx. Generalising, I hypothesise that any existential binding of $Uxy\ldots$ is a proper part of $Uxy\ldots$. A consequence is that any universals with a common part $Uxy\ldots$ also have a common part that is the 0-adic universal obtained by existential bindings.

4.3. Determinables of determinates

I begin with two pieces of terminology. The first is that if either $Uxy\ldots$ stands to $Vxy\ldots$ as determinable to determinate or $Uxy\ldots = Vxy\ldots$, then $Uxy\ldots$ is said to be within $Vxy\ldots$. The second stipulation is that $Uxy\ldots$ is said to be the core of $Vxy\ldots$ if it is within $Vxy\ldots$. and there is no universal distinct from $Uxy\ldots$ within $Vxy\ldots$.

4.4. Some fusion equivalences

Because any two universals that overlap have a 0-adic universal as a common part, fusion may be characterised in terms of 0-overlapping, that is universals sharing the same 0-adic parts. It follows that a fusion class is closed under the operation of existential binding. I hypothesise that it is also closed under the operation of reflexive binding.

I also hypothesise that the fusion class of a conjunction $Uwx\ldots$ & $Vyz\ldots$ has for its members the universals $Xwx\ldots$ & $Yyz\ldots$ such that $Xwx\ldots$ is fusion equivalent to $Uwx\ldots$ and $Ywx\ldots$ is fusion equivalent to $Vwx\ldots$. A bolder hypothesis, a conjecture even, is that if some universals have a disjunction then it is fusion equivalent to all the disjuncts, because it is their determinable.

Any determinable is, I hypothesise, fusion equivalent to its determinates. Here I include determinable structural universals because substituting a fusion equivalent property in a complex universal preserves fusion equivalence. Thus *x's having red and black parts* is fusion equivalent to *x's having scarlet and black parts.*

The fusion equivalence class of a system of determinates of a determinable property includes its existential bindings, that is the state of affairs that some determinate is instantiated. This suggests a further hypothesis about the fusion class of a universal:

1. Any fusion class has a meet, which is itself a member of the fusion classes. (Hence universals are mutual fusions iff the meet of their classes is the same.)
2. The meet of the fusion class of $Uxy\ldots$ is the total existential binding of a universal $U^*xy\ldots$ that is the core either of $Uxy\ldots$ itself or some universal $Vxy\ldots$ from which $Uxy\ldots$ is obtained by one or more reflexive bindings.
3. If $Wxy\ldots$ is the join of $Uxy\ldots$ and $Vxy\ldots$ then $W^*xy\ldots$ is the join of $U^*xy\ldots$ and $V^*xy\ldots$.

The above hypotheses about mutual fusion are all based on the time-honoured method of inability to think of a counter-example.

4.5. The mereology of narrow conjunction and incompatibility

If neither $Uwx\ldots$ nor $Vyz\ldots$ are parts of the other then they have a proper sum $(U+V)wx\ldots yz\ldots$, which a maximal proper part of any of the *narrower by one* conjunctions obtained by one operation of reflexive binding. Consider any universal Rxy of which $(U+V)wx\ldots yz\ldots$ is a maximal proper part. I hypothesise that either it is the sum of two universals of which $Uwx\ldots$ and $Vyz\ldots$ are parts or it is a narrower by one conjunction of $Uwx\ldots$ and $Vyz\ldots$.

I define the compatibility number of two universals neither part of the other as the number of their narrower by one conjunctions. Incompatible properties and relations have compatibility 0. A 0-adic universal has compatibility 0 with any other universal. If we consider the class of all properties and a totally symmetric universals their pairwise compatibility is at most 1.

4.6. The mereology of determinables and determinates

If we are given the mereological structure of a fusion class we could easily recognise non-trivial systems of determinates of a determinate, as follows. (By a trivial system I mean one with only the one member.)

For simplicity I shall concentrate on the discrete case, namely that in which if $Uxy\ldots$ is the core of $Vxy\ldots$ then for some n there is no sequence of distinct universals:

$$Uxy\ldots = U_0xy\ldots, U_1xy\ldots, \ldots, U_nxy\ldots = Vxy\ldots$$

such that $U_jxy\ldots$ is within $U_{j+1}xy\ldots$. The greatest such n indicates the level of the $Vxy\ldots$, with a core determinable being on level 1.

The mereology of a system of determinables of a determinate has a tree structure.

1. If $Uxy\ldots$ is of level 1 less than $Vxy\ldots$, then $Uxy\ldots$ is a maximal proper part of $Vxy\ldots$, and is of the same adicity.
2. $Uxy\ldots$ is meet of any two of the determinables on the next level that it is within.

3. For systems of determinables that are either properties or symmetric relations: If neither $Uxy\ldots$ is within $Vxy\ldots$ nor vice versa then $Uxy\ldots$ and Vxy are incompatible.

4.7. The product structure of a fusion class

We may consider the universals in a given fusion class **F** that are all cores of systems of determinables. This is a partially ordered system **C** ordered by parthood. We may also consider all the 0-adic members of the fusion class, which forms another partially ordered system **D**, again ordered by parthood. Consider the functions c and d from **F** onto **C** and **D** respectively, such that c maps each universal to its core and d maps each universal to its total existential binding. Then, I hypothesise, the pair of functions c and d exhibit **F** as a product $\mathbf{C} \times \mathbf{D}$ of partially ordered systems. That is, both c and d preserve parthood and: if both $c(Uxy\ldots) = c(Vxy\ldots)$ and $d(Uxy\ldots) = d(Vxy\ldots)$ then $Uxy\ldots = Vxy\ldots$.

We may think of the determinable structure exemplified by **D** stretching out horizontally in a diagram and the structure of existential and reflexive binding exemplified by **C** rising vertically above each member of **D**. For some fusion classes this will be trivial because there is no genuine system of determinates of a determinable, everything universal being of level 1. But in non-trivial cases the product structure is a perspicuous analysis.

5. Using mereology to characterise the operations

By relaxing the requirement of unique fusion the system of all universals has enough mereological structure for us to characterise the various operations. First note that the product structure enables systems of determinates of determinables to be picked out, so, we may if we choose concentrate on the absolute determinates (determinables of level 1).

5.1. The adicity of universals

The 0-adic universals are characterised as those universals incompatible with any other universal. Any universal with a maximal proper part that is 0-adic and which is not itself 0-adic is a property. (The maximal proper part is an existential binding).

For any universal $Uxy\ldots$ that is not a proper sum of universals, we may consider a sequence of narrower by one conjunctions with n properties $P_1x_1, P_2x_2, \ldots, P_nx_n$, resulting in a universal $Vxy\ldots$ such that:

1. No narrow conjunction of any two of the $P_1x_1, P_2x_2, \ldots, P_nx_n$is part of $Vxy\ldots$, and
2. $Vxy\ldots$ is not itself a proper sum.

If n is the largest integer for which there is such a sequence, then $Uxy\ldots$ is n-adic. In that case $Vxy\ldots$ is the universal $Ux_1x_2\ldots x_n$ & Q_1x_1 & Q_2x_2 & $\ldots$ & Q_nx_n, where the Q_jx are the P_ix in some order or other. Call a universal of this form *a full conjunction of* $Uxy\ldots$ with Q_1x, Q_2y etc. The stipulation that $Vxy\ldots$ is not itself a proper sum excludes cases such as that in which because $Ux = Uy$ & Uz, we could consider:

$$Vxy = (Ux \mathbin{\&} Uy) \mathbin{\&} P_1x \mathbin{\&} P_2y = (Ux \mathbin{\&} P_1x) + (Uy \mathbin{\&} P_2y)\,.$$

5.2. Characterising reflexive binding

Reflexive bindings are of two kinds, that in which the $Uxx\ldots$ is formed out of $Uxy\ldots$, and so has adicity one less than $Uxy\ldots$, and that in which the adicity of the reflexive binding is equal or greater than that of the universal that is bound. The former is characterised thus: if $Vwx\ldots$ is an $(n-1)$-adic universal with an n-adic part $Uyz\ldots$ that is maximal among n-adic parts of $Vwx\ldots$, and for which $Vwx\ldots$ is minimal among the n-adic universals with $Uyz\ldots$ as a part, then $Vwx\ldots$ is a reflexive binding of $Uyz\ldots$. In the second kind of reflexive binding $Uxy\ldots$ & $Uyz\ldots$ is formed out of $Uxy\ldots$.[11] This can be characterised using full conjunctions. The $(2n-1)$-adic $Vxy\ldots$ is the reflexive binding of the n-adic $Uxy\ldots$ if the full conjunction of $Vxy\ldots$ with $2n-1$ properties $P_1x_1, P_2x_2, \ldots, P_{n-1}x_{n-1}, Q_1y_1, Q_2y_2, \ldots, Q_{n-1}y_{n-1}$, and P_{n-1} & $Q_{n-1}z$ is the reflexive binding (of the kind just characterised) of the sum of the full conjunction of $Uxy\ldots$ and the n properties $P_1x_1, P_2x_2, \ldots, P_nx_n$ with the full conjunction of $Uxy\ldots$ and n other properties, the $Q_1y_1, Q_2y_2, \ldots, Q_ny_n$.

[11] Strictly speaking, reflexive bindings of the second kind include cases in which $Uxy\ldots$ & $Uyz\ldots$ & $Rwz\ldots$ is formed out of $Uxy\ldots$ & $Rwz\ldots$, where $Rwz\ldots$ is m-adic. In that case the $(2n+m-1)$-adic $Vxy\ldots$ & $Rwz = Uxy\ldots$ & $Uyz\ldots$ & Rwz. But these can be analysed using broad conjunction with $Rwz\ldots$ and the reflexive binding of the second kind without the & $Rwz\ldots$.

5.3. Existential binding

If a universal $Uxy\dots$ is maximal among $(n-1)$-adic parts of a n-adic universal $Vxy\dots$ and $Vxy\dots$ is minimal among n-adic universals with $Uxy\dots$ as a part then either $Vxy\dots$ is a reflexive binding of $Uxy\dots$ or $Uxy\dots$ is an existential binding of $Vxy\dots$. Having characterised reflexive binding, the remaining cases must be existential bindings.

6. Objections and replies

I have characterised various operations on universals using non-classical mereology. I now reply to various objections.

6.1. Correspondence

The operations on universals were defined using correspondence and subsequently characterised mereologically. It remains to characterise correspondence, without relying on Modal Extensionality. For some predicates, we may refer to in some way (perhaps by means of figurative language) some paradigms. In such cases the predicate corresponds to the salient universal instantiated by those particulars, if there is a unique salient universal. To say it is salient among those that are instantiated by the paradigms is to say that is the one we humans think of most easily. It is a contingent fact about human beings that we have rather limited ability to think of universals. Otherwise there might not be enough salient ones. But given this limited ability I hold there are enough correspondences to provide paradigms of the operations on universals. Given our limited powers to think of second-order universals we then can consider the second order universals corresponding to the operations on first order universals. It follows that we do not need a complete correspondence between predicates and (some) universals in order to characterise the operations on universals. It also follows that we may use these operations to extend the domain of predicates for which there is correspondence to include ones corresponding to structural universals ‘formed out of’ ‘more basic’ universals to which predicates already correspond.

6.2. The application of asymmetric predicates

If a relation lacks total symmetry then there is more than one predicate corresponding to it. So, for example, the universal *x's loving y* (Lxy)

corresponds to both the predicates 'x loves y' and 'y loves x'. The objection is that, on my proposed theory, there is nothing that makes it true that Mary loves John rather than John loves Mary.

My reply is that there will be some distinguishing monadic properties, Mx and Jx, to do with, say, Mary's not being called 'John' and vice versa. And the universal Mx & Jy & Lxy is not the same as Mx & Jy & Lyx. They are both upper bounds of the three universals Mx, Jy, and Luv, but neither is the join, which is the broad conjunction, the 4-adic, Jx & My & Luv.

Suppose the only distinguishing features of Mary and John is that Mary loves John but not vice versa — maybe they are angels of the same species. Then my previous reply fails. But in this case we do not need anything to make it true that Mary loves John, not vice versa. Mary is characterised precisely as the one of the pair that loves the other.

Maybe you posit thisnesses, so that in spite of a lack of monadic universals to distinguish Mary from John they have different thisnesses. Then, I submit, thisnesses are to be considered honorary universals and incorporated into the mereological theory as such.

6.3. The perverse convoluted ontology

The complicated mereological characterisation of the 'structure' of universals could be objected to both by a misguided appeal to simplicity and in a less obvious way, to which I must reply in detail. The misguided objection is that the complications of my proposed theory rule it out as a fundamental ontology. It is not, I reply, the proposed ontology that is shown to be complicated but the correspondence with our predicates.

The less obvious objection relies upon the way that scientific realism presupposes that scientists 'carve nature at the joints'. Hence, the predicates of established science must correspond straightforwardly to universals. For example methane and other hydrocarbons may be classified in terms of the structure of carbon, hydrogen and bonding. Again, the various atomic nuclei are classified in terms of the numbers of protons and neutrons in them. If we replace this classification by the proposed mereological analyses, we get gobbledegook.

I concede that the way we think of things does correspond rather closely to the structure of universals. I further concede that the mereological theory of structural universals does not reveal this correspondence, which may, however, be described in terms of second-order universals. Thus the second-order relation of X being the narrow conjunction of Y and Z holds between three universals $Uxy\ldots$, $Vxy\ldots$ and $Wxy\ldots$ just in case $\ldots$, where the dots are filled in by the proposed mereological analysis of narrow conjunction; and likewise for all the other operations. These second-order universals reveal the structure of complex universals, as proposed by John Bigelow in [2]. The mereological analysis of the operations has the advantage over Bigelow's of not multiplying primitive second-order universals. Nonetheless these second-order relations are real, and underpin the anti-Kantian correspondence between the way we think and the way things are in themselves, which is presupposed, I say, by scientific realism.

Conclusion

If we reject classical mereology then it is possible to develop a mereological theory of first-order universals, illustrating the power of non-classical mereology. This is just one step towards a mereological theory of the whole of analytic ontology.

Acknowledgments. Many thanks to Rafał Gruszczyński, Achille Varzi and an anonymous reader for their helpful comments.

References

[1] Armstrong, D., *Universals and Scientific Realism, Vol. II. A Theory of Universals*, Cambridge: Cambridge University Press, 1978.

[2] Bigelow, J., "Towards structural universals", *Australasian Journal of Philosophy*, 64 (1986): 94–96. DOI: 10.1080/00048408612342291

[3] Caplan, B., Ch. Tillman, and P. Reeder, "Parts of singletons", *Journal of Philosophy*, 107 (2010): 501–533.

[4] Cotnoir, A. J., "Anti-symmetry and non-extensional mereology", *The Philosophical Quarterly*, 60 (2010): 396–405. DOI: 10.1111/j.1467-9213.2009.649.x

[5] Cotnoir, A. J., and A. Bacon, "Non-wellfounded mereology", *Review of Symbolic Logic*, 5 (2012): 187–204. DOI: 10.1017/S1755020311000293

[6] Forrest, P., *The Necessary Structure of the All-pervading Aether: Discrete or Continuous? Simple or Symmetric?*, Ontos, 2012. DOI: 10.1515/9783110325928

[7] Forrest, P., "Exemplification and parthood", *Axiomathes*, 23 (2013): 323–341. DOI: 10.1007/s10516-013-9215-6
[8] Franklin, J., *An Aristotelian Realist Philosophy of Mathematics*: *Mathematics as the Science of Quantity and Structure*, Palgrave MacMillan, 2014. DOI: 10.1057/9781137400734
[9] Lewis, D., "Against structural universals", *Australasian Journal of Philosophy*, 64 (1986): 25–46. DOI: 10.1080/00048408612342211
[10] Lewis, D., *Parts of Classes*, Blackwell, 1991.
[11] Obojska, L., "Some remarks on supplementation principles in the absence of antisymmetry", *Review of Symbolic Logic*, 6 (2013): 343–347. DOI: 10.1017/S1755020312000330
[12] Sober, E., "Why logically equivalent predicates may pick out different properties", *American Philosophical Quarterly*, 19 (1982): 183–189.
[13] Thompson, J. J., "The statue and the clay", *Nous*, 32 (1998): 149–173. DOI: 10.1111/0029-4624.00094
[14] Tillman, Ch., and G. Fowler, "Propositions and parthood: The universe and antisymmetry", *Australasian Journal of Philosophy*, 90 (2012): 525–539. DOI: 10.1080/00048402.2011.611812
[15] Varzi, A., "The extensionality of parthood and composition", *The Philosophical Quarterly*, 58 (2008): 108–133. DOI: 10.1111/j.1467-9213.2007.542.x

Peter Forrest
School of Humanities
University of New England
Australia
pforrest@une.edu.au