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## IS TIME REVERSIBLE?

**Abstract.** This paper presents an elementary logical proof of time irreversibility. To this effect, we construct a simple mathematical model of a process, which can demonstrably be shown to be irreversible in the sense that it cannot be reproduced in a backward direction by the very conditions of its construction. To model this process we employ a puzzle problem (paradox) referred to in the literature as “Austin’s Dog” or “Trojan Fly”.

**Keywords:** time; process; direction of time; reversibility of time; Trojan Fly

### 1. Preliminary remarks on time reversibility

It has already become a matter of philosophical decorum when considering the problem of time, to complain about its “mysteriousness”, with a standard reference to the famous excerpts from Chapter XIV, Book 11 of St. Augustine’s *Confessions*; see e.g., [2, p. 42], [3, p. 258], [5, p. 368] or [10, p. 3]. David Deutsch, a notable physicist and popularizer of science, makes these complaints more specific by claiming that

all the mysteries of time stem from its basic, common-sense attribute, namely, that the present moment, which we call “now”, is not fixed but moves continuously in the future direction. [3, p. 258]

The problem is that the very “movement” in question, usually called *the flow of time* [11], is not at all as obvious as it may seem at first sight. Deutsch remarks that from the perspective of (classical) physics, the notion of a “time flow” is in fact redundant: “None of Newton’s physical theories refers to the flow of time, nor has any subsequent physical theory referred to, or been compatible with, the flow of time” [3, p. 266]. Moreover, it is very well known that not only in Galilean, but also in Lorentz

transformations, the time factor is deprived of any non-quantitative characteristics, such as the characteristic of directedness.

Meanwhile, many philosophical conceptions consider directedness to be the most fundamental and specific feature of time, determining its *qualitative nature*; see e.g., [12, p. 19ff]. The much celebrated Leibnizian definition of time as an *order of successions* [7, p. 14], is of a purely qualitative nature as highlighted by Samuel Clarke in his well-known objection [7, p. 19]. Hans Reichenbach also remarks that time order belongs to *directed relations*, and the direction of time can be expressed exactly by the distinction between past and future [12, p. 26–27].

The idea of directedness of time is closely related to that of its irreversibility. Assume that time normally flows in a certain (fixed) direction. Can this direction ultimately be changed to the opposite direction? Again, from the perspective of classical physics, there is no reason why such inversion could not happen. Indeed, as has been repeatedly pointed out in the literature, “most laws of nature are invariant for time reversal”, which means that “when we let the clock run backwards rather than forwards, the deterministic laws that govern the macroscopic world do not change” [14, p. 1]. This holds for Newton’s laws, Maxwell’s equations<sup>1</sup>, Schrödinger’s equation, and the equations of fluid mechanics. Even the much disputed in this respect second law of thermodynamics, can be formulated in a time-symmetric manner where “irreversibility or even time-asymmetry plays no role”, and thus, one could claim that “the second law has nothing to do with the arrow of time” [15, p. 305].

Plato in *Statesman* [9, 269c–271a] attempted to substantiate the possibility of a “pendular” development of the world as a whole, by describing a “reversal which takes place from time to time of the motion of the universe”. The situation “which takes place at the time when the transition is made to the cycle opposite to that in which we are now living” is presented here in rather expressive words:

The life of all animals first came to a standstill, and the mortal nature ceased to be or look older, and was then reversed and grew young and delicate; the white locks of the aged darkened again, and the cheeks of the bearded man became smooth, and recovered their former bloom; the bodies of youths in their prime grew softer and smaller, continually

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<sup>1</sup> “In the case of electromagnetic theory, it is immediately seen that Maxwell’s equations, like the equations of classical mechanics, are unchanged when the time direction is reversed” [16, p. 8].

by day and night returning and becoming assimilated to the nature of a newly-born child in mind as well as body; in the succeeding stage they wasted away and wholly disappeared.

This colorful description of a possible world inversion may be considered classical. As Zwart explains,

time flows back, in the sense that everything that has happened takes place again, but in the reverse direction of before. All processes reverse their directions, all systems run through the same series of states as before, only in the reverse order of succession. In this way all traces of what happened when time flowed forward are destroyed when it flows back, as if it had never happened at all. It would be like what we see when a motion picture is run off in the reverse: burning ashes become houses, pieces of glass spread over the floor jump up and join together to form a vase, etc. [...] Consequently time reversal in this conception can only be general and universal; it cannot confine itself to part of the universe exclusively, for then it would be impossible that all traces of what had happened were obliterated. This conception of time reversal is, as far as I can see, the only one that is logically tenable.

[18, p. 155]

Under this understanding, the reversibility of time is interpreted as reversibility of *time-unfolding processes*. In particular, by considering the specificity of physical processes, Zwart mentions so-called *experiments in time reversal* (see also [8]), which attempt to challenge the “principle of time-reversal invariance of the basic laws of nature”: “If time invariance were to be proved violated, even in one instance only, time reversal would be impossible” [18, p. 156–157]. Furthermore, he stresses that

the experiments in question are not really experiments in time reversal, but they are experiments in the reversibility of elementary processes. From their results, it can be concluded whether in principle time reversal would be possible. As up to now no instances of irreversible elementary processes have been found, we have to conclude that at the present moment our knowledge of nature does not make time reversal a priori impossible.

[18, p. 157]

Our aim here is not to give a comprehensive survey of the current status quo in the field of the mentioned experiments. Moreover, it should be emphasized that in this context we consider *real physical processes* with the view to finding a sample of such a process that can conclusively

be shown to be *physically* irreversible. This sort of investigation surely requires the competence of experimental physicists.

Yet, in a more general logico-philosophical sense, one can raise a question about the possibility in principle of constructing a *schematic model* of some irreversible process with no particular reference to any specific physical conditions whatsoever. In this respect, one can think of a logical process modeled along the lines of, e.g., Zeno's paradoxes. If it were possible to outline such an abstract mathematical model schematizing some irreversible process, then the problem of time reversibility would receive a purely philosophical (and in fact, *a priori*) solution.

The goal of this paper is just to present the outline of such a solution. Before proceeding, we explain some basic notions involved in the analysis of the problem at hand.

## 2. Objects, processes and moments

According to Ludwig Wittgenstein, the world is determined by facts, denoting the existence of various states of affairs. Any state of affairs is a combination of objects (entities, things) [17, 1.11; 2; 2.01]. In other words, combinations of objects constitute facts, more explicitly, facts about objects having a variety of properties, being in various relations with one another, and possibly *undergoing certain processes*.

The last detail is of particular importance in our analysis. By modifying slightly another statement by Wittgenstein [17, 2.011], one could claim that it is essential for an object not only to be a possible constituent of some state of affairs, but also to be a possible carrier of some process. In effect, every process is represented by a sequence of states of some object, or several objects, which is/are the carrier(s) or bearer(s) of this process. For the sake of brevity, we consider the states of the process itself.

Processes unfold over time, which is composed of moments (instants). The notion of a time-moment is a primitive undefinable notion analogous to that of a point in geometry. Every actualized period (time interval) represents a sequence of moments linearly ordered by the relation "later". It is assumed that it is always possible to distinguish between any two moments in such a sequence, and if needed, one can always fix (calculate) any time moment to any desired degree of precision.

It is also assumed that any process is uniquely determined by the totality of its successive states, and that there always exists a one-to-one correspondence between these states and the set of moments from some time interval. Thus, one can equivalently conceive of the states of some process as the moments of its unfolding. A process can have a starting moment (beginning), and an end point, which concludes the process. If a process has no beginning and/or ending, it is called infinite. In this paper, we deal only with finite processes.

The reversal of a process (that is, a process reproduced in reverse order) is the totality of its moments linearly ordered by a reverse relation to the one by which the initial process was ordered. Moreover, for any two moments  $a$  and  $b$  belonging to both the initial process and its reversal, the following holds:  $aRb \Leftrightarrow bR^-a$ , where  $R$  and  $R^-$  are mutually inverse relations. (In particular, relations “later” and “earlier” are mutually inverse.)

In the reversal of a process, the starting moment and the end point are swapped. The reversal of a process ends upon reaching the beginning of the initial process. A process is reversible if and only if, starting at any given moment, it can be completely reproduced in the reverse direction.

Time, in its entirety, is reversible if and only if all the processes (both existing ones and those that can exist) are reversible. Correspondingly, time as such is irreversible, if and only if there is *at least one* irreversible process possible. Thus, to prove the irreversibility of time it suffices to model some process that can never be reproduced in the reverse (backward) direction.

### 3. Process *Irr* and its irreversibility

We are now in a position to construct a specific process that can be demonstrably shown to be irreversible. To model this process, we make use of a puzzle problem referred to in the literature as “Austin’s Dog”. Martin Gardner, in one of his books on recreational mathematics, presents this puzzle as follows:

A boy, a girl and a dog are at the same spot on a straight road. The boy and the girl walk forward the boy at four miles per hour, the girl at three miles per hour. As they proceed, the dog trots back and forth between them at 10 miles per hour. Assume that each reversal of its

direction is instantaneous. An hour later, where is the dog and which way is it facing?<sup>2</sup> [4, p. 134]

The same puzzle is sometimes characterized as a “paradox” and known as the “Trojan Fly”:

Achilles travels at 8 mph but the tortoise manages only 1 mph. So Achilles has given it a start. At the point where Achilles catches the tortoise, he draws level with a fly, which proceeds to fly back and forth between them at 20 mph. After another hour, Achilles is 7 miles ahead of the tortoise, but where is the fly? [1, p. 200]

The entire array of problems associated with this puzzle and its paradoxicality is discussed in detail in [4, ch. 13] with special reference to Wesley Salmon [13] (see also [6]). Here we do not delve into these issues, as we are interested not in the puzzle itself, but in the general scheme of the presented process. More concretely, we employ a certain modification of this scheme suggested by Vladimir Shalak<sup>3</sup>, which allows one to avoid an element of uncertainty occurring at the very moment when Achilles, the tortoise and the fly find themselves at the same point.

The process of interest can be described as follows:

### **Process *Irr*.**

Two objects,  $a$  and  $b$  (by way of illustration, call these objects “Achilles” and “fly”, respectively) are at the same point  $A$ . Some distance from  $A$  there is a wall  $c$ , impenetrable for both Achilles and the fly. Let  $B$  be the nearest point of the wall to  $A$ , i.e., line  $(AB)$  is perpendicular to  $c$ . The whole process starts when Achilles and the fly set out (simultaneously) from point  $A$  towards point  $B$ , with the fly moving twice as fast as Achilles. Throughout the entire process, Achilles and the fly are in constant motion along line  $(AB)$ . Since  $c$  is impenetrable for both Achilles and the fly, it plays the role of their “motion limiter” in the

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<sup>2</sup> The puzzle, attributed to A. K. Austin, first appeared in print in the January 1971 issue of the “Mathematics Magazine”. The problem with the puzzle is that according to the suggested solution, in an hour the dog can be at any point between the boy and the girl, facing either way. The suggested solution is this: “At the end of one hour, place the dog anywhere between the boy and the girl, facing either direction. Time-reverse all motions and the three will return at the same instant to the starting point” [4, p. 134]. It is noteworthy that this argument essentially relies on the assumption of time-reversibility.

<sup>3</sup> Shalak proposed replacing the moving tortoise with an impenetrable wall (personal communication, September 2012).

sense that whenever either Achilles or the fly arrives at the wall (at point  $B$ ), their motion (at the very moment of being at point  $B$ ) is directed away from the wall towards point  $A$ . Achilles is also impenetrable for the fly (and is thus its motion limiter, in the sense that whenever the fly, moving between Achilles and the wall, arrives at the point where Achilles is, its movement is directed towards point  $B$ ), *except for the moment when both Achilles and the fly are (simultaneously) at point  $B$* . In the latter case, the movement of both Achilles and the fly is directed towards point  $A$ . The goal of Achilles is to reach point  $B$  and then retrace his steps to  $A$ , whereupon the process terminates.<sup>4</sup>

It is important to keep in mind that here we are dealing with an exact mathematical model rather than a real physical process. Objects  $a$  and  $b$  are considered not even to be point particles, but mathematical points; we abstract away not only from their size, shape, and structure, but also from their mass. Moreover, the objects can occupy the same single space position (point) at the same time. We ignore as inessential all the physical characteristics of the process and the objects involved, for example, that a physical fly would need an additional acceleration to be able to reverse the direction of its motion. The speed of the objects is taken to be constant during the whole process, and each reversal of direction is assumed to be instantaneous.

It is easy to see that as long as Achilles moves to point  $B$ , the fly oscillates between him and the wall, with the amplitude of this oscillation becoming increasingly shorter. We then have the following:

LEMMA 3.1. *At the moment Achilles arrives at point  $B$ , the fly is at this point too, i.e., Achilles and the fly reach the wall simultaneously at the same point.*

PROOF. This is obvious, considering that when Achilles is at point  $B$ , there is no other point for the fly to be other than  $B$ .  $\square$

Lemma 3.1 captures the state of process  $Irr$  at the very moment Achilles (and the fly) reaches the wall. But what happens next? The following lemma gives the answer to this question.

LEMMA 3.2. *At any moment after the one Achilles and the fly reach point  $B$ , the fly will be outside the line segment between the wall and*

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<sup>4</sup> A crucial difference between process  $Irr$  and Shalakh's original formulation is that according to Shalakh, Achilles *always* remains impenetrable for the fly, whereas in  $Irr$  there is *one distinguished moment* when Achilles is devoid of this property.

*Achilles, moving freely and rectilinearly along the line  $AB$  in a direction away from the wall, until the end of the process.*

PROOF. By Lemma 3.1, Achilles arrives at point  $B$  at the same time as the fly. According to the *Irr*-conditions, at this (and only this) moment Achilles is not a motion limiter for the fly, and the movement of both Achilles and the fly is directed away from the wall towards point  $A$ . In other words, Achilles and the fly, as equal objects, start simultaneously from point  $B$  towards point  $A$ . Note that the speed of the fly is twice that of Achilles. Hence, at any subsequent moment the fly is twice as far from the wall as Achilles is, and having no motion limiter any more, it will be moving freely and rectilinearly in a direction away from the wall, until the process terminates.  $\square$

It turns out that the proof of this lemma can be used effectively to construct an argument for the irreversibility of time. The following theorem clearly shows this.

**THEOREM 3.3.** *It is impossible to construct the reversal of process *Irr*. More concretely, as soon as Achilles and the fly arrive backwardly at point  $B$ , no subsequent state of the process will be identical to any state of the original process *Irr*.*

PROOF. A backward reproduction of *Irr* begins at the moment when Achilles reaches point  $A$ , which is the starting moment of such an attempted reversal. At this moment, by Lemma 3.2, the fly is on line  $AB$ , at a distance of  $2A$  from the wall. The reversal of the process goes smoothly until the moment when Achilles reaches the wall at point  $B$ . At this exact moment, the fly also reaches point  $B$ . Moreover, at this moment, according to the background conditions: (1) the motion of both Achilles and the fly is directed away from the wall towards point  $A$ ; and (2) Achilles is penetrable for the fly and is not a motion limiter for it. Hence, at this moment Achilles and the fly, as equal objects, start simultaneously from point  $B$  towards point  $A$ , and at any subsequent moment, the fly is once again twice as far from the wall as Achilles is. That is, the fly is outside the interval between Achilles and the wall, contrary to where it was in the original process *Irr*. Thus, oscillation of the fly between Achilles and the wall is no longer possible, which implies the impossibility of the reverse reproduction of *Irr* in general.  $\square$



In summary, point  $B$  represents a kind of “point of no return” for the whole process  $Irr$ ; after passing this exact point, no reversal of the process is possible.

#### 4. Conclusion

Irreversibility of time can be construed in different ways. In the most limited sense, irreversibility means the impossibility of reversal of *any* process at *any* moment of its development. Clearly, such total irreversibility is not feasible, since it is not difficult to find even a physical process at some moment of its development, the subsequent state of which is identical to the previous one. Moreover, this may well be some local processes that can be reproduced in reverse order, in other words, from finish to start. Hence, we can imagine certain (restricted) areas where all the processes can go backwards for a while.

A “partial reversibility” of this kind would not, however, mean the reversibility of time as such, since time reversibility in a general sense claims the possibility of the universal backward development for all areas and time intervals of arbitrary length. Therefore, even a single irreversible process is a sufficient guarantee against all-around time reversibility.

This paper just established the impossibility of such general time-reversibility. We constructed a concrete process that, by the very conditions of its construction, cannot be reproduced in a backward direction. The key among these conditions is that in a certain distinguished moment, one of the involved objects is devoid of some important property, which is inherent to the object in all other moments of the process irrespective of its direction. The property in question is the ability of an object to be a motion limiter for another object involved in the process. This property is formulated in a fully symmetric way, and therefore, it should hold as it occurs both in the original process  $Irr$  and in any attempt at its reversal.

Inasmuch as process  $Irr$  itself represents an abstract (mathematical) model, the proof of its irreversibility is of a purely logical (or even mathematical) nature. This proof involved no physical properties or relations inherent in any real physical objects. Thus, the main result of this paper does not depend on any factual features of the physical universe and

holds true for any “possible world”, and not only for the best one, in which, according to Leibniz, all of us are lucky enough to live.

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