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REFLECTIONS ON TEMPORAL AND MODAL LOGIC

Abstract. The most popular method of incorporating time into a formal logic is based on the work of Arthur Prior. It treats tenses as operators on sentences. In this essay I show a serious problem with that approach, a confusion of scheme versus proposition, which makes any system built in that way incoherent. I will compare how other formal logics deal with the scheme versus proposition distinction and find that only for formal modal logics does the same problem arise. I then compare Prior's approach to other ways of taking time into account in formal logics.

Keywords: temporal logic; modal logic

Introduction

There are three ways we take account of time in our reasoning. We relate sentences with temporal connectives like "before," "after," and "at the same time as." We use time-markers, indicators of specific times such as "June 6, 1970," or "1983," along with words that pick out unnamed times, such as "sometime" or "always." And we use tenses, which are markers for relative times. In some languages the latter are attached to the verb or predicate; in other languages a marker is attached to a sentence, a paragraph, or even an entire story.

The most popular method now of incorporating time into a formal logic is based on the work of Arthur Prior. It treats tenses as operators on sentences. In this essay I will set out a problem about how such systems treat sentences as schema or propositions. I will compare how other formal logics deal with the same issue. Then I'll compare Prior's

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approach to other formal ways of taking time into account in logical systems.[1]

Prior's Approach to Temporal Logic

Consider the sentence:

(1) John loves Mary.

This is not a proposition. That is, it is not true or false.[2] In order to be true or false, references must be supplied for the words "John" and "Mary." But more, we need to know what time (1) is meant to describe. If "John" means the person John Paul Jones, and "Mary" means Mary Magdalene, and the time is meant to be November 16, 2012, then (1) is false. If "John" means the man who married Mary Schwartz Rodrigues of Socorro in 2010, and "Mary" refers to Mary Schwartz Rodrigues, then it is a true proposition about January 16, 2011 but is a false one if it is meant to be about April 13, 2008.

Even with fixed references for the names, then, we cannot take (1) to be a proposition unless a time is specified. Otherwise (1) could be true at some times and false at others. Yet a proposition is true or false, not both true and false, nor true sometimes and false another. When we encounter a sentence that appears to be true at some time and false at another, we know we have an ambiguity. We can resolve that ambiguity by specifying a time which the sentence is meant to describe. Thus, we have to view (1) as a *scheme* awaiting references for the names and a designated time in order to become a proposition. Let's assume in what follows that references have been supplied for "John" and "Mary."

To take account of relative time in reasoning, Arthur Prior suggested that we treat "in the past" and "in the future" as adverbs of sentences as wholes. In his approach there are two "temporal operators":

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[1] There are many good survey articles and books on the topics discussed here. So in this essay I will refer only to works that are needed for the particular points under discussion.

[2] The discussion that follows can be modified to apply to propositions taken to be abstract objects. The issue then is whether the sentence under discussion stands for, represents, points to, or somehow indicates a proposition. Similar comments apply to all the formalisms discussed here.

The discussion can also be modified to apply to reasoning with many "truth-values" by using the dichotomy of designated vs. undesignated as a true/false division, as I discuss in my [6, 2].

P meant to be understood as "in the past"
or "in the past it is true that"

F meant to be understood as "in the future"
or "in the future it is true that"

Those readings are ambiguous between "sometime in the past" and "always in the past" and between "sometime in the future" and "always in the future." Normally, "sometime in the past" and "sometime in the future" are what are meant in the formal systems.

Using these, we can convert (1) into two distinct propositions:

P(John loves Mary)

F(John loves Mary)

Each is true or false. Taking the default time indication to be the present, such a system treats (1) as being about now, the ambiguity between that and the scheme being resolved by context.

Let's consider more carefully:

(2) P(John loves Mary)

This is meant to be understood and analyzed as:

(3) At some time in the past, "John loves Mary" is true.

In this methodology "P" is said to be a operator on propositions. But the sentence (1) as it appears in (2) cannot be a proposition. If (1) were a proposition within (2) it would have to be about a time, and prefacing it with "in the past" would make no sense. For example, if (1) in (2) were taken to be a proposition about now as I'm writing, February 11, 2013, then (4) would mean:

In the past (John loves Mary on February 11, 2013)

And that's just nonsense.

Rather, "P" takes (1) as a scheme and converts that into the proposition (2), where we understand the past as relative to the time I am writing this. That's why I've used quotation marks around (1) in (3).

The sentence (3) gives the conditions under which (2) is true. It says that there is some time in the past that could be assigned to (1) that makes it into a true proposition. The formal semantics then build on that. I'll describe those briefly.

First, some conception of time is given. Let's assume for this discussion that time is linear and that it is made up of points, whether those be tiny, such as a nanosecond, or large, like last week. So long as those are linearly ordered, we have a timeline T. Implicit in (3) is that at each time each atomic scheme, such as (1), is either true or false. That's formalized by assigning to each atomic scheme p a subset of the timeline: those times at which the scheme is true when interpreted as being about that time. Let's notate the assignment of times to atomic schema by ν. Further, we need to take some time $\mathfrak{n}$ as being the now of our timeline. Then in the formal semantics:

P(John loves Mary) iff there is some t in T such that t is before $\mathfrak{n}$ and t is in ν(John loves Mary)

For "F(John loves Mary)" the condition is the same except "after" replaces "before."

Already a problem needs to be resolved. Suppose that "January 6, 1447" is a time in T. At that time, the people to whom "John" and "Mary" refer did not exist. Some would say that "John loves Mary" evaluated at that time is then nonsense. But that is not allowed here. On this approach, any atomic sentence that is evaluated at a time at which one or more names lacks reference must be treated as false. Falsity is the default truth-value.

We can extend these semantics to make each time a model not just of the atomic schema but of combinations of those using the formal connectives $\wedge$ for "and" and $\neg$ for "not," using, for example, the classical interpretation of those. Thus, for every time t in the timeline we will have a model $\mathfrak{M}_t$ of classical propositional logic. The collection of those models constitutes the model $\mathfrak{M}$. This is clear enough. But in this approach the operators P and F can be iterated, as in:

P P(John loves Mary)
P F(John loves Mary)
F P(John loves Mary)
F F(John loves Mary)
P P P(John loves Mary)
P F P(John loves Mary)

Consider, for example,

(4) F P(John loves Mary)

It's truth conditions should be:

At some time in the future, "P(John loves Mary)" is true.

But "P(John loves Mary)" is a proposition, evaluated by (3). It is either true or false, and that can't change by prefacing it with "F".

The usual presentation of the semantics for such a system obscure this issue. The truth-conditions for (4) are said to be:

(5) At some time in the future, at some time in the past relative to that time, "John loves Mary" is true.

Using $\models$ to stand for "true in the model," condition (5) is given formally as:

There is a time $t > \mathfrak{n}$ such $\mathfrak{M}_t \models$ P(John loves Mary)
which is iff there is a time $t > \mathfrak{n}$ such that there is a time $t' < t$
such that $\mathfrak{M}_{t'} \models$ John loves Mary.

The "P" in (4) is no longer meant to be relative to now as in (3) but to a time in the future.

This again illustrates that the operators "P" and "F" are not propositional operators. They convert propositional schema into propositions. But that means that we have to view "P(John loves Mary)" as a proposition in (2) and as a scheme in (4). It seems that we have an endemic ambiguity of scheme vs. proposition in this formal logic. Does the same ambiguity occur throughout formal logics?

Classical Propositional Logic

In classical propositional logic we start with a few "connectives" from our ordinary language and formalize those. Typically we start with "and," "or," "not," and "if ... then ...". For our purposes here it's enough to consider just "and" and "not." Let's suppose, then, that we are reasoning with these sentences:

John is a man.
Mary is a woman.
John loves Mary.

Let's assume as before that the references of "John" and "Mary" are fixed. And we can assume that the sentences are about now, since they're in the present tense. So each is true or false. That is, we can treat each as a proposition. So consider:

(6) John is a man and Mary is a woman.

Our use of "and" is fairly regular in English when it is used in this way, coming between two sentences. Usually, though not always, we take it to mean that both sentences are true, so that (6) is true iff both "John is a man" is true and "Mary is a woman" is true. In that case, (6) is a proposition which is formed from two other propositions by joining them with "and."

We can also form:

Mary is not a woman.
John doesn't love Mary.
It's not the case that John is a man.

Our use of "not" or "no" is irregular, sometimes attached to a verb, sometimes appearing as an auxiliary, sometimes prefacing the sentence. But generally it is used to transform one proposition into another which has the opposite truth-value. Thus, "Mary is not a woman" is true iff it's not the case that "Mary is a woman" is true.

There is no scheme vs. proposition ambiguity in our use of these "connectives" in English. Each creates a new proposition from one or more other propositions.

We create a formal logic when we abstract from these ordinary connectives, writing $\wedge$ for "and" and $\neg$ for "not". We regularize the use of the latter by taking it to precede the sentence it is transforming, as in "Not: John is a man." We then take what are called either *sentence letters* or *propositional variables* to stand for any sentences we might want to reason about as propositions. So we have p_0, p_1, p_2, p_3, ... going on as far as we wish. We make clear the rules for how to form a new proposition from old ones using connectives, so that we have in the *formal language*, for example:

(7) $p_0 \wedge p_1$
$\neg(p_{36})$
$p_{189} \wedge \neg(p_{23})$
$\neg(p_{4318} \wedge \neg(p_2))$
$\neg\neg\neg(p_{4318} \wedge \neg p_2)$
$(p_{4318} \wedge \neg(p_2)) \wedge p_{800}$

Such formal inscriptions are not true or false. They are schema in the sense of giving us the forms of the propositions that we can investigate with this formal logic. They are *formal schema of propositions.*

To have propositions we can reason about with this logic we must realize the propositional variables as propositions. Thus, we might take p_0 to be realized, that is stand for, "John is a man" and take p_1 to be realized, that is stand for, "Mary is a woman." These are the atomic propositions we are considering. Then $p_0 \wedge p_1$ is realized as "John is a man ∧ Mary is a woman," which is a formal version of (6). If we realize some or all of the propositional variables, that is, if we have a list of which propositions are assigned to which propositional variables, then we can take the formal schema with those assignments to be the semi-formal language. Corresponding to the forms at (7) we might have in the semi-formal language:

(8) John is a man ∧ Mary is a woman
¬ (Ralph is a dog)
Sheila is a herring ∧ ¬ (Romulo is a wolf)
¬(Edgar is a dog ∧ ¬ (John loves Edgar))
¬¬¬(Edgar is a dog ∧ ¬ (John loves Edgar))
(No one who hates dog is a good person ∧ ¬ (Mary is a dog)) ∧ John loves Mary

Still, these are not propositions until we say how we will interpret the formal symbols. In classical propositional logic we say that if A and B are semi-formal propositions, then $A \wedge B$ is true iff both A and B are true; $\neg A$ is true iff A is not true. Then each of the semi-formal sentences at (8) is a proposition, either true or false. We can actually determine which they are if we know the truth-values of the atomic propositions in our realization. If we list out those truth-values, we have a *model.* This gives us a way to reason with our atomic propositions using the connectives "and" and "not" in a clearer, more rigorous way than in ordinary English. There is no confusion of scheme and proposition.

In our reasoning we might not know whether a particular atomic proposition such as "John loves Mary" is true or false. Or we might be interested in evaluating whether one proposition follows from one or more others. Letting $A_0, A_1, \ldots, B$ stand for any propositions, we say:

$A_0, A_1, \ldots,$ therefore B
is a *valid inference* if there is no possible way the world could be such that all of the *premises* $A_0, A_1, \ldots$ are true and the *conclusion* B is false.

We can investigate whether an ordinary language inference is valid by considering it's formalization. Thus, we might ask whether the following is valid:

$$
(9)\quad \frac{\begin{array}{l}\text{Ralph is a dog.}\\ \neg\ (\text{Ralph is a dog} \wedge \text{Ralph is a cat})\end{array}}{\text{Therefore, } \neg\ (\text{Ralph is a cat})}
$$

Is there any way in which the premises could be true and conclusion false? What do we mean by a "way the world could be"?

In this context, concerned with only the truth or falsity of atomic propositions and propositions formed from the atomic propositions with $\wedge$ and $\neg$, a way the world could be is a *model*, completely determined by which of the atomic propositions are true and which are false. So to say that the inference at (9) is valid is to say, within the limited context we are considering, that there is no model, no way to assign truth-values to the atomic propositions in it in which all the premises are true and the conclusion false.

When analyzing whether an inference such as (9) is valid, the sentences that realize the propositional variables are not taken as propositions. Either "Ralph is a dog" is true or it is false if it is a proposition. Yet in evaluating (9) we consider ways in which it could be true or could be false. Each of the models we survey in making that determination does take "Ralph is a dog" to be a proposition. But in surveying those models, we take "Ralph is a dog" to be a scheme, awaiting an assignment of a truth-value in order to be viewed as a proposition. The ambiguity of scheme vs. proposition here arises only when we stand back from any particular use of the sentences realizing the propositional variables and look at all possible uses of them in the context we are considering. We have no choice but to do this in order to make an evaluation of whether an inference is valid.

The ambiguity does not arise just because when we assert "Ralph is a dog" we mean it to be a proposition, yet in (9) we are not asserting it. When in our ordinary language we say "Ralph is a dog, therefore Ralph is not a cat" we are using "Ralph as a dog" just as much as when we simply say "Ralph is a dog." We mean it to be a proposition. It is our interest in deciding whether we can conclude that Ralph is not a cat from "Ralph is a dog" that makes us treat "Ralph is a dog" as a sentence that could be true in some circumstances and false in others in evaluating that inference.

We can take "Ralph is a cat" to be a proposition and not know whether it is true or false. We use inferences such as (9) to help us determine which it is. In doing so we do not forget that "Ralph is a cat" is a proposition, but we consider that sentence to be sufficiently meaningful to be considered true or false in other ways the world might be.

There are other motives for us to survey all models. For example, we might wish to know whether a sentence such as "¬ (Edgar is a dog ∧ ¬ (Edgar is a dog))" is true or false due solely to its form. When we do so, we are treating as a scheme what we took to be a proposition.

The scheme vs. proposition ambiguity that arises is not introduced by any of our methods of formal analysis but arises from our reflecting on our reasoning. It is clear when we are treating a sentence as a proposition and when we are treating it as a scheme. We treat a sentence that is a proposition as a scheme only when we are concerned with ways in which it could be true.

Classical Predicate Logic

When we formalize reasoning based on the view that the world is made up of things, and we quantify over things, then it seems there is an ambiguity of scheme vs. proposition. Consider:

Someone loves Mary.

This is a proposition. We can make clearer our assumption that we're reasoning about things with this sentence by rewriting it as:

(10) There is something, and that thing loves Mary.

The whole is a proposition. But what is the status of "that thing loves Mary"? It can't be a proposition. It is like (1), "John loves Mary," before references are supplied for "John" and "Mary." When a reference is supplied for "that thing" it becomes a proposition. Over the last hundred and fifty years logicians have managed to clarify the role of "that thing" in sentences like (10) using formal methods.

First, a formal language is offered in which instead of "that thing" we use variables: x_0, x_1, x_2, x_3, ... Then we focus on only two ways of talking in English about how many things: "all" and "some." We understand "some" to mean "at least one, but possibly more." And we understand "all" as "all, even if there isn't even one." We use the symbol

$\forall$ for this reading of "all," and we use the symbol $\exists$ for this reading of "some." Readings of "all" as "all and there is at least one" and of "some" as "at least one and not all" can be devised from these and the other resources of the formal and semi-formal languages.

So we can formalize (10) as:

(11) $\exists x_1(x_1 \text{ loves Mary})$

And we can formalize "Everyone loves Mary" as "$\forall x_1(x_1 \text{ loves Mary})$" These are propositions since we previously established the reference of Mary.

Consider, then:

(12) $\forall x_1 \exists x_2(x_2 \text{ loves } x_1)$

This formalizes "Everyone is loved by someone." Here "x_2 loves x_1" is a scheme, awaiting either references or stipulations that we're talking about some or all in order to be a proposition. But also "$\exists x_2(x_2 \text{ loves } x_1)$" is a scheme. It is neither true nor false unless we give a reference for "x_1" or stipulate that we're talking about some or all. It is only (12) that is a proposition.

There is no ambiguity. The operators "$\forall x_1$" and "$\exists x_2$" are used only on *open sentences*, that is, sentences where a variable awaits reference or a quantification; those uses of variables we call *free*. A quantifier converts an open sentence into another open sentence if there is a free variable remaining, and into a proposition if there is no free variable remaining.[3]

Consider, though:

(13) $\forall x_1 \exists x_2((x_2 \text{ loves } x_1) \wedge \text{ John loves Mary})$

Here a scheme (open sentence) "$(x_2 \text{ loves } x_1)$" is connected by the propositional connective $\wedge$ to a proposition "John loves Mary." Doing so creates another scheme "$((x_2 \text{ loves } x_1) \wedge \text{ John loves Mary})$." But there is no ambiguity. What is a proposition remains a proposition; what is a

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[3] Most logicians allow in their formalisms sentences such as "$\forall x_2 \exists x_1(x_1 \text{ loves } x_1)$", where $\forall x_2$ prefaces a proposition. But that is only for convenience in presenting the syntax. Such superfluous quantifications are not needed, as I show in [7]. Without superfluous quantification it is always clear whether we are dealing with a scheme or a proposition (relative to a model), and at no point in the work do we require a sentence to be read as a scheme in one context and a proposition in another.

Some logicians allow "$(x_1 \text{ loves } x_2)$" to be read as a proposition, too, understanding that to mean "for all x_1 and all $x_2(x_1 \text{ loves } x_2)$" But to do so does create a confusing ambiguity of scheme vs. proposition.

scheme continues to be a scheme; and the connection of the two is a scheme. In any case, such sentences are always equivalent to ones in which the quantifiers are attached only to open sentences. In this case, the sentence is equivalent to:

$\forall x_1 \exists x_2 (x_2$ loves $x_1) \wedge$ John loves Mary

Complicating the syntax a bit we could eliminate problematic sentences such as (13).

For generality we devise a fully formal language of predicate logic by taking, in addition to the propositional connectives, the variables, and the quantifiers, *name symbols* a_0, a_1, a_2, a_3, ... and *predicate symbols* $P_0^1, P_1^1, P_2^1, \ldots, P_0^2, P_1^2, P_2^2, \ldots$, where the subscript gives the number of the predicate symbol and the superscript indicates how many variables are needed for it. For example, "— is a dog" is unary and "— loves —" is binary.

When we realize the name symbols as names, such as "John" and "Mary," and predicate symbols as predicates, such as "— loves —" and "— is a dog," we have a sentence such as (12) of the *semi-formal language*. The closed sentences, that is, ones with no free variable, would seem to be the propositions we are investigating.

But there is a significant problem in trying to make clear what we mean by saying that x_1 stands for a particular thing while pointing to it if we are talking about all things (see [3]). So for a sentence such as (12) to be a proposition, we require first that we specify exactly what things we're quantifying over. For example, we could stipulate that we're talking about all animate creatures, or we could stipulate that we're talking about all dogs, or all people, or .... It is only when we specify such a *universe* of quantification that (12) can be treated as a proposition.

The formal methods are meant to clarify quantification by making more explicit the truth-conditions of a sentence such as (12). We take a *model* of the semi-formal language to be a universe, specific references for all the names, and a stipulation of which atomic sentences are true. Only here the atomic sentences are not just ones like "John loves Mary" but also ones like "(x_2 loves x_1)" when references from the universe are supplied for x_1 and x_2. We make explicit our picking out references as when we say "John loves that thing" by allowing any thing in the universe to be a reference for any variable. So if our universe is all animate creatures, then "(x_2 loves x_1)" is an atomic proposition when

x_1 is stipulated to refer to me and x_2 is stipulated to refer to my dog Birta.

Then in the model, we have:

$\forall x_1 \exists x_2(x_2$ loves $x_1)$ is true
iff for any reference supplied for x_1, $\exists x_2(x_2$ loves $x_1)$ is true
iff for any reference supplied for x_1, and then for some reference supplied for x_2, $(x_2$ loves $x_1)$ is true.

There is, however, an oddity here with certain sentences when we think of scheme vs. proposition. Consider:

(14) $\forall x_1 \neg(\exists x_2(x_1$ loves $x_2 \wedge \neg$ $(x_1$ loves $x_1)))$

Here the propositional connective $\wedge$ is joining not propositions but schema. Yet in the analysis of the truth-conditions for (14) we do see the propositional connective joining propositions:

$\forall x_1 \neg(\exists x_2(x_1$ loves $x_2 \wedge \neg$ $(x_1$ loves $x_1)))$ is true
iff for any reference supplied for x_1, $\neg(\exists x_2(x_1$ loves $x_2 \wedge$ $\neg$ x_1 loves $x_1))$ is true
iff for any reference supplied for x_1, and then for some reference supplied for x_2, $(x_1$ loves $x_2 \wedge \neg$ $(x_1$ loves $x_1))$ is true.

In the last line $\wedge$ joins two propositions because the variables are supplied with references. We are justified in using the propositional connectives to join schema or to join a scheme and a proposition because in the final line of the semantic analysis a reference must be supplied for each variable, so that in the end the propositional connectives do operate on propositions (see [3]).

One important motive in devising classical predicate logic is to investigate the validity of inferences. Consider, for example:

(15)
All dogs bark.
Ralph is a dog.
____
Therefore, Ralph barks.

This is valid: there is no way the premises could be true and conclusion false. That is, it is not possible for “Ralph is a dog” and “All dogs bark” to be true and “Ralph barks” to be false at the same time and in the same way. But in this analysis we are not treating “Ralph is a dog” as a proposition, for it is false, and there’s no possibility about that. Rather, we are considering ways those same words in that same order understood in the same way might result in a true proposition. That is, in analyzing whether (15) is valid, we treat "Ralph is a dog" as a scheme, not a proposition. It becomes a proposition only upon an indication of a way the world could be, whether that be a particular time in the past, an imagined time in the future, another "world" at this very time, ...

We can formalize (15), writing $A \supset B$ for $\neg(A \wedge \neg B)$:[4]

$$(16)\qquad \begin{array}{l} \forall x_1(x_1 \text{ is a dog} \supset x_1 \text{ barks}). \\ \underline{\text{Ralph is a dog.}\qquad\qquad\qquad} \\ \text{Therefore, Ralph barks.} \end{array}$$

If we understand a possibility in the context of our formal analyses of reasoning to be a model of predicate logic, then we can show that (16) is valid. If "$\forall x_1(x_1$ is a dog $\supset x_1$ barks)" is true in the model, then no matter what object we let x_1 stand for, "x_1 barks" is true. So in particular, if we let the thing that is Ralph be what x_1 stands for, and Ralph is a dog, that is "x_1 is a dog" is true, then "x_1 barks" is true, too. A possibility is a model of the logic. Again, "Ralph is a dog" is treated as a proposition when we wish to consider whether it is true or false, and it is treated as a scheme, awaiting a specification of a model, when we are investigating whether an inference in which it appears is valid.

We have exactly the same situation as with classical propositional logic except that the indications that can be supplied to turn a scheme into a proposition are more ample. What is held constant in making (1) into a proposition is that "John" and "Mary" are names and "loves" is understood the same regardless of the time. What is held constant in (9) is that "Ralph" is a name and "is a dog" is meant to be understood the same.

Yet that is not really accurate, since in many analyses we say that "dog" might mean a creature similar to what we call dogs but which is different in some specified way. One major problem in understanding and using formal logic is deciding what is held constant across different possibilities, that is, across different ways the world could be. But it is a problem not just for formal logic. It shows up already in evaluating whether (15) is valid. We might consider a way the world could be in which "is a dog" means is a walrus, and "barks" means that it has tusks. If we do, then it is really clear that (15) is only a scheme and not a collection of propositions. However, invoking such a possibility in evaluating inferences is never done in ordinary reasoning (see [4]). We

[4] Why this formalization is apt is explained in [3].

impose some kind of informal limits on what counts as an acceptable interpretation of the words in (15). Making those explicit is rarely done and would seem to be quite difficult. If we allow unlimited scope for what we mean by a possibility, then (16) is no different from:

$$(17)\quad \begin{array}{l} \forall x_1(P_7(x_1) \supset P_9(x_1)) \\ P_7(a_1) \\ \hline \text{Therefore, } P_9(a_1) \end{array}$$

This really is a scheme of propositions.

The nature of possibilities is a big subject, which I have discussed in [5]. But whether we treat (16) as a completely general scheme, no different from (17), or as a collection of meaningful sentences, it is clear that the sentences in it are not treated as propositions in analyzing whether (16) is valid. They are treated as schema, though the limits on how we turn those into propositions might not be clear.

Just as with classical propositional logic, we treat a semi-formal sentence as a proposition when we are considering one way the world could be, that is, when we are considering just one model. When we are surveying models in order to investigate the validity of an inference, we treat the sentences of the semi-formal language as schema. The contexts are different, and there is no confusion of scheme vs. proposition.

Classical Modal Logic

Formal propositional logics of possibility and necessity have been devised to clarify how to reason with sentences when considering possibilities.

In formal modal logic, there are two operators besides the usual propositional connectives $\neg$ and $\wedge$:

- $\Diamond$ meant to be understood as "possibly," "it is possible that" or "it is possibly true that",
- $\Box$ meant to be understood as "necessarily," "it is necessary that" or "it is necessarily true that".

Consider, then:

(18) Ralph is a dog.

This is a proposition, a false one, understood about here and now. In the formal syntax we can form:

$\Diamond$(Ralph is a dog)
$\Box$(Ralph is a dog)

These are taken to be propositions.

Let's consider the first:

(19) $\Diamond$(Ralph is a dog)

This is meant to be understood and analyzed as:

It is possible that "Ralph is a dog" is true.

If (18) is a proposition, then it is not incorporated in (19), for then we would have:

It is possible that "Ralph is a dog" is true right here and now.

That's nonsense, for either "Ralph is a dog" is true or it is false now, and there's no possibility about that. In (19) "Ralph is a dog" is a scheme, true in some ways the world could be, false in others.

In the syntax of formal modal logic we can attach the possibility operator or the necessity operator to any sentence that we have in classical propositional logic. Thus, for example, we can form:

$\Diamond$(Ralph is a dog $\wedge$ $\neg$(Ralph barks))
$\Box$($\neg$(Ralph is a dog $\wedge$ $\neg$(Ralph barks))

These sentences formalize how we might reason about inferences, considering whether there is a way the premises could be true and conclusion false.

It would seem that there is no confusion of scheme vs. proposition here. Thinking of the system as formalizing how we reason about inferences and possibilities, all the sentences are schema. So we can iterate the possibility and necessity operators and nest one within another as in:

(20) $\Box\Diamond$(Ralph is a dog)
$\Box$(Ralph is a dog $\wedge$ $\Diamond$(Ralph barks))

All the sentences are schema, and the possibility and necessity operators transform schema into schema. We are never concerned with whether a particular atomic proposition such as "Ralph is a dog" or a particular complex proposition such (19) or those at (20) are true.

But that's just wrong. We are concerned with whether sentences such as those are true or false. The inference (15) is valid iff the following is false:

(21) $\Diamond$(All dogs bark $\wedge$ (Ralph is a dog) $\wedge$ $\neg$(Ralph barks))

Perhaps the formal semantics for such sentences can clarify whether we are talking of schema or propositions.

Let's consider one particular semantics, the semantics of what is called *classical logical necessity* (that is, S5). We proceed just as we did when using classical propositional logic to analyze whether (15) is valid. We take a possibility to be a model of classical propositional logic, that is, an assignment of truth values to the atomic schema where $\wedge$ and $\neg$ are are interpreted as in classical propositional logic. We designate one of those models as being the world as it is here and now, the *actual world*. Then we use all of those "sub-models" together to make a model by using the following truth-conditions:

$\Box A$ is true iff in every possibility, that is, in every sub-model, A is true.

$\Diamond A$ is true iff there is some possibility, that is, there is some sub-model, in which A is true.

Then we evaluate every sentence relative to the actual world. So in this model, "Ralph is a dog" is false. But "$\Diamond$(Ralph is a dog)" is true because there is a way we can assign truth-values to the atomic schema in which "Ralph is a dog" is assigned to be true.

And now we do have a confusion of scheme and proposition. We treat "Ralph is a dog" as a proposition in this model, but we treat that sentence as a scheme in "$\Diamond$(Ralph is a dog)". And though we treat the latter as a proposition, we treat it as a scheme in:

(22) $\Box$($\Diamond$(Ralph is a dog))

The truth-conditions for this are:

(23) $\Box$($\Diamond$(Ralph is a dog)) is true
iff in in every sub-model "$\Diamond$(Ralph is a dog)" is true
iff in every sub-model, there is some model relative to that in which "Ralph is a dog" is true.

But there's no "relative to" here: we're always looking at all possibilities. So (22) is equivalent to "$\Diamond$(Ralph is a dog)." Any iteration of the possibility and necessity operators can be replaced by the use of a single operator, and sentences with nesting of operators as in the last sentence at (20) are equivalent to ones with no operator within the scope of another. All the formal mechanism reduces to just an investigation of sentences of the form $\Box A$ and $\Diamond B$ where A and B are sentences of classical propositional logic. This is a lot of work to get us back to where we were when we first began investigating (15). Only we have not clarified our informal analyses of (15). By trying to formalize that process we have brought together what we had previously kept in separate contexts: the use of a sentence as a proposition in a particular model and as a scheme when surveying models. Formal modal logic does not clarify; it introduces confusion by trying to meld many models into one.

In other modal logics the confusion is worse.[5] A sentence such as (22) cannot be reduced to one that is simpler. The formal semantics give truth-conditions for it in terms of an accessibility relation between possibilities, that is between sub-models. So in the truth-conditions at (23) "relative to" does matter. So we are talking about schema being true in our actual world by considering how they are true in other models, which requires considering other models relative to those, and so on. Once again everything appears to be a scheme, except we really do want to know whether (21) is true.

Worse, we have no way to say whether the formal semantics are apt because we have no prior intuition, indeed no idea at all of the conditions under which (22) is true. We simply don't talk that way. In our ordinary speech we would consider "It's necessary that it's possible that Ralph is a dog" to be nonsense. Yes, we can put the words together in that way, but they make no sense. The formal modal logic is pointed to as a way to clarify what such a sentence means, allowing philosophers to feel confident in their analyses of possibilities. But that confidence is built on a serious confusion, an attempt to meld into one system the use of sentences as propositions and the investigation of whether inferences are valid.

Often it's said that formal modal logic is based on a use-mention confusion. In "It's possible that Ralph is a dog" the sentence "Ralph is a dog" is mentioned not used, and what we should really say is "It's possible that "Ralph is a dog" is true." But that's wrong because in "It's

---

[5] See my [2] for a full development of those and the logic of classical logical necessity, S5.

possible that Ralph is a dog" the proposition "Ralph is a dog" is neither used nor mentioned. The problem, which appears to be at the heart of formal modal logic, is a confusion of scheme vs. proposition.

Arthur Prior developed his approach to formal temporal logics in conscious analogy with formal modal logics. So let's return to the problem of scheme vs. proposition in his work.

Resolving the Ambiguity in Prior's Approach to Temporal Logic?

Recall that in Prior's approach to temporal logic we take "John loves Mary" as a proposition, yet in the proposition "P(John loves Mary)" we take that sentence to be a scheme. And the latter is a scheme not a proposition in "F P(John loves Mary)." The approach does not segregate uses of sentences as propositions and uses of sentences as schema. Can we resolve this ambiguity?

We do not form a new proposition "F P(John loves Mary)" from the proposition "P(John loves Mary)". Rather, what we have, and what the formal semantics assume, is that each of P, F, P P, P F, F P, F F, P F P, ... is a distinct "operator" which when prefixed to a proposition-scheme forms a proposition. Thus, each of "John loves Mary," "P(John loves Mary)," "F(John loves Mary)," "P F(John loves Mary)," and so on is an atomic proposition. In the usual formulations of a propositional logic, an atomic proposition is taken to be a unitary whole with no internal structure. In this approach an atomic proposition does have internal structure: a proposition-scheme + tense marker. The axioms of a temporal logic in Prior's tradition can then be understood as relating propositions in terms of the internal structure of atomic propositions.

Before we go further, we should ask why we would want to have such a proliferation of temporal scheme-into-proposition operators.

One view of logic is as a guide to reasoning well. We investigate how we talk, how we reason, and we resolve ambiguities, clarify inferences, and sometimes see more deeply into how our assumptions about the world affect our reasoning. Formal tools are seen as abstractions from our ordinary reasoning, in some cases helping us to find what we have good reason to believe. If that is how we view logic, how do we arrive at Prior's approach? Neither in English nor in any Indo-European language do we say "In the past John loves Mary." Prior notes that but says it is only an accident of our grammar that we don't treat "in the past" and "in the future" as adverbs of sentences.

> I want to suggest that putting a verb into a past or future tense is exactly the same sort of thing as adding an adverb to the sentence. "I *was* having my breakfast" is related to "I am having my breakfast" in exactly the same way as "I am *allegedly* having my breakfast" is related to it, and it is only an historical accident that we generally form the past tense by modifying the present tense, e.g. by changing "am" to "was", rather than by tacking on an adverb. In a rationalized language with uniform constructions for similar functions we could form the past tense by prefixing to a given sentence the phrase "It was the case that", or "It has been the case that" (depending on what sort of past we meant), and the future tense by prefixing "It will be the case that". For example, instead of "I will be eating my breakfast" we could say
>
> It will be the case that I am eating my breakfast,
>
> and instead of saying "I was eating my breakfast" we could say
>
> It was the case that I am eating my breakfast.
>
> The nearest we get to the latter in ordinary English is "It was the case that I was eating my breakfast", but this is one of the anomalies like emphatic double negation. The construction I am sketching embodies the truth behind Augustine's suggestion of the "secret place" where past and future times "are", and his insistence that wherever they are, they are not there as past or future but as present. The past is not the present but it is the past present, and the future is not the present but is the future present.
>
> There is also, of course, the past future and the future past. For these adverbial phrases, like other adverbial phrases, can be applied repeatedly — the sentences to which they are attached do not have to be simple ones; it is enough that they be sentences, and they can be sentences which already have tense-adverbs, as we might call them, within them. Hence we can have such a construction as
>
> It will be the case that (it has been the case that
> (I am taking off my coat)),
>
> or in plain English, "I will have taken off my coat". We can similarly apply repeatedly such specific tense-adverbs as "It was the case forty-eight years ago that". [11, pp. 40–41]

Prior is trying, it seems, to clarify how to reason about time by treating "in the past" and "in the future" analogously to how we use "it is possible that" and "it is necessary that." Yet his approach is hardly a clarification if it introduces more confusion than we had before.

Still, if I understand correctly, there are some languages that treat "in the past" and "in the future" as operators on sentences. In Chinese, in particular, a phrase roughly translated as "before" can precede a sentence, a paragraph, or even a whole story. In American Sign Language words like "yesterday" or "next month" can precede a sentence, as in "Yesterday, John loves Mary," though the more general operators of "in the past" and "in the future," again if I understand correctly, attach only to verbs. But I know of no language in which an operator like "in the past" or "in the future" can preface a sentence that already begins with one of those. Iterations are not allowed.

And why would we want to use such iterations? We do so in order to move around the time relative to which the operators are meant to pick out a time, as in (4) explicated in (5). But we never do this all by itself in ordinary speech. We only do it for one sentence relative to another. Consider:

(24) Mary had loved Hubert before John loved Mary.

The truth conditions for this are:

> There is a time in the past at which "John loves Mary" is true, and, relative to that time, there is some time in the past at which "Mary loves Hubert" is true.

This we can model in Prior's approach, though not by iterating temporal connectives. Rather, we would use:

(25) P(John loves Mary $\wedge$ P(Mary loves Hubert))

The conditions for this to be true are:

> At some time in the past "John loves Mary" is true and, relative to that time, there is some time in the past at which "Mary loves Hubert" is true.

In the formal models, we have:

P(John loves Mary $\wedge$ P(Mary loves Hubert)) is true
iff there is a time $t < \mathfrak{n}$ such $\mathfrak{M}_t \models$ John loves Mary $\wedge$
  P(Mary loves Hubert)
iff there is a time $t < \mathfrak{n}$ such $\mathfrak{M}_t \models$ John loves Mary and
  $\mathfrak{M}_t \models$ P(Mary loves Hubert)

iff there is some time $t < \mathfrak{n}$ such $\mathfrak{M}_t \models$ John loves Mary and
some time $t' < t$ such that $\mathfrak{M}_{t'} \models$ Mary loves Hubert

But to model this is to once again have an ambiguity of scheme vs. proposition. We have agreed to take the temporal operators as converting schema to propositions, yet in (25) "P(Mary loves Hubert)" has to be understood as a scheme. If this is a major motive for using Prior's approach to formal systems of temporal logic, we need to ask whether the scheme vs. proposition confusion is really needed or whether there are more straightforward, simpler ways in which we can formalize reasoning with propositions such as (24).

Tenses for Predicates

Let's start by considering another approach to temporal logic that I present in [8]. My goal there is to show how to formalize reasoning from English and similar languages. In English we have a part of speech we call an infinitive. We do not use an infinitive as the main verb in a sentence. Rather, we add a tense to the infinitive using a suffix and/or auxiliary words to get what we call a predicate. For example, in English from the infinitive "to talk" we have:

talk(s)	simple present
talked	simple past
will talk	simple future
is talking	present progressive
was talking	past progressive
will be talking	future progressive
have (has) talked	present perfect
had talked	past perfect will
have talked	future perfect
have (has) been talking	present perfect progressive
had been talking	past perfect progressive
will have been talking	future perfect progressive

Though there are many irregularities in forming tensed verbs in this way, and many variations, these are the twelve basic ways to form a predicate from an infinitive in English (see [1]).

In English, "John to talk" is not a sentence, nor (does it represent) a proposition. But "John talked," "John talks," and so on are sentences.

They are (or represent) propositions when an indication is made of what counts as the present, or what counts as the present in the past, or what counts as the present in the future.

We can formalize the use of tenses in this manner. In predicate logic we include in the formal language symbols for infinitives, I_1, I_2, I_3, ... We also include in our formal language the following *tense markers*: simple present, simple past simple future, present, progressive past, progressive future, progressive, present perfect, present perfect, progressive past perfect, past perfect, progressive future, perfect, future perfect progressive.
Then for each n the following is a predicate symbol: I_n-simple present: I_n-simple past, I_n-simple future, I_n-present progressive, I_n-past progressive, I_n-future progressive, I_n-present perfect, I_n-present perfect progressive, I_n-past perfect, I_n-past perfect progressive, I_n-future perfect, I_n-future perfect progressive.

To utilize this formalism for reasoning, we realize some or all of the infinitive symbols. For example, we can realize I_1 as "to talk," I_2 as "to run," I_3 as "to give," I_4 as "to take."[6] A complex of such a realization with a tense marker then plays the role that a tensed infinitive does in English and that a predicate does in the realization. Examples of sentences in a semi-formal language are then:

Ralph (to bark-present)
Lemuel (to give-past perfect)
$\forall x_1(x_1\,$(to give-past perfect)$\,\supset \exists x_2(x_2\,$(to take-past)$))$

In the usual formulation of predicate logic, an atomic predicate is taken to be a unitary whole with no internal structure. In this system an atomic predicate does have internal structure: an infinitive + tense marker. We can create a formal temporal logic by formulating axioms relating propositions in terms of the internal structure of atomic predicates, as I do in [8].

Were we to view logic not as a guide for how to reason well but as a formal system that codifies truths about the world, whether abstract or not, we might choose instead of these twelve basic tense markers some more general approach to tenses. We could have markers:

Present, P, F

[6] I discuss only unary predicates here. In [8] I deal with predicates of any arity.

Then, for example, we could take as a predicate any of:

to talk-Present
to talk-$\mathrm{P}^{i_1}\,\mathrm{F}^{j_1}\,\mathrm{P}^{i_2}\,\mathrm{F}^{j_2}\ldots\mathrm{P}^{i_n}\,\mathrm{F}^{j_n}$
to talk-$\mathrm{F}^{i_1}\,\mathrm{P}^{j_1}\,\mathrm{F}^{i_2}\,\mathrm{P}^{j_2}\ldots\mathrm{F}^{i_n}\,\mathrm{P}^{j_n}$

where

$\mathrm{P}^0 = \mathrm{F}^0 =$ nothing and $i_1 \neq 0$
$\mathrm{P}^1 = \mathrm{P}$ and $\mathrm{F}^1 = \mathrm{F}$
$\mathrm{P}^{n+1} = \mathrm{P}\,\mathrm{P}^n$ and $\mathrm{F}^{n+1} = \mathrm{F}\,\mathrm{F}^n$

More generality could be gained by adding progressive markers. It is clear, I hope, how we could proceed to give a formal language and define realizations of that.

This would be similar to the way of resolving the ambiguity in Prior's approach by taking P, F, P F, F P, P P, F F, ... as distinct schemeinto-proposition-operators. But here there is no confusion of scheme and proposition. There is, just as in our ordinary speech, propositions about times relative to other times which can be joined with connectives such as "and" and "not." We could formalize (24) with:

Mary (to love-past perfect) Hubert $\wedge$ John (to love-simple past) Mary

That may be right. But it misses a key part of (24): the connective "before."

Temporal Propositional Connectives

Esperanza Buitrago-Díaz and I in [9] have shown how to formalize the use of temporal connectives like "before." We start with the usual propositional connectives $\neg$ and $\wedge$ and sentences which are to be taken as propositions in a model, just as in classical propositional logic. We add to this language connectives meant to formalize (roughly) the ordinary language connectives "before," "after," and "at the same time as." Thus, we might have:

(26) Mary loves Hubert before John loves Mary.

For a model of the logic, we first take a collection T of instants that is linearly ordered to be the timeline. Each atomic sentence is meant to be about a particular time, which is an interval of T; that is, it is meant to describe the world at that time. For example, we could assign the entire day January 18, 1953 to "John loves Mary"and assign to "Mary loves Hubert" the year 1946. Then for each atomic sentence we say whether it is true or false of the time assigned to it. The connectives $\neg$ and $\wedge$ are evaluated in the usual way. And we have, roughly:

"p before q" is true
iff the time assigned to p is before the time assigned to q
and both p and q are true

"p after q" is true
iff the time assigned to p is after the time assigned to q
and both p and q are true

"p at the same time as q" is true
iff the time assigned to p is the same as the time assigned to q
and both p and q are true.

For example,

"Mary loves Hubert before John loves Mary" is true
iff the time assigned to "Mary loves Hubert" comes before the time assigned to "John loves Mary," and both "Mary loves Hubert" and "John loves Mary" are true of those times.

There is only one model. There is no ambiguity of scheme vs. proposition. There are only propositions and propositional connectives. A proposition is about some time, and it is either true or false.

But (26) does not formalize (24), for (26) does not require that the times assigned to the constituent propositions be in the past in order for the whole to be true. To accomplish that, we need to designate one or a collection of atomic propositions as being about the present. Perhaps "Spot is barking" will do. Then noting that "$\neg$(Spot is barking $\wedge$ $\neg$ Spot is barking)" is true no matter the time assigned to "Spot is barking," we can say that a proposition p is about the past if the following is true:

$\neg(p \wedge \neg p)$ before $\neg$(Spot is barking $\wedge$ $\neg$ Spot is barking)

We can abbreviate this as:

P(p)

Similarly we can have abbreviations P(p) and Present (p) which are true, respectively, if p is about a time in the past, or is true about a time in the future, or is true about the present. These are abbreviations, ways for us to say that a proposition is about the past. For example, consider:

John loves Mary $\wedge$ P(John loves Mary)

This is true iff John loves Mary at some time in the past. Superficially there is a similarity to what is done in Prior's approach, but there is no ambiguity, no confusion. We are making propositions from other propositions, and we can stand outside the system to note whether those are about the present, the past, or the future. After all, what is present, past, or future in a language of only relative time markers must be designated from outside the system, which we do here by designating a particular proposition or collection of propositions as about the present.

If we add these temporal propositional connectives to the predicate logic of tensed atomic predicates as described above, we can then formalize (24):

Mary (to love-past perfect) Hubert before
John (to love-simple past) Mary

The syntax is clear and unambiguous, the formal semantics are clear and unambiguous, and we can add axioms governing the relations of sentences with tensed atomic predicates to temporal connectives. We can even prove completeness theorems for the logic of tensed atomic predicates, for the logic of temporal connectives, and for the two logics melded together (see [8]).

We can clarify and give a guide to how to reason well. Or, if you consider logic to be a description of the most general truths of the universe, we can see our way to an "intellectual intuition" of those truths that are formalized in the logics. There is no impediment of confusion to stop us.

Quantifying Over Times

Another way to formalize reasoning that takes account of time is to use specific time markers such as January 6, 1947, or 2:55 pm March 3, 2013, or 306 BCE. Let's assume that we have a timeline composed of instants and names for some of those.

To what do we attach such markers? Consider the sentence:

John loved Mary on June 6, 1970.

We could formalize this by attaching a time-marker to the sentence as a whole as in:

(John loves Mary) (June 6, 1970)

To do so, however, leads us into the same confusion of scheme vs. proposition we had with Prior's approach to temporal logic.

Instead, in [8] I attach time markers to predicates. That is, a unary predicate such as "— to talk" is now construed as a binary predicate, "— to talk (—)," where the blank in parentheses is to be filled with a time marker. Similarly, "— loves —" is now construed as a predicate with three blanks to be filled "— loves (—) —", the one in parentheses to be filled by a time marker. So we could have the semi-formal sentence:

John loves (June 6, 1970) Mary

We don't need to use tenses or any relative time markers in this system because we can compare the time markers. Thus, "June 6, 1970 is before June 6, 1982" would be one of the sentences of the semi-formal language. But not all the times we want to talk about have names. So we introduce variables that are meant to range over times: t_0, t_1, t_2, t_3, ... Then a semi-formal proposition might be:

$\exists t_1 \exists x_3$(John loves (t_1) x_3)

To formalize talk of the past and the future we have to designate a time marker for the present, say "March 3, 2013." Let's call that n. Then we can formalize

John loved Mary

as

$\exists t_1$((John loves (t_1) Mary) $\wedge$ t_1 is before n)

We don't speak like this in ordinary English. But the method is close to how we sometimes incorporate time indications in our speech, and it is clear and easy to deal with formally. Moreover, it helps us see the kinds of assumptions about the world that we base our reasoning on, as you can read in that volume.

To give a formal logic along these lines we have to adopt axioms for how times relate, whether time is linear, for example, or branching. We have to adopt axioms for how the time markers and variables interact with the names and variables for objects. Such a logic for quantifying over times is clear, creates no scheme vs. proposition confusion, is close enough to ordinary speech for us to have clear intuitions about how to proceed in giving truth-conditions for sentences, and helps us uncover assumptions about how we understand time and objects in our reasoning.

Conclusion

Prior devised his approach to tense logic in conscious analogy with formal modal logics.

By trying to coalesce many models of classical propositional logic into one model, modal logics create a scheme vs. proposition confusion. It is not clear what is true, what is true relative to, nor what is the status of any sentence in the semi-formal language. The many analyses of possibility and necessity that are made using formal modal logic, analyses that purport to clarify our assumptions about the nature of what is possible and what is necessary, are all suspect, based as they are on an underlying confusion that infects all the work.

That confusion is much worse in the formal systems of temporal logic built on Prior's work. There is no need to coalesce many models of classical propositional logic into one when taking account of time in our reasoning. To do so leaves us with no clear idea whether we are talking of scheme or proposition. The many analyses of problems in reasoning which first Prior and then others made using his systems of temporal logic are suspect, based as they are on an underlying confusion that infects all the work.

From the very beginning of Prior's work on temporal logic this confusion of scheme and proposition is evident. In "Tense-Logic and an Analogue of S4" in *Time and Modality* [10], Prior begins by taking sentences such as (1) to be schema of propositions:

> In the logic of tenses, the ordinary statement-variables p, q, r, etc., are used to stand for statements in what is not now the ordinary sense of the term "statement", though it was the ordinary sense in ancient and medieval logic. They are used to stand for "statements" in the sense in which the truth-value of a statement may be different at different times [...]. The statement "It will be the case that Professor Carnap is flying to the moon", as I understand it, is not a statement about the statement "Professor Carnap is flying to the moon", but a new statement about Professor Carnap, formed from the simpler one by means of the operator F. [10, p. 8]

But then Prior begins to talk of those sentences as propositions:

> The idea that it is necessary to introduce a special present-tense operator would, moreover, have extremely awkward formal consequences. For to say that such an operator is necessary is to say that the expressions to which we attach it would not be propositions, that tense operators

> do not form propositions out of propositions, at all events out of tensed propositions; rather, they form propositions out of merely juxtaposed nouns and verbs, or they form tensed propositions out of untensed ones. And from this in turn it would follow that tense operators cannot be iterated or attached to propositions to which tense-operators are already attached; that is, we would have to rule out such forms as "It will be the case that it has been the case that p". And to rule this would be practically to destroy tense logic before we have started to build it.
>
> [10, p. 10]

I agree. But I take this to mean that tense logic as he does it is wrong, not that we must persevere in the mistake.

We talk lots of nonsense and confusion. We often reason badly. As logicians we try to bring clarity to our reasoning. We have a responsibility not to add to our confusion.

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